A-REI.C.6: Solve Linear Systems Algebraically and by Graphing

SYSTEMS
A.REI.C.6: Solve Linear Systems

C. Solve systems of equations.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Solutions to systems of equations
Solutions to systems of equations are those values of variables which solve all equations in the system simultaneously (at the same time). A system of equations may have one, two, or more solutions.

EXAMPLE: The system $\begin{matrix} 2x - y = 3 \\ x + y = 3 \end{matrix}$ has a common solution of $(2,1)$.

When $x = 2$ and $y = 1$, both equations balance, which means both equations are true.

You can verify this by substituting the values $(2,1)$ into both equations.

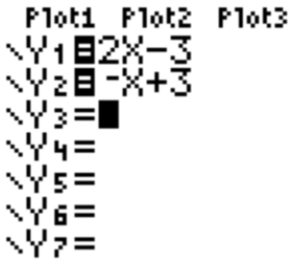
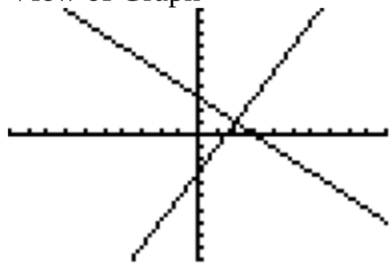
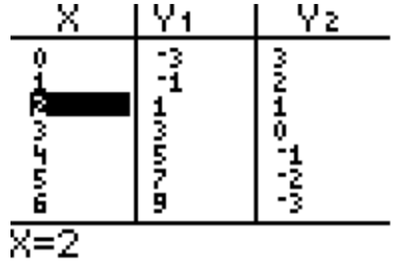
$$2(\not{x}2) - (\not{y}1) = 3$$
$$4 - 1 = 3$$
$$3 = 3 \; check$$
$$(\not{x}2) + (\not{y}1) = 3$$
$$2 + 1 = 3$$
$$3 = 3 \; check$$

You can also verify this by looking at the graphs of both equations.

STEP #1. Put both equations into slope intercept form.

2x - y = 3	x + y = 3
-y=-2x+3	y=-x+3
y=2x-3	

STEP #2. Graph both equations on the same coordinate plane.

<table>
<tr>
<td>Input for transformed equations in a graphing calculator.

</td>
<td>View of Graph

You can see that the graphs of the two equations intersect at $(2,1)$ This is the solution for this system of equations.</td>
<td>Table of Values


You can also see in the table of values that when $x = 2$, the value of the dependent variable is the same in both equations.</td>
</tr>
</table>

Elimination Method (See A.REI.C.5)

Substitution Method

Strategy: Find the easiest variable to isolate in either equation, and substitute its equivalent expression into the other equation. This results in a new equation with only one variable.

EXAMPLE:

Solve the system of equations $\begin{aligned} 3C + 4M &= 12.50 \\ 3C + 2M &= 8.50 \end{aligned}$ by isolating one variable in one equation and substituting its equivalent expression into the other equation.

STEP #1 Isolate one variable in one equation. Normally, you should pick the equation and the variable that seems easiest to isolate.

$$\begin{aligned} &Eq.\#1 \\ 3C + 4M &= 12.50 \\ 3C &= 12.50 - 4M \\ C &= \frac{12.50 - 4M}{3} \end{aligned}$$

STEP #2. Substitute the equivalent expression for the variable in the other equation.

$$Eq.\#2$$

$$3\left(\not{C}\,\frac{12.50 - 4M}{3}\right) + 2M = 8.50$$

Note that , after the substitution, equation #2 has only one variable.

STEP #3. Solve the other equation with one variable, which in this case is M.

$$Eq.\#2$$

$$\not{3}\left(\frac{12.50-4M}{\not{3}}\right)+2M=8.50$$

$$1\left(\frac{12.50-4M}{1}\right)+2M=8.50$$

$$12.50-4M+2M=8.50$$

$$12.50-8.50=4M-2M$$

$$4.00=2M$$

$$\boxed{\frac{4.00}{2}=M=2.00}$$

STEP #4. Substitute the value of the variable you found in the first equation and solve for the second variable.

$$Eq.\#1$$

$$3C+4\left(\not{M}\ 2.00\right)=12.50$$

$$3C+8=12.50$$

$$3C=4.50$$

$$\boxed{C=\frac{4.50}{3}=1.50}$$

One again, these are the same values you found using the tables method, so you do not have to check them. Normally, you would do a check.

Lesson Plan

Graphing Method

STEP #1.

Put the equations into slope-intercept form (Y=mx+b) and identify slope (m) and the y-intercept (b).

STEP #2.

Graph both equations on the same coordinate plane. Pick either equation to start.

STEP #3.

Identify the location of the point or points where the two lines intersect. This is the point(s) that makes both equations balance. This is the solution to the system of equations. Write its address on the coordinate plane as an ordered pair, as in (x,y).

STEP #4.

Check your solution by substituting it into the original equations. If both equations balance, you have the correct solution and you are done. If not, find your mistake.

