Lesson Plan

A.REI.D.12: Graph Systems of Inequalities

SYSTEMS

A.REI.D.12: Graph Systems of Inequalities

D. Represent and solve equations and inequalities graphically.

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
    Selected problem set(s)
- facilitate a summary and share out of student work
    Homework – Write the Math Assignment

BIG IDEAS

A linear inequality describes a region of the coordinate plane that has a **boundary line**. Every point in the region is a **solution of the inequality**.

Two or more linear inequalities together form a **system of linear inequalities**. Note that there are two or more boundary lines in a system of linear inequalities.

A **solution of a system of linear inequalities** makes each inequality in the system true. The graph of a system shows all of its solutions.

Graphing a Linear Inequality

Step One. Change the inequality sign to an equal sign and graph the boundary line in the same manner that you would graph a linear equation.

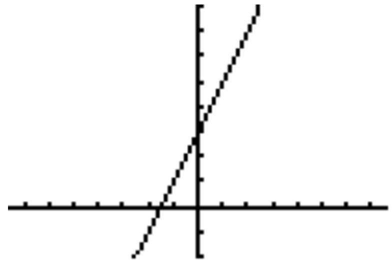
- When the inequality sign **contains** an equality bar beneath it, use a solid line for the boundary.
- When the inequality sign **does not contain** an equality bar beneath it, use a dashed or dotted line for the boundary

Step Two. Restore the inequality sign and test a point to see which side of the boundary line the solution is on. The point (0,0) is a good point to test since it simplifies any multiplication. However, if the boundary line passes through the point (0,0), another point not on the boundary line must be selected for testing.

- If the test point makes the inequality true, shade the side of the boundary line that includes the test point.
- If the test point makes the inequality not true, shade the side of the boundary line does not include the test point.

Example Graph $y < 2x + 3$

First, change the inequality sign an equal sign and graph the line: $y = 2x + 3$. This is the boundary line of the solution. Since there is no equality line beneath the inequality symbol, use a dashed line for the boundary.



Next, **test a point** to see which side of the boundary line the solution is on. Try (0,0), since it makes the multiplication easy, but remember that any point will do.

$y < 2x + 3$

$0 < 2(0) + 3$

$0 < 3$ True, so the solution of the inequality is the region that contains the point $(0,0)$.

Therefore, we shade the side of the boundary line that contains the point (0,0).

Note: The TI-83+ graphing calculator does not have the ability to distinguish between solid and dashed lines on a graph of an inequality. The less than and greater than symbols are input using the far-left column of symbols that can be accessed through the Y= feature.

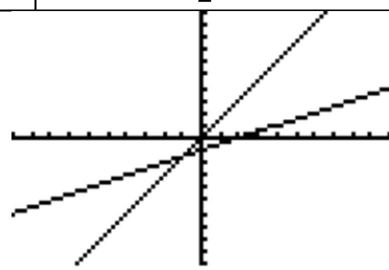
Graphing a System of Linear Inequalities. Systems of linear inequalities are graphed in the same manner as systems of equations are graphed. The solution of the system of inequalities is the region of the coordinate plane that is shaded by both inequalities.

Example: Graph the system:

$$4y \geq 6x$$
$$-3x + 6y \leq -6$$

First, convert both inequalities to slope-intercept form and graph.

$4y \ge 6x$ $\frac{4y}{4} \ge \frac{6x}{4}$ $y \ge \frac{3}{2}x$ m=$\frac{3}{2}$, b=0	$-3x + 6y \le -6$ $6y \le -6 + 3x$ $6y \le 3x - 6$ D_2 $\frac{6y}{6} \le \frac{3x}{6} - \frac{6}{6}$ $y \le \frac{1}{2}x - 1$ m=$\frac{1}{2}$, b=-1



Next, test a point in each inequality and shade appropriately.

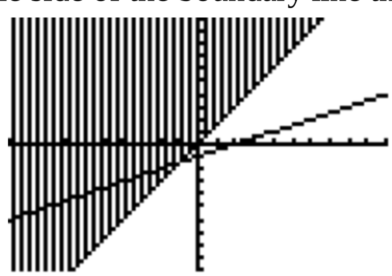
- Since point (0,0) is on the boundary line of $y \ge \frac{3}{2}x$, select another point, such as (0,1).

