A.CED.A.3: Interpret Solutions

EQUATIONS AND INEQUALITIES

A.CED.A.3: Interpret Solutions

A. Create equations that describe numbers or relationships.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods (linear).

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

Selected problem set(s)

- facilitate a summary and share out of student work

Homework – Write the Math Assignment

Big Idea - A Linear Inequality with Two Variables

A linear inequality describes a region of the coordinate plane that has a **boundary line**. The **boundary line** divides the plane into two equal halves.

Every point on one side of the boundary line is a **solution of the inequality**.

Points on the boundary line are solutions of the inequality *if, and only if*, the inequality sign contains an equal sign (egs. $\leq$ or $\geq$).

Points on the boundary line are *not* solutions of the inequality if the inequality sign does *not* contain an equal sign (egs. $<$ or $>$).

Two or more linear inequalities together form a **system of linear inequalities**. Note that there are two or more boundary lines in a system of linear inequalities.

A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

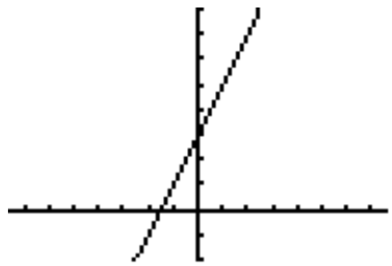
- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed or dotted line for the boundary

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



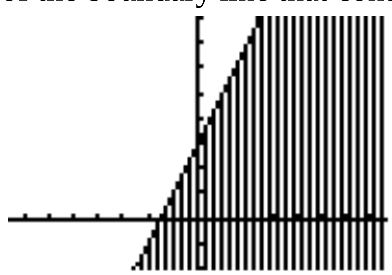
Next, **test a point** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).



Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the Y= feature.

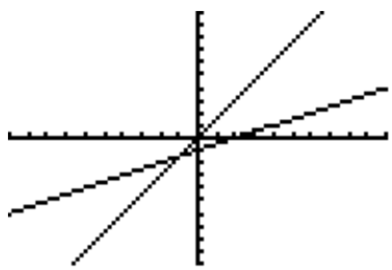
Big Idea - Systems of Linear Inequalities

Graphing a System of Linear Inequalities. Systems of linear inequalities are graphed in the same manner as systems of equations are graphed. The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

Example: Graph the system: $\begin{array}{l} 4y \geq 6x \\ -3x+6y \leq -6 \end{array}$

First, convert both inequalities to slope-intercept form and graph.

$4y \geq 6x$ $\frac{4y}{4} \geq \frac{6x}{4}$ $y \geq \frac{3}{2}x$ m=$\frac{3}{2}$, b=0	$-3x+6y \leq -6$ $6y \leq -6+3x$ $6y \leq 3x-6$ D_2 $\frac{6y}{6} \leq \frac{3x}{6}-\frac{6}{6}$ $y \leq \frac{1}{2}x-1$ m=$\frac{1}{2}$, b=-1

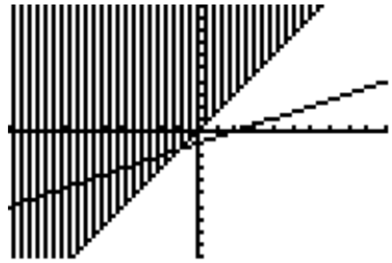


Next, test a point in each inequality and shade appropriately.

- Since point (0,0) is on the boundary line of $y \geq \frac{3}{2}x$, select another point, such as (0,1).

$$y \geq \frac{3}{2}x$$
$$\text{test point } (0,1) \quad 1 \geq \frac{3}{2}(0)$$
$$1 \geq 0$$

The test of point $(0,1)$ makes the inequality true, so the point $(0,1)$is in the solution set of the inequality. Shade the side of the boundary line that contains point $(0,1)$.



- Since (0,0) is not on the boundary line of $y \le \frac{1}{2}x - 1$, we can use (0,0) as our test point, as follows:

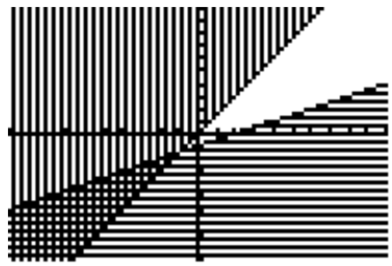
  $y \le \frac{1}{2}x - 1$

  Test (0,0)

  $0 \le \frac{1}{2}(0) - 1$

  $0 \le -1$ This is not true, so the point (0,0) is not in the solution set of this inequality.

