Lesson Plan

A-REI.C.5: Prove Equivalent Forms of Systems

SYSTEMS

A.REI.C.5: Solve Systems by Elimination

C. Solve systems of equations.

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

A **term** is a number {1,2,3,…}, a variable {x,y,z,a,b,c…}, or the product of a number and a variable {2x, 3y, ½ a, etc.). Terms are separated by + or – signs in an expression, and the + or – signs are part of each term. (Everything inside parenthesis is treated as one term until the parentheses are removed.)

A **variable** is a letter that represents an unknown value(s). When we are asked to solve an equation, it usually means that we must isolate the variable and find its value.

A **coefficient** is a number that comes in front of a variable. A coefficient can be an integer, a decimal, or a fraction. A coefficient multiplies the variable. Every variable has a coefficient. If a variable appears to have no coefficient, it's coefficient is an "invisible 1"

An **expression** is a mathematical statement consisting of one or more terms.

An **equation** is two expressions that have an equal (=) sign between them.

A **solution to a system of equations** is the set of values for each variable that solve all equations in the system simultaneously (at the same time). A system of equations may have one, two, or more solutions.

BIG IDEA

An equation describes a relationship between its terms and expressions. If every term is multiplied or divided by the same factor, the relationship is unchanged.

Solving Systems by Elimination

Strategy: Multiply or divide one or both equations so that the coefficients of one variable are the same or opposites. Then, eliminate that variable by adding or subtracting both equations. The result is a new equation with one variable instead of two variables.

EXAMPLE #1:

Solve the following system of equations by elimination.

$$4M + 3C = 12.00$$

$$5C + 6M = 19.00$$

STEP #1 Line up the like terms in columns.

$$3C + 4M = 12.00$$

$$5C + 6M = 19.00$$

STEP #2. Multiply or divide one or both equations to ensure that one of the variables has the same or opposite coefficients. In this example, the C variable has a coefficient of 3 in the first equation and a coefficient of 5 in the second equation, so we can make the coefficient of C be 15 in both equations by multiplying the first equation by 5 and the second equation by 3.

$$5(3C + 4M = 12.50) \rightarrow 15C + 20M = 60.00$$

$$3(5C + 6M = 19.00) \rightarrow 15C + 18M = 57.00$$

STEP #3. After ensuring that one of the variables has the same or opposite coefficients, add or subtract the like terms in the two equations to form a third equation, in which the coefficient of one of the variables is zero. In this example, we will subtract the second equation from the first, as follows:

$$15C + 20M = 60.00$$

$$\underline{-(15C + 18M = 57.00)}$$

$$2M = \ \ 3.00$$

Note that, after this step, the new equation has only one variable.

STEP #4. Solve the new equation with one variable.

$$2M = \ \ 3.00$$

$$M = 1.50$$

STEP #5. Substitute the value of the known variable into either equation and solve for the second variable.

$$3C + 4M = 12.00$$

$$3C + 4(1.50) = 12.00$$

$$3C + 6.00 = 12.00$$

$$3C = 6.00$$

$$C = 2.00$$

Step #6. Check your answers in both equations:

$$4M + 3C = 12.00$$
$$4(1.50) + 3(2.00) = 12.00$$
$$6.00 + 6.00 = 12.00$$
$$12.00 = 12.00$$

$$5C + 6M = 19.00$$
$$5(2.00) + 6(1.50) = 19$$
$$10.00 + 9.00 = 19.00$$
$$19.00 = 19.00$$

Cookie Analogy

A cookie recipe describes a relationship between the different ingredients in the cookies. If the amount of every ingredient is multiplied by the same amount, the relationship between the ingredients will be unchanged and the cookies will taste the same. If you want to make twice the number of cookies, you double the recipe by multiplying everything by two. If you want to make three times the number of cookies, you multiply all the ingredients by three. You can make half the number of cookies by dividing all the ingredients by two. The secret is to multiply or divide *everything* by the same number. Your cookies will not be very good if you multiply only some of the ingredients and don't multiply all of the ingredients. The same is true with equations. You can multiply or divide any equation by any number, so long as you multiply or divide every term and expression by the same number, and the relationships between the terms and expressions will be unchanged.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Albert says that the two systems of equations shown below have the same solutions.

First System	Second System
$8x + 9y = 48$ $12x + 5y = 21$	$8x + 9y = 48$ $-8.5y = -51$

Determine and state whether you agree with Albert. Justify your answer.

2. Which system of equations has the same solution as the system below?

$$2x + 2y = 16$$
$$3x - y = 4$$

a. $2x + 2y = 16$
   $6x - 2y = 4$

b. $2x + 2y = 16$
   $6x - 2y = 8$

c. $x + y = 16$
   $3x - y = 4$

d. $6x + 6y = 48$
   $6x + 2y = 8$

3. Which pair of equations could *not* be used to solve the following equations for *x* and *y*?

$$4x + 2y = 22$$
$$-2x + 2y = -8$$

a. $4x + 2y = 22$
   $2x - 2y = 8$

b. $4x + 2y = 22$
   $-4x + 4y = -16$

c. $12x + 6y = 66$
   $6x - 6y = 24$

d. $8x + 4y = 44$
   $-8x + 8y = -8$

A-REI.C.5: Prove Equivalent Forms of Systems
Answer Section

1. ANS:

Albert is correct. Both systems have the same solution $\left(\frac{-3}{4}, 6\right)$.

Strategy: Solve one system of equations, then test the solution in the second system of equations.

STEP 1. Solve the first system of equations.

$$Eq.\ 1\quad 8x + 9y = 48$$
$$Eq.\ 2\quad 12x + 5y = 21$$

Multiply Eq. 1 by 3 and Multiply Eq. 2 by 2.

