Lesson Plan

A.REI.B.4: Use Appropriate Strategies to Solve Quadratics

POLYNOMIALS AND QUADRATICS
A.REI.B.4: Use Appropriate Strategies to Solve Quadratics

B. Solve equations and inequalities in one variable.
4. Solve quadratic equations in one variable.
a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

VOCABULARY

Standard Form of a Quadratic: $ax^2 + bx + c = 0$

Vertex Form of a Quadratic: $f(x) = a(x-h)^2 + k$, where (h, k) is the vertex of the parabola.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

BIG IDEAS

Completing the square and the quadratic formula can be used to solve any quadratic.

The quadratic formula is derived from the standard form of a quadratic by completing the square.

Derivation of the Quadratic Formula Given $ax^2+bx+c=0$

STEP 1. Isolate the variables

$$ax^2+bx+c=0$$

$$ax^2+bx=-c$$

$$\frac{ax^2}{a}+\frac{bx}{a}=\frac{-c}{a}$$

$$x^2+\frac{b}{a}x=\frac{-c}{a}$$

STEP 2. Complete the Square

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$$

STEP 3. Factor the trinomial.

$$\left(x+\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{-c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\left(\frac{4a}{4a}\right)\frac{-c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

STEP 4. Take the square roots of both expressions.

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Solve the equation $4x^2 - 12x = 7$ algebraically for *x*.

2. A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

$$x^2 + 6x = 13$$

The next step in the student's process was $x^2 + 6x + c = 13 + c$. State the value of *c* that creates a perfect square trinomial. Explain how the value of *c* is determined.

3. Which equation has the same solutions as $x^2 + 6x - 7 = 0$?

a. $(x+3)^2 = 2$	c. $(x-3)^2 = 16$
b. $(x-3)^2 = 2$	d. $(x+3)^2 = 16$

4. Find the zeros of $f(x) = (x-3)^2 - 49$, algebraically.

5. If $4x^2 - 100 = 0$, the roots of the equation are

a. -25 and 25	c. -5 and 5
b. -25, only	d. -5, only

6. When directed to solve a quadratic equation by completing the square, Sam arrived at the equation $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$.

   Which equation could have been the original equation given to Sam?

   a. $x^2 + 5x + 7 = 0$
   b. $x^2 + 5x + 3 = 0$
   c. $x^2 - 5x + 7 = 0$
   d. $x^2 - 5x + 3 = 0$

7. What are the solutions to the equation $x^2 - 8x = 24$?

   a. $x = 4 \pm 2\sqrt{10}$
   b. $x = -4 \pm 2\sqrt{10}$
   c. $x = 4 \pm 2\sqrt{2}$
   d. $x = -4 \pm 2\sqrt{2}$

8. Write an equation that defines $m(x)$ as a trinomial where $m(x) = (3x - 1)(3 - x) + 4x^2 + 19$. Solve for *x* when $m(x) = 0$.

9. Solve the equation for *y*: $(y - 3)^2 = 4y - 12$

10. What is the solution of the equation $2(x + 2)^2 - 4 = 28$?

    a. 6, only
    b. 2, only
    c. 2 and -6
    d. 6 and -2

11. Amy solved the equation $2x^2 + 5x - 42 = 0$. She stated that the solutions to the equation were $\frac{7}{2}$ and -6. Do you agree with Amy's solutions? Explain why or why not.

12. Fred's teacher gave the class the quadratic function $f(x) = 4x^2 + 16x + 9$.

a) State two different methods Fred could use to solve the equation $f(x) = 0$.

b) Using one of the methods stated in part *a*, solve $f(x) = 0$ for *x*, to the *nearest tenth.*

A.REI.B.4: Use Appropriate Strategies to Solve Quadratics
Answer Section

1. ANS:
Strategy 1: Solve using factoring by grouping.

$$4x^2 - 12x = 7$$

$$4x^2 - 12x - 7 = 0$$

$$|ac| = 28$$

The factors of 28 are

1 and 28

2 and 14 (use these)

$$4x^2 - 14x + 2x - 7 = 0$$

$$\left(4x^2 - 14x\right) + (2x - 7) = 0$$

$$2x(2x - 7) + 1(2x - 7) = 0$$

$$(2x + 1)(2x - 7) = 0$$

$$x = -\frac{1}{2}$$

$$x = \frac{7}{2}$$

Strategy 2: Solve by completing the square.