  $y \ge \frac{3}{2}x$

  Test (0,1)

  $1 \ge \frac{3}{2}(0)$

  $1 \ge 0$ This is true, so the point (0,1) is in the solution set of this inequality. Therefore, we shade the side of the boundary line that includes point (0,1).



- Since (0,0) is not on the boundary line of $y \le \frac{1}{2}x - 1$, we can use (0,0) as our test point, as follows:

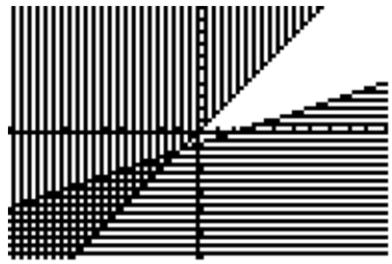
$y \le \frac{1}{2}x - 1$

Test (0,0)

$0 \le \frac{1}{2}(0) - 1$

$0 \le -1$ This is not true, so the point (0,0) is not in the solution set of this inequality.

We therefore must shade the side of the boundary line that does not incude the point (0,0).



Note that the system of inequalities divides the coordinate plane into four sections. The solution set for the system of inequalities is the area where the two shaded regions overlap.

Remember The Big Rule for Solving Inequalities:

All the rules for solving equations apply to inequalties – plus one more:

When an inequality is multiplied or divided by any negative number, the direction of the inequality sign changes.

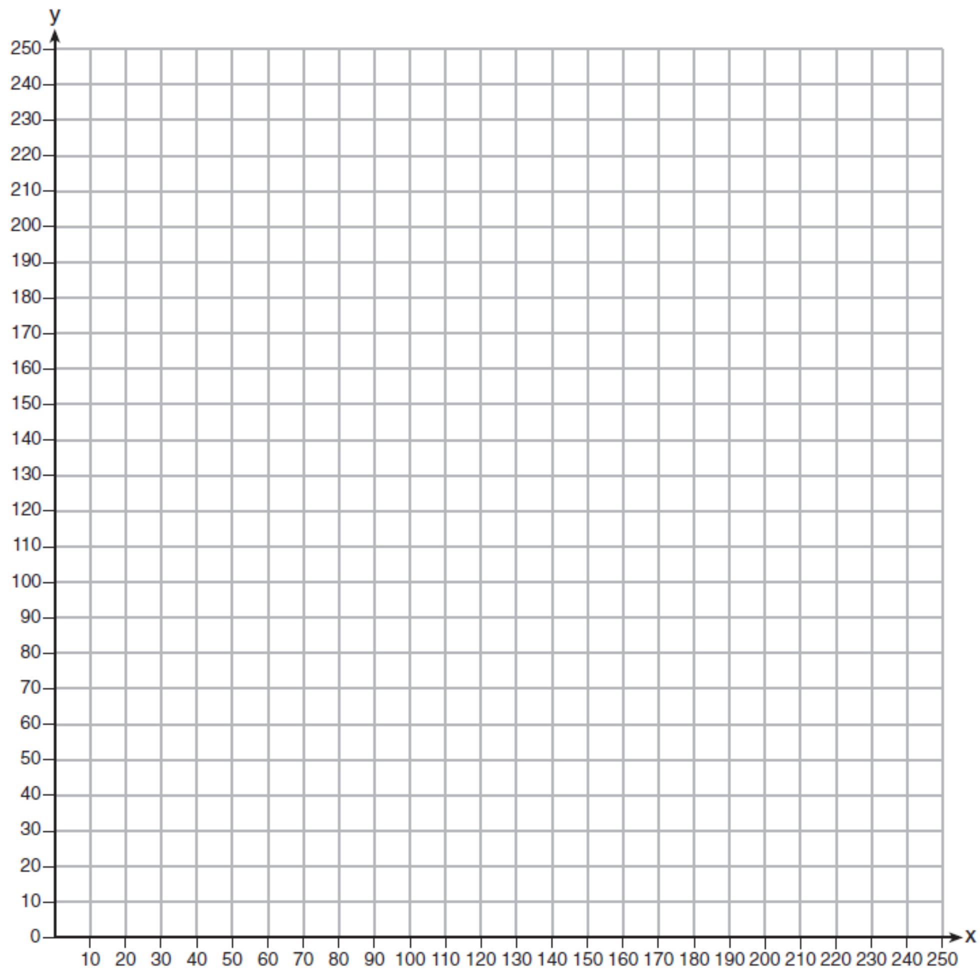
REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