Then solve for the first variable

$$24x + 27y = 144$$
$$\underline{24x + 10y = 42}$$
$$17y = 102$$
$$y = 6$$

Solve for the second variable.

$$8x + 9(6) = 48$$
$$8x = -6$$
$$x = -\frac{3}{4}$$

The solution is $\left(\frac{-3}{4}, 6\right)$

STEP 2: Test the second system of equations using the same solution set.

$8x + 9y = 48$ $8\left(\frac{-3}{4}\right) + 9(6) = 48$ $-6 + 54 = 48$ $48 = 48$	$-8.5y = -51$ $-8.5(6) = -51$ $-51 = -51$

DIMS? Does It Make Sense? Yes. The solution $\left(\frac{-3}{4}, 6\right)$ makes both equations balance.

PTS: 4 REF: 061533ai NAT: A.REI.C.5 TOP: Solving Linear Systems

2. ANS: B

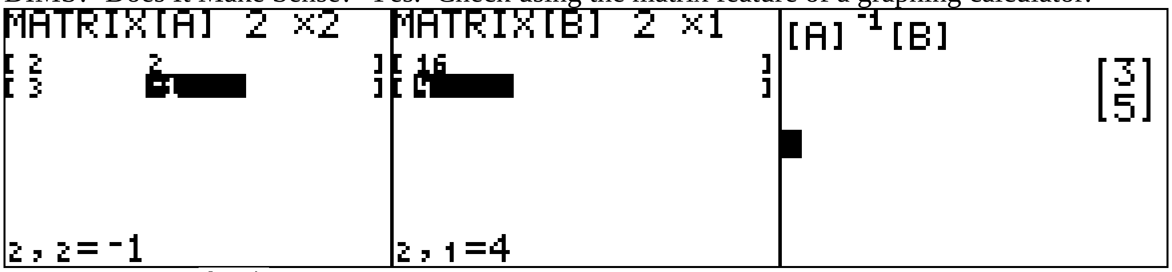
Strategy: Find equivalent forms of the system and eliminate wrong answers.

STEP 1. Eliminate answer choices *c* and *d* because the first equation in each system is not a multiple of any equation in the original system.

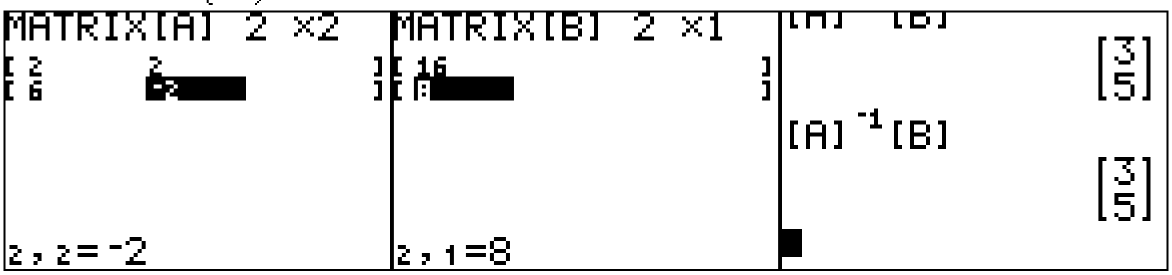
STEP 2. Eliminate answer choice *a* because $6x - 2y = 4$ is not a multiple of $3x - y = 4$.

Choose answer choice *b* as the only remaining choice.

DIMS? Does It Make Sense? Yes. Check using the matrix feature of a graphing calculator.



The solution set $(3,5)$ also works for the system in answer choice *b*.



PTS: 2 REF: 061414ai NAT: A.REI.C.5 TOP: Solving Linear Systems

3. ANS: D

Strategy: Eliminate wrong answers by deciding which systems of equations are made of multiples of the original system of equations and which system is made of equations that are not multiples of the orginal system of equations.

Choice (a) is a multiple of the original system of equations.

$$\begin{pmatrix} 4x + 2y = 22 \\ 2x - 2y = 8 \end{pmatrix} = \begin{pmatrix} 1(4x + 2y = 22) \\ -1(-2x + 2y = -8) \end{pmatrix}$$

Choice (b) is a multiple of the original system of equations.

$$\begin{bmatrix} 4x + 2y = 22 \\ -4x + 4y = -16 \end{bmatrix} = \begin{bmatrix} 1(4x + 2y = 22) \\ 2(-2x + 2y = -8) \end{bmatrix}$$

Choice (c) is a multiple of the original system of equations.

$$\begin{bmatrix} 12x + 6y = 66 \\ 6x - 6y = 24 \end{bmatrix} = \begin{bmatrix} 3(4x + 2y = 22) \\ -3(-2x + 2y = -8) \end{bmatrix}$$

Choice (d) is ***not*** a multiple of the original system of equations.

$$\begin{bmatrix} 8x + 4y = 44 \\ -8x + 8y = -8 \end{bmatrix} \neq \begin{bmatrix} 8x + 4y = 44 \\ -8x + 8y = -8 \end{bmatrix}$$

PTS: 2 REF: 011621ai NAT: A.REI.C.5 TOP: Solving Linear Systems

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. <u>The Problem</u>

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. <u>What is the problem asking?</u>

Find the salary of the fifth employee.

Part 1c. <u>Answer</u>

The salary of the fifth employee is $350 per week.

Part 1d. <u>Explanation of Strategy</u>

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. <u>A New Problem</u>

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b <u>What is the new problem asking?</u>

Find Joseph's score on the missing exam.

Part 2c. <u>Answer to New Problem</u>

Joseph received a score of 85 on the missing examination.

Part 2d. <u>Explanation of Strategy</u>

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.