NOTE: Graphing solutions are best performed with the aid of a graphing calculator. Input both equations in the [Y=] feature of the TI-83+ and identify the solution in either the graph or table of values views. In the graph view, input

[2nd] [calculate] [5.intersection] [enter] [enter] [enter]

and the intersection of the two linear equations will appear on the screen.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Guy and Jim work at a furniture store. Guy is paid $185 per week plus 3% of his total sales in dollars, *x*, which can be represented by $g(x) = 185 + 0.03x$. Jim is paid $275 per week plus 2.5% of his total sales in dollars, *x*, which can be represented by $f(x) = 275 + 0.025x$. Determine the value of *x*, in dollars, that will make their weekly pay the same.

2. The line represented by the equation $4y + 2x = 33.6$ shares a solution point with the line represented by the table below.

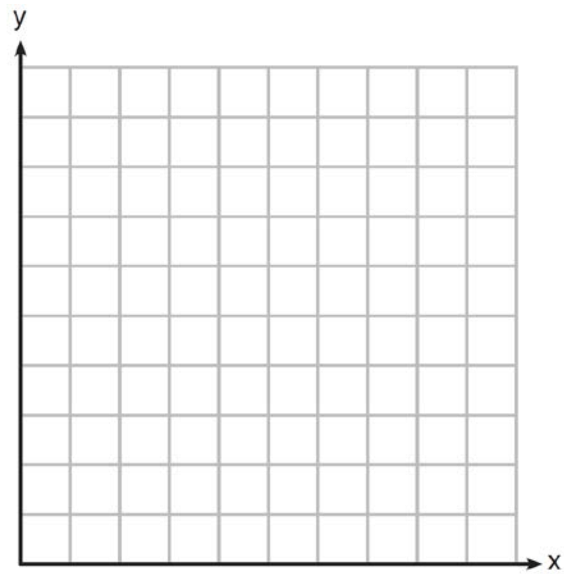
x	y
–5	3.2
–2	3.8
2	4.6
4	5
11	6.4

The solution for this system is

a. $(-14.0, -1.4)$
b. $(-6.8, 5.0)$
c. $(1.9, 4.6)$
d. $(6.0, 5.4)$

Lesson Plan

3.Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies for $19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for $24. Let *x* equal the price of one package of cupcakes and *y* equal the price of one package of brownies. Write a system of equations that describes the given situation. On the set of axes below, graph the system of equations.



Determine the exact cost of one package of cupcakes and the exact cost of one package of brownies in dollars and cents. Justify your solution.

A-REI.C.6: Solve Linear Systems Algebraically and by Graphing
Answer Section

1. ANS:
$18,000

Strategy: Set both function equal to one another and solve for *x*.

STEP 1. Set both functions equal to one another.

$$g(x) = 185 + 0.03x$$
$$f(x) = 275 + 0.025x$$
$$185 + 0.03x = 275 + 0.025x$$
$$0.03x - 0.025x = 275 - 185$$
$$0.005x = 90$$
$$x = 18,000$$

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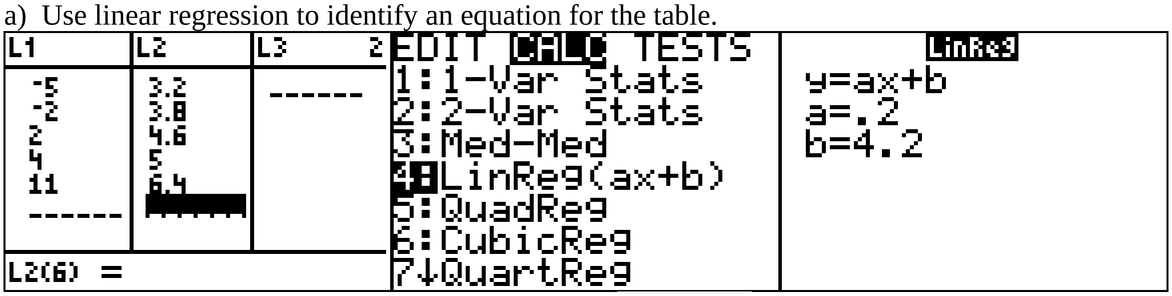
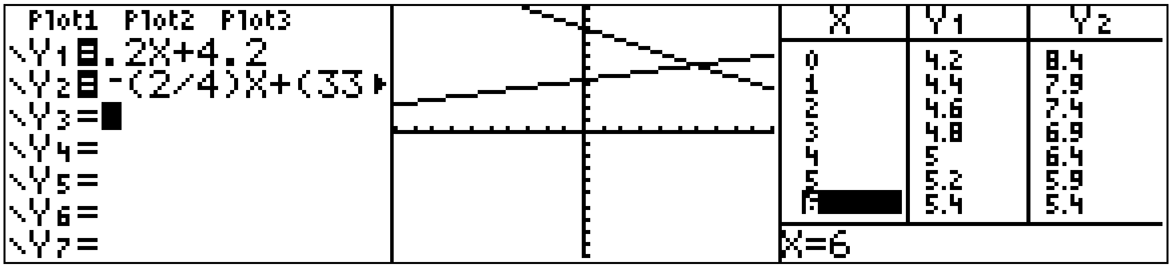
2. ANS: D
Step 1. Understand that this question is asking for the coordinates of the intersection of two different lines: the first line is represented by the equation $4y + 2x = 33.6$ and the second line is represented by the table.
Step 2. Strategy: a) Identify the function rule for the data in the table; b) transform $4y + 2x = 33.6$ into $y = mx + b$ format; and c) input both equations into a graphing calculator to find their intersection.
Step 3. Execution of strategy:
a) Use linear regression to identify an equation for the table.