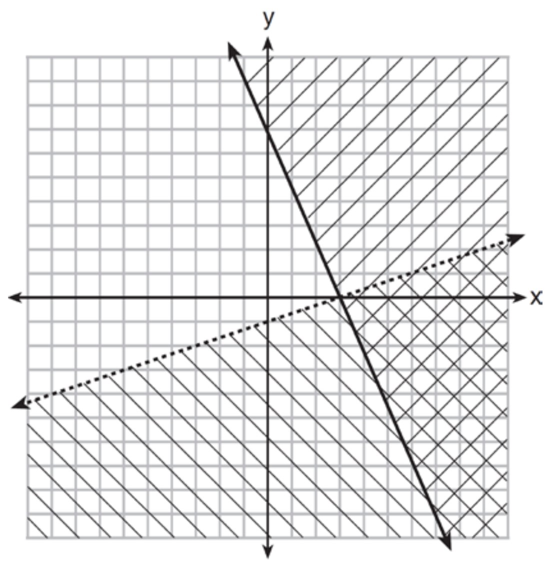
   Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, *x*, and child tickets, *y*, that would satisfy the cinema's goal.

   Graph the solution to this system of inequalities on the set of axes below. Label the solution with an *S*.

   Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.



2. What is one point that lies in the solution set of the system of inequalities graphed below?



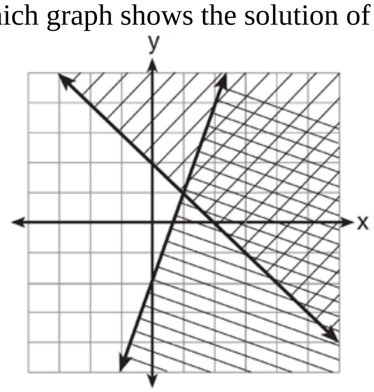
a. $(7, 0)$
b. $(3, 0)$
c. $(0, 7)$
d. $(-3, 5)$

3. Given: $y + x > 2$

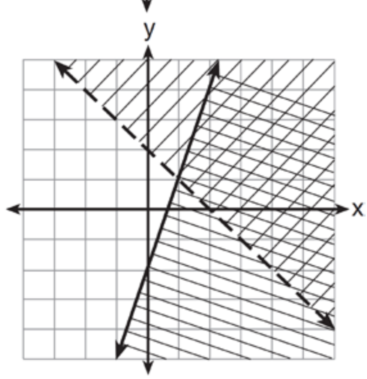
$y \le 3x - 2$

Which graph shows the solution of the given set of inequalities?

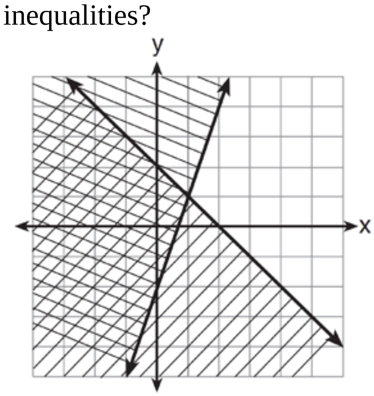
a.



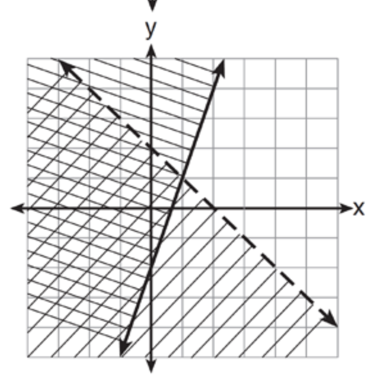
b.



c.

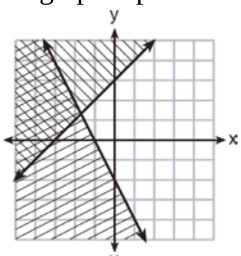


d.

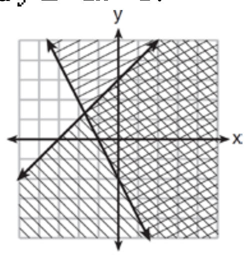


4. Which graph represents the solution of $y \leq x + 3$ and $y \geq -2x - 2$?

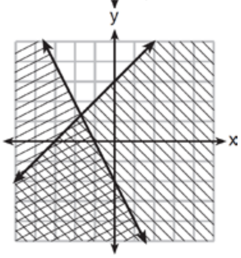
a.



