

A.REI.B.3: Solve Linear Equations and Inequalities in One Variable.

EQUATIONS AND INEQUALITIES

A.REI.B.3: Solving Linear Equations and Inequalities in One Variable

B. Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters (linear equations and inequalities only).

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
 - explain vocabulary and/or big ideas associated with the lesson
 - connect assessment practices with curriculum
 - model an assessment problem and solution strategy
 - facilitate guided discussion of student activity
 - facilitate guided practice of student activity
- [Selected problem set\(s\)](#)
- facilitate a summary and share out of student work
- Homework – Write the Math Assignment

Vocabulary

A **term** is a number $\{1,2,3,\dots\}$, a variable $\{x,y,z,a,b,c,\dots\}$, or the product of a number and a variable $\{2x, 3y, \frac{1}{2}a, \text{etc.}\}$. Terms are separated by $+$ or $-$ signs in an expression, and the $+$ or $-$ signs are part of each term. (Everything inside parenthesis is treated as one term until the parentheses are removed.)

A **variable** is a letter that represents an unknown value(s). When we are asked to solve an equation, it usually means that we must isolate the variable and find its value.

A **coefficient** is a number that comes in front of a variable. A coefficient can be an integer, a decimal, or a fraction. A coefficient multiplies the variable. Every variable has a coefficient. If a variable appears to have no coefficient, it's coefficient is an "invisible 1"

An **expression** is a mathematical statement consisting of one or more terms.

An **equation** is two expressions that have an equal ($=$) sign between them.

The Big Rule for Solving Inequalities:

All the rules for solving equations apply to inequalities – plus one:

When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

- Which value of x satisfies the equation $\frac{7}{3} \left(x + \frac{9}{28} \right) = 20$?
 - 8.25
 - 8.89
 - 19.25
 - 44.92

- What is the value of x in the equation $\frac{x-2}{3} + \frac{1}{6} = \frac{5}{6}$?
 - 4
 - 6
 - 8
 - 11

- The inequality $7 - \frac{2}{3}x < x - 8$ is equivalent to
 - $x > 9$
 - $x > -\frac{3}{5}$
 - $x < 9$
 - $x < -\frac{3}{5}$

- Given $2x + ax - 7 > -12$, determine the largest integer value of a when $x = -1$.

- Given that $a > b$, solve for x in terms of a and b :
$$b(x - 3) \geq ax + 7b$$

- Solve for x algebraically: $7x - 3(4x - 8) \leq 6x + 12 - 9x$
If x is a number in the interval $[4, 8]$, state all integers that satisfy the given inequality. Explain how you determined these values.

A.REI.B.3: Solve Linear Equations and Inequalities in One Variable. Answer Section

1. ANS: A

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3} \left(x + \frac{9}{28} \right)$	=	20
Divide both expressions by $\frac{7}{3}$ (Division property of equality)	$\frac{\frac{7}{3} \left(x + \frac{9}{28} \right)}{\frac{7}{3}}$	=	$\frac{20}{\frac{7}{3}}$
Cancel and Simplify	$x + \frac{9}{28}$	=	$\frac{60}{7}$
Subtract $\frac{9}{28}$ from both expressions (Subtraction property of equality)	x	=	$\frac{60}{7} - \frac{9}{28}$
Simplify	x	=	$\frac{231}{28}$
Simplify	x	=	8.25

or

Notes	Left Expression	Sign	Right Expression
Given	$\frac{7}{3} \left(x + \frac{9}{28} \right)$	=	20
Distributive Property	$\frac{7}{3}x + \frac{7}{3} \left(\frac{9}{28} \right)$	=	20
Cancellation	$\frac{7}{3}x + \frac{1}{3} \left(\frac{9}{4} \right)$	=	20
Simplification	$\frac{7}{3}x + \frac{3}{4}$	=	20
Subtract $\frac{3}{4}$ from both expressions (Subtraction Property of Equality)	$\frac{7}{3}x$	=	$20 - \frac{3}{4}$
Simplification	$\frac{7}{3}x$	=	$\frac{77}{4}$
Multiply both expressions by 12 (Multiplication property of equality)	$\frac{12}{1} \left(\frac{7x}{3} \right)$	=	$\frac{12}{1} \left(\frac{77}{4} \right)$

Lesson Plan

Cancel	$\frac{4}{1} \left(\frac{7x}{1} \right)$	=	$\frac{3}{1} \left(\frac{77}{1} \right)$
Simplify	$28x$	=	231
Divide both expressions by 28 (Division property of equality)	$\frac{28x}{28}$	=	$\frac{231}{28}$
Simplify	x	=	8.25