  We therefore must shade the side of the boundary line that does not incude the point (0,0).



Note that the system of inequalities divides the coordinate plane into four sections. The solution set for the system of inequalities is the area where the two shaded regions overlap.

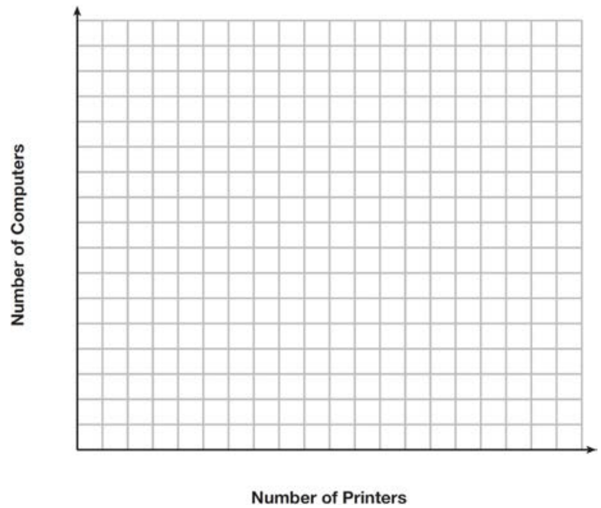
<u>Remember The Special Rule for Solving Inequalities</u>:

All the rules for solving equations apply to inequalties – plus one:

<u>When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.</u>

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. An on-line electronics store must sell at least $2500 worth of printers and computers per day. Each printer costs $50 and each computer costs $500. The store can ship a maximum of 15 items per day. On the set of axes below, graph a system of inequalities that models these constraints.



Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.

2. Edith babysits for *x* hours a week after school at a job that pays $4 an hour. She has accepted a job that pays $8 an hour as a library assistant working *y* hours a week. She will work both jobs. She is able to work no more than 15 hours a week, due to school commitments. Edith wants to earn at least $80 a week, working a combination of both jobs. Write a system of inequalities that can be used to represent the situation. Graph these inequalities on the set of axes below.

Determine and state one combination of hours that will allow Edith to earn *at least* $80 per week while working *no more than* 15 hours.

3. A cell phone company charges $60.00 a month for up to 1 gigabyte of data. The cost of additional data is $0.05 per megabyte. If *d* represents the number of additional megabytes used and *c* represents the total charges at the end of the month, which linear equation can be used to determine a user's monthly bill?

a. $c = 60 - 0.05d$
b. $c = 60.05d$
c. $c = 60d - 0.05$
d. $c = 60 + 0.05d$

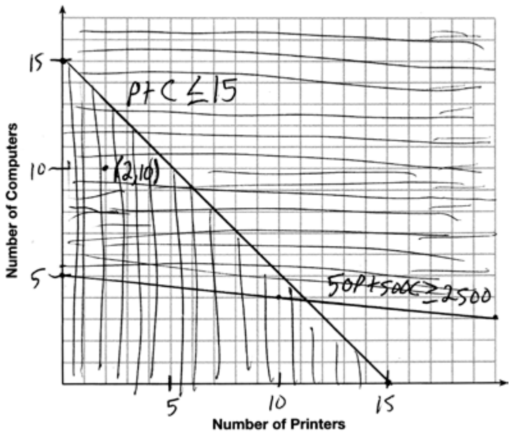
4. An animal shelter spends $2.35 per day to care for each cat and $5.50 per day to care for each dog. Pat noticed that the shelter spent $89.50 caring for cats and dogs on Wednesday. Write an equation to represent the possible numbers of cats and dogs that could have been at the shelter on Wednesday. Pat said that there might have been 8 cats and 14 dogs at the shelter on Wednesday. Are Pat's numbers possible? Use your equation to justify your answer. Later, Pat found a record showing that there were a total of 22 cats and dogs at the shelter on Wednesday. How many cats were at the shelter on Wednesday?

5. Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for $1.75 per pound and peaches for $2.50 per pound. If she made $337.50, how many pounds of peaches did she sell?
   a. 11
   b. 18
   c. 65
   d. 100

6. The Celluloid Cinema sold 150 tickets to a movie. Some of these were child tickets and the rest were adult tickets. A child ticket cost $7.75 and an adult ticket cost $10.25. If the cinema sold $1470 worth of tickets, which system of equations could be used to determine how many adult tickets, *a*, and how many child tickets, *c*, were sold?
   a. $a + c = 150$
      $10.25a + 7.75c = 1470$
   b. $a + c = 1470$
      $10.25a + 7.75c = 150$
   c. $a + c = 150$
      $7.75a + 10.25c = 1470$
   d. $a + c = 1470$
      $7.75a + 10.25c = 150$

A.CED.A.3: Interpret Solutions
Answer Section

1. ANS:



a)

b) A combination of 2 printers and 10 computers meets all the constraints because $(2, 10)$ is in the solution set of the graph.