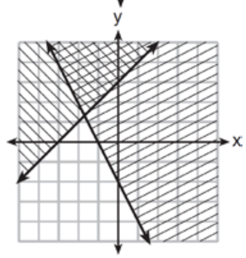
c.



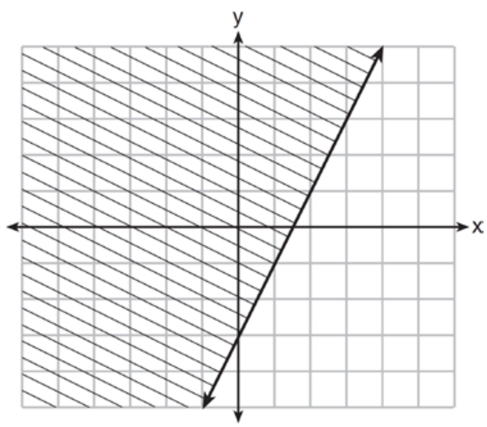
b.



d.

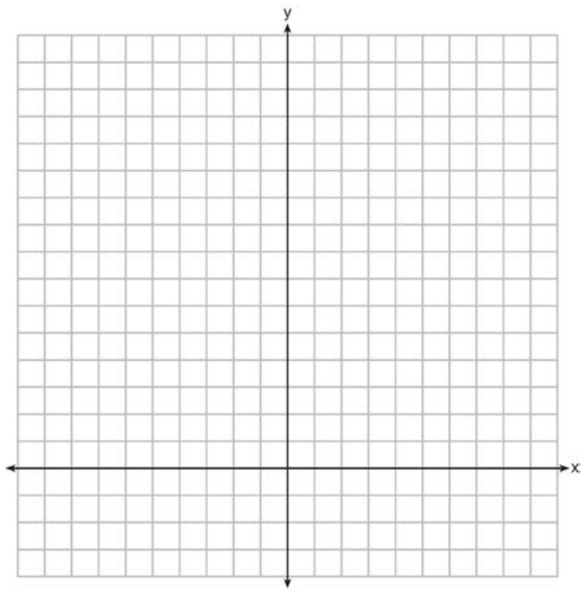


5. The graph of an inequality is shown below.



a) Write the inequality represented by the graph.
b) On the same set of axes, graph the inequality $x + 2y < 4$.
c) The two inequalities graphed on the set of axes form a system. Oscar thinks that the point $(2, 1)$ is in the solution set for this system of inequalities. Determine and state whether you agree with Oscar. Explain your reasoning.

6. The sum of two numbers, x and y, is more than 8. When you double x and add it to y, the sum is less than 14. Graph the inequalities that represent this scenario on the set of axes below.



Kai says that the point $(6, 2)$ is a solution to this system. Determine if he is correct and explain your reasoning.

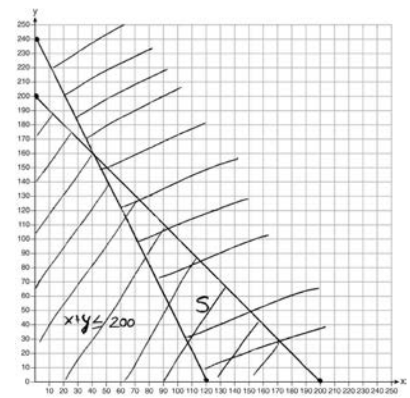
A.REI.D.12: Graph Systems of Inequalities
Answer Section

1. ANS:
System of Inequalities
Let x represent the number of adult tickets and let y represent the number of child tickets.

$$x + y \le 200$$

$$12.5x + 6.25y \ge 1500$$

Graph of the System



Marta is incorrect because the coordinates (30, 80) are not in the solution area.

Check: Marta is incorrect because $\$12.50(30) + \$6.25(80) = \$875.00$.. This is less than the cinema's goal of selling at least $1500 worth of tickets.

PTS: 6 REF: 011637ia NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities
KEY: graph

2. ANS: A
Strategy: Visually estimate whether a point falls in the solution area and eliminate wrong answers.

a. (7,0) clearly falls in the solutions area for both the solid line and the dotted line.
b. (3,0) appears to be in the solution area for the solid line, but not for the dotted line.
c. (0,7) is clearly not in the solution area for the dotted line.
d. (-3,5) is clearly not in the solution area for either the solid line or the dotted line.

