

A.CED.A.4: Transform Formulas

EQUATIONS AND INEQUALITIES

A.CED.A.4: Transform Formulas

A. Create equations that describe numbers or relationships.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm’s law $V = IR$ to highlight resistance R .*

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

[Selected problem set\(s\)](#)

- facilitate a summary and share out of student work

Homework – Write the Math Assignment

Vocabulary

Formula: A **formula** is an equation that shows the relationship between two or more variables.

Transform: To **transform** something is to change its form or appearance. In mathematical equations, a **transformaion** changes form and appearance, but does not change the relationships between variables. To **transform** a formula or equation usually means to isolate a specific variable.

Big Idea #1

Properties and operations can be used to transform **formulas** to isolate different variables in the same ways that equations are manipulated to isolate a variable.

Example: The **formula** $P = 2l + 2w$ can be used to find the perimeter of a rectangle. In English, $P = 2l + 2w$ translates as “The *perimeter equals two times the length plus two times the width.*” In the **formula** $P = 2l + 2w$, the P variable is already isolated. You can isolate the l variable or the w variables, as follows. (*Note that the steps and operations are the same as with regular equations.*)

<p>To isolate the l variable: Start with the formula: $P = 2l + 2w$Move the term $2w$ to the left expression. $P - 2w = 2l$Divide both sides of the equation by 2. $\frac{P - 2w}{2} = l$You now have a formula for l in terms of P and w.</p>	<p>To isolate the w variable: Start with the formula: $P = 2l + 2w$Move the term $2l$ to the left expression. $P - 2l = 2w$Divide both sides of the equation by 2. $\frac{P - 2l}{2} = w$You now have a formula for w in terms of P and l.</p>
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REGENTS PROBLEMS TYPICAL OF THIS STANDARD

- The equation for the volume of a cylinder is $V = \pi r^2 h$. The positive value of r , in terms of h and V , is
 - $r = \sqrt{\frac{V}{\pi h}}$
 - $r = \sqrt{V\pi h}$
 - $r = 2V\pi h$
 - $r = \frac{V}{2\pi}$
- The formula for the sum of the degree measures of the interior angles of a polygon is $S = 180(n - 2)$. Solve for n , the number of sides of the polygon, in terms of S .
- The volume of a large can of tuna fish can be calculated using the formula $V = \pi r^2 h$. Write an equation to find the radius, r , in terms of V and h . Determine the diameter, to the nearest inch, of a large can of tuna fish that has a volume of 66 cubic inches and a height of 3.3 inches.
- The formula for the area of a trapezoid is $A = \frac{1}{2} h(b_1 + b_2)$. Express b_1 in terms of A , h , and b_2 . The area of a trapezoid is 60 square feet, its height is 6 ft, and one base is 12 ft. Find the number of feet in the other base.

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Answer Section

1. ANS: A

Strategy: Use the four column method to isolate r .

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

PTS: 2

REF: 011516ai

NAT: A.CED.A.4

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2. ANS:

$$S = 180(n - 2)$$

$$S = 180n - 360$$

$$S + 360 = 180n$$

$$\frac{S + 360}{180} = n$$

or

$$\frac{S}{180} + 2 = n$$

PTS: 2

REF: 061631ai

NAT: A.CED.A.4

TOP: Transforming Formulas

3. ANS:

a) $r = \sqrt{\frac{V}{\pi h}}$

b) 5 inches

Strategy: Use the four column method to isolate r and create a new formula, then use the new formula to answer the problem.

Notes	Left Expression	Sign	Right Expression
Given	V	=	$\pi r^2 h$
Divide both expressions by πh	$\frac{V}{\pi h}$	=	$\frac{\pi r^2 h}{\pi h}$
Simplify	$\frac{V}{\pi h}$	=	r^2
Take square root of both expressions.	$\sqrt{\frac{V}{\pi h}}$	=	r

Substitute the values from the problem into the new equation.

Lesson Plan

$$V = 66, h = 3.3$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$r = \sqrt{\frac{66}{\pi(3.3)}}$$

$$r = \sqrt{\frac{20}{\pi}}$$

$$r \approx \sqrt{6.4}$$

$$r \approx 2.52$$

If the radius is approximately 2.5 inches, the diameter is approximately 5 inches.

PTS: 4 REF: 081535ai NAT: A.CED.A.4 TOP: Transforming Formulas

4. ANS:

a) $b_1 = \frac{2A}{h} - b_2$

b) The other base is 8 feet.

Strategy: Use the four column method to isolate b_1 and create a new formula, then use it to find the length of the other base.

Notes	Left Expression	Sign	Right Expression
Given	A	=	$\frac{1}{2}h(b_1 + b_2)$
Multiply both expressions by 2	$2A$	=	$h(b_1 + b_2)$
Divide both expressions by h	$\frac{2A}{h}$	=	$\frac{h(b_1 + b_2)}{h}$
Simplify	$\frac{2A}{h}$	=	$b_1 + b_2$
Subtract b_2 from both expressions	$\frac{2A}{h} - b_2$	=	b_1

Substitute the values stated in the problem in the formula.

Lesson Plan

$$A = 60, h = 6, b_2 = 12$$

$$b_1 = \frac{2A}{h} - b_2$$

$$b_1 = \frac{2(60)}{6} - 12$$

$$b_1 = \frac{120}{6} - 12$$

$$b_1 = 20 - 12$$

$$b_1 = 8 \text{ feet}$$

PTS: 4

REF: 081434ai

NAT: A.CED.A.4

TOP: Transforming Formulas

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
 PART 1b. State your understanding of **what the problem is asking**.
 PART 1c. **Answer** the problem.
 PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
 PART 2b. State your understanding of **what the new problem is asking**.
 PART 2c. **Answer** the new problem.
 PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.