$$4x^2 - 12x = 7$$

$$\frac{4x^2}{4} - \frac{12x}{4} = \frac{7}{4}$$

$$x^2 - 3x = \frac{7}{4}$$

$$x^2 - 3x + \left(\frac{-3}{2}\right)^2 = \frac{7}{4} + \left(\frac{-3}{2}\right)^2$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{4}$$

$$x - \frac{3}{2} = \pm 2$$

$$x = \frac{3}{2} \pm 2$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

Strategy 3. Solve using the quadratic formula, where $a = 4$, $b = -12$, and $c = -7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{8}$$

$$x = \frac{12 \pm \sqrt{256}}{8}$$

$$x = \frac{12 \pm 16}{8}$$

$$x = \frac{3 \pm 4}{2}$$

$$x = -\frac{1}{2} \text{ and } \frac{7}{2}$$

PTS: 2 REF: 011529ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

2. ANS:

The value of c that creates a perfect square trinomial is $\left(\frac{6}{2}\right)^2$, which is equal to 9.

The value of c is determined by taking half the value of *b*, when $a = 1$, and squaring it. In this problem, $b = 6$, so $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$.

PTS: 2 REF: 081432ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

3. ANS: D

Strategy: Use the distributive property to expand each answer choice, the compare the expanded trinomial to the given equation $x^2 + 6x - 7 = 0$. Equivalent equations expressed in different terms will have the same solutions.

a. $(x+3)^2 = 2$ $(x+3)(x+3) = 2$ $x^2 + 6x + 9 = 2$ $x^2 + 6x + 7 = 0$ (Wrong Choice)	c. $(x-3)^2 = 16$ $(x-3)(x-3) = 16$ $x^2 - 6x + 9 = 16$ $x^2 - 6x - 7 = 0$ (Wrong Choice)
b. $(x-3)^2 = 2$ $(x-3)(x-3) = 2$ $x^2 - 6x + 9 = 2$ $x^2 - 6x + 7 = 0$ (Wrong Choice)	d. $(x+3)^2 = 16$ $(x+3)(x+3) = 16$ $x^2 + 6x + 9 = 16$ $x^2 + 6x - 7 = 0$ (Correct Choice)

PTS: 2 REF: 011517ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

4. ANS:

The zeros occur when $x = 10$ and $x = -4$.

$$f(x) = (x-3)^2 - 49$$
$$0 = (x-3)^2 - 49$$
$$49 = (x-3)^2$$
$$\sqrt{49} = \sqrt{(x-3)^2}$$
$$\pm 7 = x - 3$$
$$3 \pm 7 = x$$
$$x = 10$$
$$x = -4$$

Check

$f(x) = (x-3)^2 - 49$	$f(x) = (x-3)^2 - 49$
$f(10) = (10-3)^2 - 49$	$f(-4) = (-4-3)^2 - 49$
$f(10) = (7)^2 - 49$	$f(-4) = (-7)^2 - 49$
$f(10) = 49 - 49$	$f(-4) = 49 - 49$
$f(10) = 0$	$f(-4) = 0$

PTS: 2 REF: 081631ai NAT: A.REI.B.4

5. ANS: C

Strategy: Solve using root operations.

$$4x^2 - 100 = 0$$
$$4x^2 = 100$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$

Answer choice *c* is correct.

PTS: 2 REF: 081403ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

6. ANS: D

Strategy: Assume that Sam’s equation is correct, then expand the square in his equation and simplify.

$$x^2 - 5x + 3 = 0$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$\left(x - \frac{5}{2}\right)\left(x - \frac{5}{2}\right) = \frac{13}{4}$$

$$x^2 - 5x + \frac{25}{4} = \frac{13}{4}$$

$$x^2 - 5x = \frac{13}{4} - \frac{25}{4}$$

$$x^2 - 5x = \frac{-12}{4}$$

$$x^2 - 5x = -3$$

$$x^2 - 5x + 3 = 0$$

PTS: 2 REF: 061518ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

7. ANS: A

Strategy 1: Use the quadratic equation to solve $x^2 - 8x = 24$, where $a = 1$, $b = -8$, and $c = -24$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-24)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{160}}{2}$$

$$x = \frac{8 \pm \sqrt{16}\sqrt{10}}{2}$$

$$x = \frac{8 \pm 4\sqrt{10}}{2}$$

$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

Strategy 2. Solve by completing the square.

$$x^2 - 8x = 24$$
$$(x-4)^2 = 24 + (-4)^2$$
$$(x-4)^2 = 24 + 16$$
$$(x-4)^2 = 40$$
$$\sqrt{(x-4)^2} = \sqrt{40}$$
$$x - 4 = \pm 2\sqrt{10}$$
$$x = 4 \pm 2\sqrt{10}$$

Answer choice *a* is correct.