PTS: 2 REF: 061405ai NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

2. ANS: A

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given:	$\frac{x-2}{3}$	=	$\frac{4}{6}$
Multiply both expressions by 6 (Multiplication property of equality)	$\frac{6}{1} \left(\frac{x-2}{3} \right)$	=	$\frac{6}{1} \left(\frac{4}{6} \right)$
Cancel and Simplify	$\frac{2}{1} \left(\frac{x-2}{1} \right)$	=	$\frac{1}{1} \left(\frac{4}{1} \right)$
Simplify	$2x-4$	=	4
Add +4 to both expressions (Addition property of equality)	$2x$	=	8
Divide both expressions by 2 (Division property of equality)	x	=	4

PTS: 2 REF: 081420ai NAT: A.REI.B.3 TOP: Solving Linear Equations
KEY: fractional expressions

3. ANS: A

Strategy: Use the four column method for solving and documenting an equation or inequality.

Notes	Left Expression	Sign	Right Expression
Given:	$7 - \frac{2}{3}x$	<	$x - 8$
Add +8 to both expressions (Addition property of equality)	$15 - \frac{2}{3}x$	<	x
Add $+\frac{2}{3}x$ to both expressions (Addition property of equality)	15	<	$x + \frac{2}{3}x$

Lesson Plan

Simplify	15	<	$\frac{5}{3}x$
Divide both expressions by $\frac{5}{3}$ (Division property of equality)	$\frac{15}{\frac{5}{3}}$	<	$\frac{\frac{5}{3}x}{\frac{5}{3}}$
Simplify	9	<	x
Rewrite	x	>	9

PTS: 2 REF: 011507ai NAT: A.REI.B.3 TOP: Solving Linear Inequalities

4. ANS:

The largest integer value for a is 0.

Strategy: Use the four column method.

Notes	Left Expression	Sign	Right Expression
Given	$2x + ax - 7$	>	-12
Substitute -1 for x	$2(-1) + a(-1) - 7$	>	-12
Simplify	$-2 - a - 7$	>	-12
Combine like terms	$-a - 9$	>	-12
Add +9 to both expressions (Addition property of equality)	$-a$	>	-3
Divide both expressions by -1 and reverse the sign	a	<	3

Since a must be less than 1, the largest integer value that is less than 1 is zero.

PTS: 2 REF: 061427ai NAT: A.REI.B.3 TOP: Solving Linear Inequalities

5. ANS:

$$x \leq \frac{10b}{b-a}$$

Strategy: Use the four column method. Remember that $a > b$.

Notes	Left Expression	Sign	Right Expression
Given	$b(x - 3)$	\geq	$ax + 7b$
Distributive Property	$bx - 3b$	\geq	$ax + 7b$
Transpose	$bx - ax$	\geq	$10b$
Factor	$x(b - a)$	\geq	$10b$
Divide by $(b - a)$	x	\leq See NOTE below	$\frac{10b}{b-a}$

NOTE: Since $a > b$, the expression $(b - a)$ must be a negative number. When dividing an inequality by a negative number, the direction of the inequality sign must be reversed.

PTS: 2 REF: 011631ai NAT: A.REI.B.3 TOP: Solving Linear Inequalities

6. ANS:

6, 7, 8 are the numbers greater than or equal to 6 in the interval.

Lesson Plan

Strategy: Use the four column method to solve the inequality, then interpret the solution.

STEP 1: Solve the inequality.

Notes	Left Expression	Sign	Right Expression
Given	$7x - 3(4x - 8)$	\leq	$6x + 12 - 9x$
Clear parentheses (Distributive property)	$7x - 12x + 24$	\leq	$6x + 12 - 9x$
Simplify (Combine like terms)	$-5x + 24$	\leq	$-3x + 12$
Add $5x$ to both expressions (Addition property of equality)	24	\leq	$2x + 12$
Subtract 12 from both expressions (Subtraction property of equality)	12	\leq	$2x$
Divide both expressions by 2 (Division property of equality)	6	\leq	x
Rewrite	x	\geq	6

STEP 2: Interpret the solution set for the interval $[4, 8]$.

The interval $[4, 8]$ contains the integers 4, 5, 6, 7, and 8.

If $x \geq 6$, then the solution set of integers is $\{6, 7, 8\}$.

PTS: 4

REF: 081534ai

NAT: A.REI.B.3

TOP: Solving Linear Inequalities

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.

PART 1b. State your understanding of **what the problem is asking**.

PART 1c. **Answer** the problem.

PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.

PART 2b. State your understanding of **what the new problem is asking**.

PART 2c. **Answer** the new problem.

PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.