Strategy: Write a system of inequalities, transform and input both inequalities into a graphing calculator, draw the graph on the paper using the table of values view in the calculator, then use the graph to answer the question.

STEP 1. Write the system of inequalities.

Let p represent the number of printers shipped each day.

Let c represent the number of computers shipped each day.

Write:

$$Eq.1 \quad p + c \le 15$$

$$Eq.2 \quad \$50p + \$500c \ge \$2500$$

STEP 2. Transform both equations and input them into the graphing calculator.

$$Eq.1 \quad p + c \le 15$$

$$c \le 15 - p$$

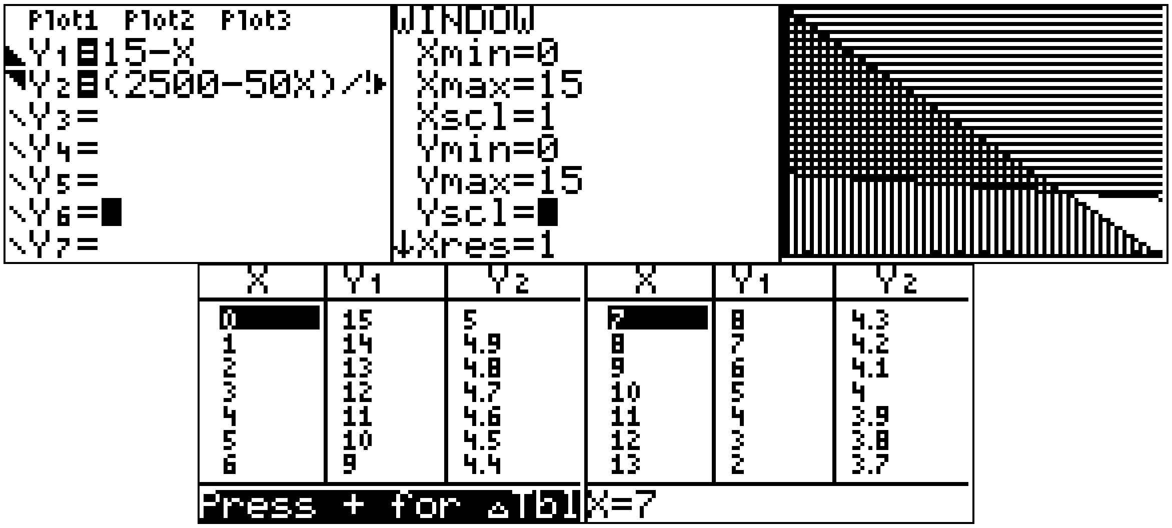
$$y \le 15 - x$$

$$Eq.2 \quad \$50p + \$500c \ge \$2500$$

$$\$500c \ge \$2500 - \$50p$$

$$c \ge \frac{\$2500 - \$50p}{\$500}$$

$$y \ge \frac{\$2500 - \$50x}{\$500}$$



X	Y1	Y2
0	15	5
1	14	4.9
2	13	4.8
3	12	4.7
4	11	4.6
5	10	4.5
6	9	4.4

X	Y1	Y2
7	8	4.3
8	7	4.2
9	6	4.1
10	5	4
11	4	3.9
12	3	3.8
13	2	3.7

STEP 3. Use information from the graphing calculator to construct the graph (see above).

STEP 4. Select (2, 10), or any other point in the heavily shaded area, as a combination of printers and computers that would allow the electronics store to meet all of the constraints.

DIMS? Does It Make Sense? Yes. The point (2, 10) satisfies both inequalities, as shown below:.