PTS: 2 REF: 061523ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: completing the square

8. ANS:
$x = -8$ and $x = -2$

Strategy: Transform the expression $(3x-1)(3-x) + 4x^2 + 19$ to a trinomial, then set the expression equal to 0 and solve it.

STEP 1. Transform $(3x-1)(3-x) + 4x^2 + 19$ into a trinomial.

$$(3x-1)(3-x) + 4x^2 + 19$$
$$9x - 3x^2 - 3 + x + 4x^2 + 19$$
$$x^2 + 10x + 16$$

STEP 2. Set the trinomial expression equal to 0 and solve.

$$x^2 + 10x + 16 = 0$$
$$(x+8)(x+2) = 0$$
$$x = -8 \text{ and } -2$$

PTS: 4 REF: 061433ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

9. ANS:
The solutions are $y = 3$ and $y = 7$.

$$(y-3)^2 = 4y-12$$
$$y^2-6y+9 = 4y-12$$
$$y^2-10y+21 = 0$$
$$(y-7)(y-3) = 0$$
$$y-7 = 0$$
$$y = 7$$
$$y-3 = 0$$
$$y = 3$$

PTS: 2 REF: 011627ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring

10. ANS: C
Step 1. Understand that solving the equation means isolating the value of x.
Step 2. Strategy. Isolate x.
Step 3. Execution of strategy.

$$2(x+2)^2-4 = 28$$
$$2(x+2)^2 = 28+4$$
$$2(x+2)^2 = 32$$
$$\frac{2(x+2)^2}{2} = \frac{32}{2}$$
$$(x+2)^2 = 16$$
$$x+2 = \sqrt{16}$$
$$x+2 = \pm 4$$
$$x = -2 \pm 4$$
$$x = 2$$
$$x = -6$$

Step 4. Does it make sense? Yes. The values 2 and -6 satisfy the equation $2(x+2)^2-4=28$.

x=2	x=-6
$2(x+2)^2-4=28$	$2(x+2)^2-4=28$
$2(2+2)^2-4=28$	$2(-6+2)^2-4=28$
$2(4)^2-4=28$	$2(-4)^2-4=28$
$2(16)-4=28$	$2(16)-4=28$
$32-4=28$	$32-4=28$
$28=28$	$28=28$

PTS: 2 REF: 061619ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: taking square roots

11. ANS:
Yes. I agree with Amy's solution. I get the same solutions by using the quadratic formula.

$$2x^2 + 5x - 42 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-42)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 + 336}}{4}$$

$$x = \frac{-5 \pm \sqrt{361}}{4}$$

$$x = \frac{-5 \pm 19}{4}$$

$$x = \frac{14}{4} = \frac{7}{2}$$

$$x = \frac{-24}{4} = -6$$

NOTE: Acceptable explanations could also be made by: 1) substituting Amy's solutions into the original equation and showing that both solutions make the equation balance; 2) solving the quadratic by completing the square and getting Amy's solutions; or 3) solving the quadratic by factoring and getting Amy's solutions.

PTS: 2 REF: 061628ai NAT: A.REI.B.4 TOP: Solving Quadratics
KEY: factoring NOT: NYSED classifies this as A.REI.A

12. ANS:
a) Quadratic formula and completing the square.

b) -0.7 and -3.3

Complete the Square Method	Quadratic Formula Method
	$f(x) = 4x^2 + 16x + 9$ a=4, b=16, c=9

Lesson Plan

$f(x)=4x^2+16x+9$ $4x^2+16x+9=0$ $4x^2+16x=-9$ $\frac{4x^2}{4}+\frac{16x}{4}=\frac{-9}{4}$ $x^2+4x=-\frac{9}{4}$ $x^2+4x+(2)^2=-\frac{9}{4}+(2)^2$ $(x+2)^2=-\frac{9}{4}+4$ $(x+2)^2=-\frac{9}{4}+\frac{16}{4}$ $(x+2)^2=\frac{7}{4}$ $x+2=\pm\sqrt{\frac{7}{4}}$ $x+2=\pm\frac{\sqrt{7}}{2}$ $x=-2\pm\frac{\sqrt{7}}{2}$ $x=-2+\frac{\sqrt{7}}{2}=-0.677=-0.7$ $x=-2-\frac{\sqrt{7}}{2}=-3.322=-3.3$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-16\pm\sqrt{(16)^2-4(4)(9)}}{2(4)}$ $x=\frac{-16\pm\sqrt{112}}{8}$ $x=\frac{-16+\sqrt{112}}{8}=\frac{-5.416}{8}=-.677=-0.7$ $x=\frac{-16-\sqrt{112}}{8}=\frac{-26.583}{8}=-3.322=-3.3$

PTS: 1 REF: 011634ai NAT: A.REI.A.1

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.