H – Quadratics, Lesson 2, Using the Discriminant (r. 2018)

QUADRATICS
Using the Discriminant

<table>
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<th>Common Core Standard</th>
<th>Next Generation Standard</th>
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<td>A-REI.4b</td>
<td>AI-A.REI.4b</td>
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<tr>
<td>Solve quadratic equations by inspection (e.g., for ( x^2 = 49 )), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as ( a + bi ), ( a - bi ) for real numbers ( a ) and ( b ).</td>
<td>Solve quadratic equations by: i) inspection, ii) taking square roots, iii) factoring, iv) completing the square, v) the quadratic formula, and vi) graphing. Recognize when the process yields no real solutions.</td>
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<tr>
<td>PARCC: Tasks do not require students to write solutions for equations that have roots with non-zero imaginary parts. Tasks can require the student to recognize cases in which a ( c ) equation has no real solutions.</td>
<td>(Shared standard with Algebra II)</td>
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Notes:
- Solutions may include simplifying radicals or writing solutions in simplest radical form.
- An example for inspection would be \( x^2 = 49 \), where a student should know that the solutions would include 7 and -7.
- When utilizing the quadratic formula, there are no coefficient limits.
- The discriminant is a sufficient way to recognize when the process yields no real solutions.

LEARNING OBJECTIVES

Students will be able to:

1) Identify the number and characteristics of solutions to quadratic equations based on analysis of the discriminant.

Overview of Lesson

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<th>Student Centered Activities</th>
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<td>Overview of Lesson</td>
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<td>- activate students' prior knowledge</td>
<td>Teacher: anticipates, monitors, selects, sequences, and connects student work</td>
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<td>- vocabulary</td>
<td>- developing essential skills</td>
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<td>- learning objective(s)</td>
<td>- Regents exam questions</td>
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<td>- big ideas: direct instruction</td>
<td>- formative assessment assignment (exit slip, explain the math, or journal entry)</td>
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<td>- modeling</td>
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VOCABULARY

discriminant
real solutions
imaginary solutions

standard form of a quadratic solution
root
x-axis intercept

zero
BIG IDEAS

Standard Form of a Quadratic: \[ a x^2 + bx + c = 0 \]

Quadratic Formula: \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**Discriminant** = \[ b^2 - 4ac \]

**Analyzing the Discriminant**

The discriminant can be used to determine the number of and type of solutions to a quadratic equation.

Every quadratic can have zero, one, or two solutions.

Solutions can be real or imaginary numbers.

<table>
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<th>If the Value of the Discriminant Is:</th>
<th>Characteristics and Number of Solutions of the Quadratic Equation Are:</th>
<th>Examples</th>
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<tr>
<td><strong>Negative</strong> [ 0 &gt; b^2 - 4ac ]</td>
<td>If the value of the discriminant is negative, then there will be two imaginary number solutions and no x-axis intercepts.</td>
<td>[ y = x^2 + 2x + 5 ] [ b^2 - 4ac = 2^2 - 4(1)(5) ] [ b^2 - 4ac = -16 ]</td>
</tr>
<tr>
<td><strong>Zero</strong> [ 0 = b^2 - 4ac ]</td>
<td>If the value of the discriminant is zero, then there will be one real solution and the graph will touch the x-axis at one and only one point.</td>
<td>[ y = x^2 + 2x + 1 ] [ b^2 - 4ac = 2^2 - 4(1)(1) ] [ b^2 - 4ac = 0 ]</td>
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</tbody>
</table>
**DEVELOPING ESSENTIAL SKILLS**

Determine the number and characteristics of the following quadratic equations by analyzing the discriminant.

1. \(-3n^2 + 4n + 6 = 6\)
2. \(-p^2 + 4p - 7 = -3\)
3. \(-x^2 + 5x - 3 = -3\)
4. \(3v^2 + 3v + 2 = 2\)
5. \(6v^2 - 2v + 6 = 4\)
6. \(p^2 - 4p - 1 = -5\)
7. \(-3x^2 - 2x + 4 = 4\)
8. \(2x^2 + 4x + 11 = 5\)
9. \(6x^2 + 6x + 3 = 3\)
10. \(3a^2 - a - 4 = -2\)

** Answers **

1. 16; two real solutions
2. 0; one real solution
3. 25; two real solutions
4. 9; two real solutions
5. -44; two imaginary solutions
6. 0; one real solution
7. 4; two real solutions
8. -32; two imaginary solutions
9. 36; two real solutions
10. 25; two real solutions
A.REI.B.4: Using the Discriminant

206) How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

207) How many real-number solutions does $4x^2 + 2x + 5 = 0$ have?
1) one
2) two
3) zero
4) infinitely many

SOLUTIONS

206) ANS: No Real Solutions

Strategy 1. Evaluate the discriminant $b^2 - 4ac$ for $a = 1$, $b = -2$, and $c = 5$.

$b^2 - 4ac$

$(-2)^2 - 4(1)(5)$

$4 - 20$

$-16$

Because the value of the discriminant is negative, there are no real solutions.

Strategy 2.
Input the equation in a graphing calculator and count the x-intercepts.

The graph does not intercept the x-axis, so there are no real solutions.

Strategy 3
Solve the quadratic to see how many real solutions there are.
Both solutions involve imaginary numbers, so there are no real solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Using the Discriminant

207) ANS: 3

Strategy: Use the discriminant, which is \( b^2 - 4ac \).
If the discriminant is > 0, then the quadratic has two real-number solutions.
If the discriminant is = 0, then the quadratic has one real-number solution.
If the discriminant is < 0, then the quadratic has no real-number solutions.

STEP 1. Identify the values of \( a, b, \) and \( c \) in the quadratic equation \( 4x^2 + 2x + 5 = 0 \).
\[
\begin{align*}
a &= 4 \\
b &= 2 \\
c &= 5
\end{align*}
\]

STEP 2. Substitute the values into \( b^2 - 4ac \) and evaluate.
\[
\begin{align*}
b^2 - 4ac &= (2)^2 - 4(4)(5) \\
&= 4 - 80 \\
&= -76
\end{align*}
\]

The quadratic has zero real-number solutions.
CHECK by inputting the quadratic equation in a graphing calculator and looking at the graph view.

The number of solutions is equal to the number of x-axis intercepts. In this case, the parabola opens upward and does not cross the x-axis, which means it has zero real-number solutions.

PTS: 2 NAT: A.REI.B.4 TOP: Using the Discriminant
KEY: AI