$Eq.1 \quad p + c \le 15$

$2 + 10 \le 15$

$12 \le 15$

$Eq.2 \quad \$50p + \$500c \ge \$2500$

$\$50(2) + \$500(10) \ge \$2500$

$\$100 + \$5000 \ge \$2500$

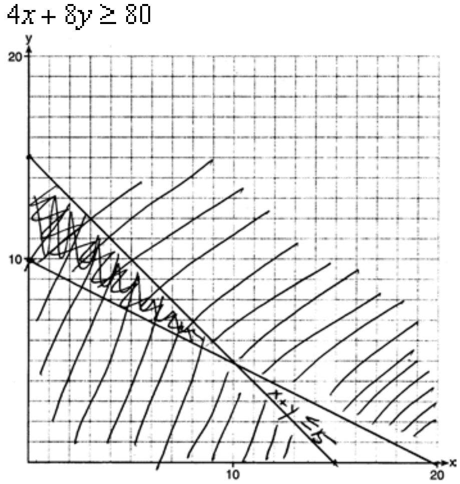
$\$5100 \ge \$2500$

PTS: 4 REF: 061535ai NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

2. ANS:

a) $x + y \le 15$

$4x + 8y \ge 80$



b)

c) Zero hours at school and 15 hours at the library.

Strategy: Write two inequalities, then input them into a graphing calculator and transfer the graph view to the paper, then answer the questions.

Lesson Plan

STEP 1. Write two inequalities.

Let x represent the number of hours Edith babysits.

Let y represent the number of hours Edith works at the library.

Write: Eq. 1 $x + y \leq 15$

Eq.2 $4x + 8y \geq 80$

STEP 2. Transform both inequalities for input into a graphing calculator

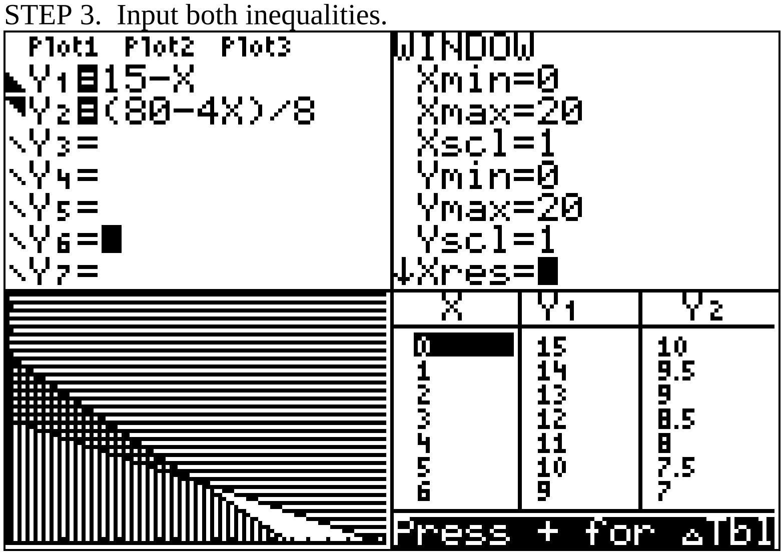
Eq. 1 $x + y \leq 15$

$y \leq 15 - x$

Eq.2 $4x + 8y \geq 80$

$y \geq \frac{80 - 4x}{8}$

STEP 3. Input both inequalities.



Use data from the table of values to construct the graph on paper.

STEP 3. Test one combination of hours in the solution set (the dark shaded area).

Test (0, 15).

Eq. 1 $x + y \leq 15$

$0 + 15 \leq 15$

$15 \leq 15$

Eq.2 $4x + 8y \geq 80$

$4(0) + 8(15) \geq 80$

$120 \geq 80$

DIMS? Does It Make Sense? Yes.

PTS: 6 REF: 081437ai NAT: A.CED.A.3 TOP: Modeling Systems of Linear Inequalities

3. ANS: D

Strategy: Translate the words into algebraic terms and expressions. Then eliminate wrong answers.

The problem tells us to:
Let *c* represent the total charges at the end of the month.
Let 60 represent the cost of 1 gigabyte of data.
Let d represent the cost of each megabyte of data after the first gigabyte.

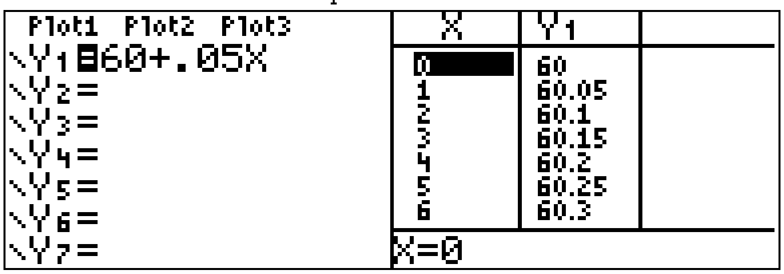
The total charges equal 60 plus .05d.
Write $c = 60 + .05d$. This is answer choice d.