The table values can be represented by the equation $y = .2x + 4.2$
b) Transform $4y + 2x = 33.6$ into $y = mx + b$ format.

$$4y + 2x = 33.6$$
$$4y = -2x + 33.6$$
$$y = -\frac{2}{4}x + \frac{33.6}{4}$$

cI Input both equations in a graphing calculator.



The lines intersect at (6, 5.4). Choice d) is the correct answer.

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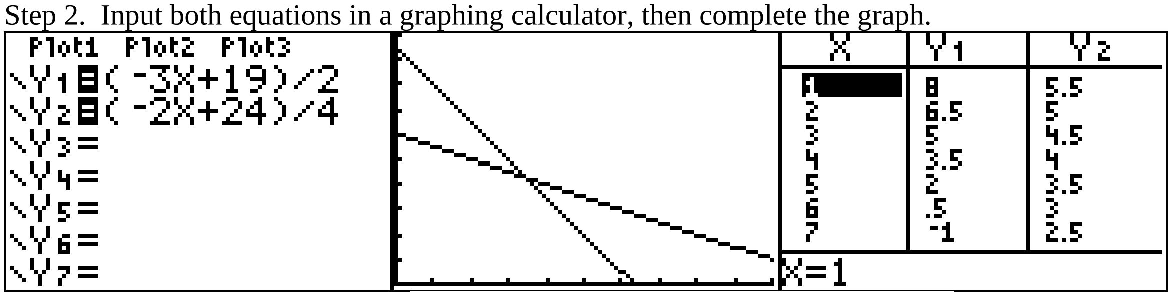
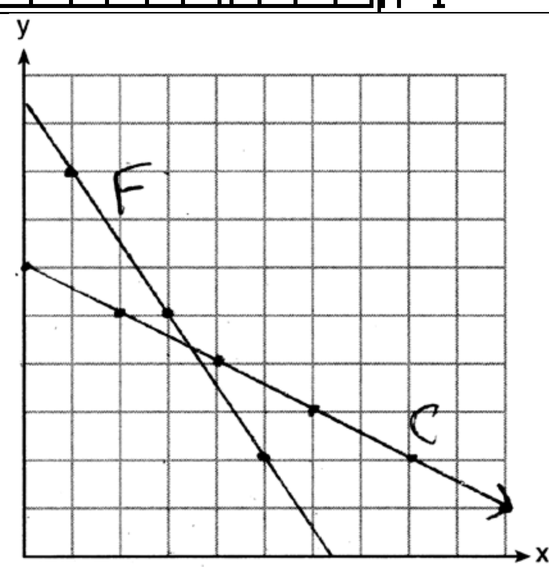
3. ANS:

Step 1. Write two equations.

Franco's Purchase: Franco bought 3 packages of cupcakes (3x) and 2 packages of brownies (2y) for $19.	Caryl's Purchase: Caryl bought 2 packages of cupcakes (2x) and 4 packages of brownies (4y) for $24.
$3C + 2B = 19$	$2C + 4B = 24$
$3x + 2y = 19$	$2x + 4y = 24$
$2y = -3x + 19$	$4y = -2x + 24$
$y = \frac{-3x + 19}{2}$	$y = \frac{-2x + 24}{4}$

Step 2. Input both equations in a graphing calculator, then complete the graph.





Determine the <u>exact cost of one package of cupcakes</u> and the <u>exact cost of one package of brownies</u> in dollars and cents. Justify your solution.

Cupcakes	Brownies
$y = \frac{-3x+19}{2}$ $\quad y = \frac{-2x+24}{4}$ $\frac{-3x+19}{2} = \frac{-2x+24}{4}$ $4(-3x+19) = 2(-2x+24)$ $-12x+76 = -4x+48$ $76-48 = 12x-4x$ $28 = 8x$ $3.5 = x$ A package of cupcakes costs \$3.50.	$y = \frac{-2x+24}{4}$ $y = \frac{-2(3.5)+24}{4}$ $y = \frac{-7+24}{4}$ $y = \frac{17}{4}$ $y = 4.25$ A package of brownies costs \$4.25

Check by inserting both values in both equations.

Franco	Caryl
$3C + 2B = 19$ $3(3.50) + 2(4.25) = 19$ $10.50 + 8.50 = 19$ $19 = 19$	$2C + 4B = 24$ $2(3.50) + 4(4.25) = 24$ $7.00 + 17.00 = 24$ $24 = 24$

PTS: 6 REF: 061637ai NAT: A.REI.C.6 TOP: Graphing Linear Systems

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.