PTS: 2 REF: 081407ai NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities

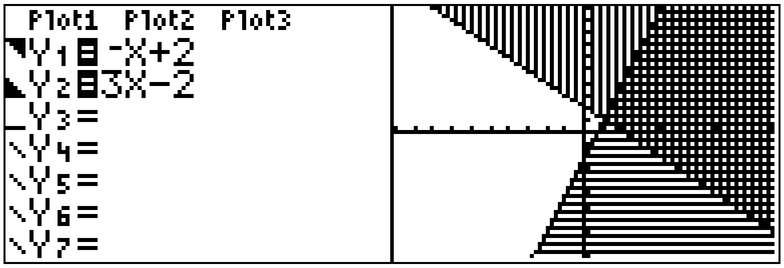
3. ANS: B
Strategy: Transpose the first inequality to slope intercept form ($y = mx + b$), then input both inequalities into a graphing calculator and eliminate wrong answers.

STEP 1. Transpose the first inequality to slope intercept form ($y = mx + b$).

$$y + x > 2$$

$$y > -x + 2$$

STEP 2. Input both inequalities into a graphing calculator and inspect the graphs.

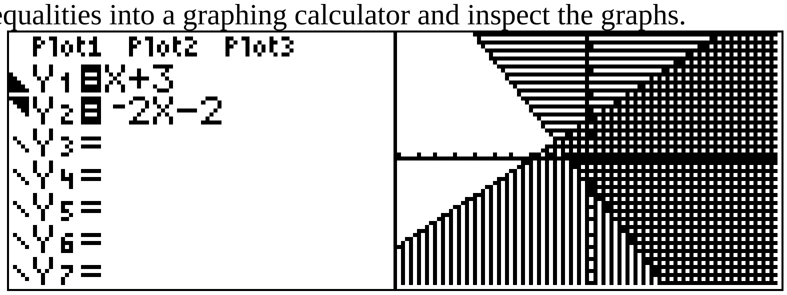


Eliminate answer choices *c* and *d*.

STEP 3. Decide between answer choices *a* and *b*.
Eliminate answer choice *a* because it shows two solid lines. The graph for $y > -x + 2$ must have a dotted line. Answer choice *b* is the correct answer.

PTS: 2 REF: 061404ai NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities
KEY: bimodalgraph

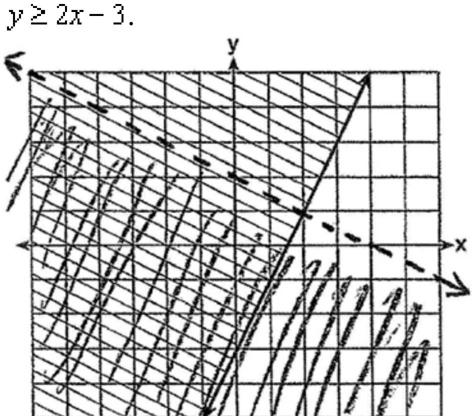
4. ANS: C
Strategy: Input both inequalities into a graphing calculator and inspect the graphs.



Answer choice *c* is the correct answer.

PTS: 2 REF: 081506ai NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities

5. ANS:
a) $y \ge 2x - 3$.



b)
c Oscar is wrong. The point (2,1) is not in the solution set of both inequalities.

Strategy: Use information from the graph together with the slope intercept form of a line ($y = mx + b$) to write the inequality $y \ge 2x - 3$, where 2 is the slope (*m*) and -3 is the y-intercept b. Then, transform the new equation and put both equations in a graphing calculator. Use the graph and the table of values to finish the system of inequaulities on paper. Finally, determine if Oscar is right or wrong.