DIMS? Does It Make Sense? Yes. $c = 60 + .05d$ could be used to represent the user's monthly bill. First, transpose the formula for input into the graphing calculator:

$$c = 60 + .05d$$

$$0 = 60 + .05x$$

$$Y_1 = 60 + .05x$$



The table of values shows that the monthly charges increase 5 cents for every additional megabyte of data.

PTS: 2 REF: 061422ai NAT: A.CED.A.2 TOP: Modeling Linear Equations

4. ANS:
a) $2.35c + 5.50d = 89.50$
b) Pat's numbers are not possible, bercause the equation does not balance using Pat's numbers.
c) There were 10 cats in the shelter on Wednesday

Strategy: Use information from the first two sentences to write the equation, then use the equation to see if Pat is correct, then modify the equation for the last part of the question.

STEP 1: Write the equation
Let *c* represent the number of cats in the shelter.
Let *d* represent the number of dogs in the shelter.

$$2.35c + 5.50d = 89.50$$

STEP 2: Use the equation to see if Pat is correct.

$$2.35c + 5.50d = 89.50$$

$$2.35(8) + 5.50(14) \neq 89.50$$

$$18.80 + 77.00 \neq 89.50$$

$$95.80 \neq 89.50$$

STEP 3: Modify the equation to reflect the total number of animals in the shelter.
Let *c* represent the number of cats in the shelter.
Let *(22-c)* represent the number of dogs in the shelter.

Lesson Plan

$$2.35c + 5.50(22 - c) = 89.50$$
$$2.35c + 121 - 5.50c = 89.50$$
$$-3.15c = -31.50$$
$$c = 10$$

DIMS? Does It Make Sense? Yes. If there were 10 cats in the shelter and 12 dogs, the total costs of caring for the animals would be $89.50.

$$2.35c + 5.50d = 89.50$$
$$2.35(10) + 5.50(12) = 89.50$$
$$23.50 + 66 = 89.50$$
$$89.50 = 89.50$$

PTS: 4 REF: 061436ai NAT: A.CED.A.2 TOP: Modeling Linear Equations

5. ANS: C

Strategy: Write and solve a system of equations to represent the problem.

Let *a* represent the number pounds of apples sold.
Let *p* represent the number of pounds of peaches sold.

STEP 1. Write a system of equations.
Eq. 1 $a + p = 165$
Eq. 2 $\$1.75a + \$2.50p = \$337.50$

STEP 2. Solve the system.

$$Eq.1 \quad a + p = 165$$
$$Eq.2 \quad 1.75a + 2.5p = 337.50$$
$$\text{Multiply Eq. 1 by 1.75}$$
$$Eq.1a \quad 1.75a + 1.75p = 1.75(165)$$
$$\text{Subtract Eq.1a from Eq.2}$$
$$.75p = 337.5 - 1.75(165)$$
$$.75p = 48.75$$
$$p = \frac{48.75}{.75}$$
$$p = 65$$

DIMS? Does It Make Sense? Yes. If $p = 65$, then $a = 100$, and these values make both equations balance.

Eq. 1 $a + p = 165$	Eq. 2 $\$1.75a + \$2.50p = \$337.50$
$100 + 65 = 165$	$\$1.75(100) + \$2.50(65) = \$337.50$
$165 = 165$	$\$175.00 + \$162.50 = \$337.50$
	$\$337.50 = \$337.50$

PTS: 2 REF: 061506ai NAT: A.REI.C.6 TOP: Solving Linear Systems

6. ANS: A

Step 1. Recognize this problem as having two variables, a and c.
Step 2. Strategy: Write a system of equations to model the problem.
Step 3. Use information from the first two sentences to write the first equation.

The Celluloid Cinema sold 150 tickets to a movie.
Some of these were child tickets and the rest were adult tickets.

$$a + c = 150$$

Eliminate answer choices b) and d).

Use information from the next two sentences to write the second equation.

A child ticket cost \$7.75 and an adult ticket cost \$10.25.
If the cinema sold \$1470 worth of tickets, ...

$$10.25a + 7.75c = 1470$$

Eliminate choice c). The answer is choice a).

Step 4. Does it make sense? Yes. Answer choice a) shows that the number of adult tickets added to the number of children tickets equals 150, and the income from the adult tickets added to the income from the children tickets equals 1470.

PTS: 2 REF: 061605ai NAT: A.REI.C. TOP: Modeling Linear Systems

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.