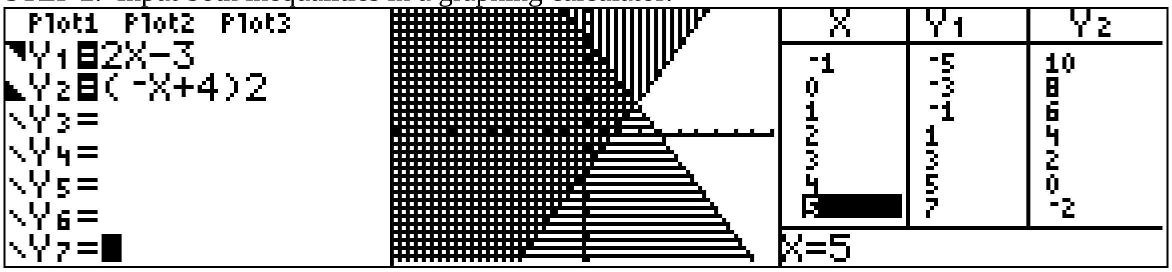
STEP 1. Transform $x + 2y < 4$ for input into a graphing calculator.

$$x + 2y < 4$$

$$2y < -x + 4$$

$$y < \frac{-x+4}{2}$$

STEP 2. Input both inequalities in a graphing calculator.



STEP 3. Use the graph view and the table view to transfer the graph to paper. Be sure to make the line dotted for $y < \frac{-x+4}{2}$. The line for $y \geq 2x - 3$ should be solid.

STEP 4. Test the point (2,1) in both equations.

$y < \frac{-x+4}{2}$ $1 < \frac{-2+4}{2}$ $1 < \frac{2}{2}$ $1 < 1$ Not True	$y \geq 2x - 3$ $1 \geq 2(2) - 3$ $1 \geq 1$ True

Oscar is wrong.

PTS: 4 REF: 011534ai NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities

6. ANS:

Strategy: Write two inequalities, then input them in a graphing calulcator and reproduce the graph on paper.

First Inequality: The sum of two numbers, *x* and *y*, is more than 8.

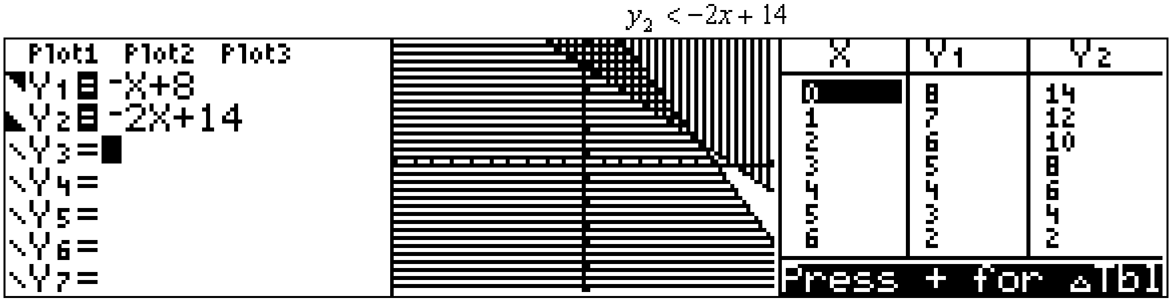
$$x + y_1 > 8$$

$$y_1 > -x + 8$$

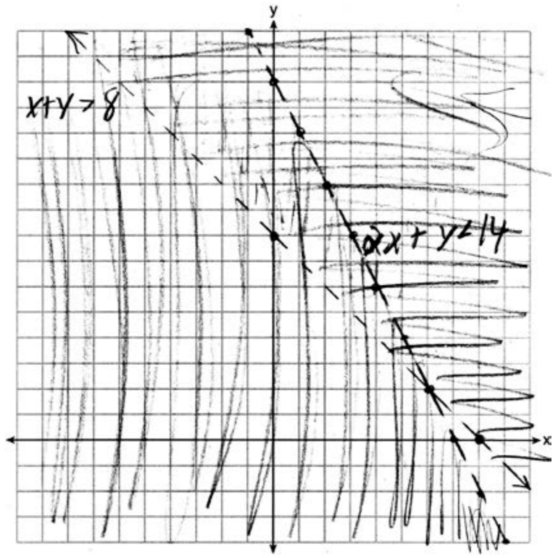
Second Inequality: When you double *x* and add it to *y*, the sum is less than 14.

$$2x + y_2 < 14$$

$$y_2 < -2x + 14$$



Lesson Plan



Kai is not correct,. (6, 2) is not a solution because it falls on the boundary lines of both inequalities and the boundary lines are not part of the solution set of this system of inequalities.

PTS: 4 REF: 061634ai NAT: A.REI.D.12 TOP: Graphing Systems of Linear Inequalities
KEY: graph

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.