Lesson Plan

A.SSE.A.2: Factor Polynomials

POLYNOMIALS AND QUADRATICS

A.SSE.A.2: Factor Polynomials

A. Interpret the structure of expressions.

2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ (linear, exponential, quadratic). Does not include factoring by grouping and factoring the sum and difference of cubes.*

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Big Idea:

Factoring polynomials is one of four general methods taught in the Regents mathematics curriculum for finding the roots of a quadratic equation. The other three methods are the quadratic formula, completing the square and graphing.

- The roots of a quadratic equation can found using the **factoring** method when the discriminant's value is equal to either zero or a perfect square.

Factoring Monomials:

$$204x^2 = 2\left(102x^2\right) = 2 \bullet 2\left(51x^2\right) = 2 \bullet 2 \bullet 3\left(17x^2\right) = 2^2 \bullet 3 \bullet 17 \bullet x^2$$

Factoring Binomials: *NOTE: This is the inverse of the distributive property.*

$$3(x+2) = 3x+6$$

$$2x^2 + 6x = 2x(x+3)$$

Special Case: Factoring the Difference of Perfect Squares.

General Rule

$$\left(a^2 - b^2\right) = (a+b)(a-b)$$

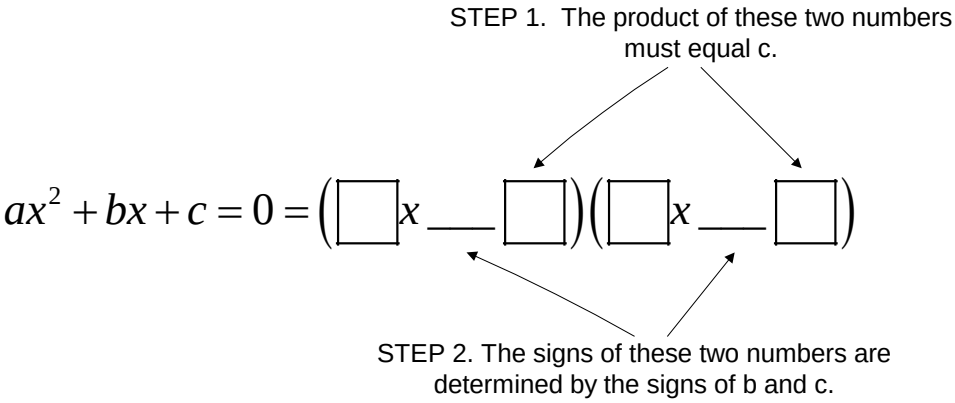
Examples

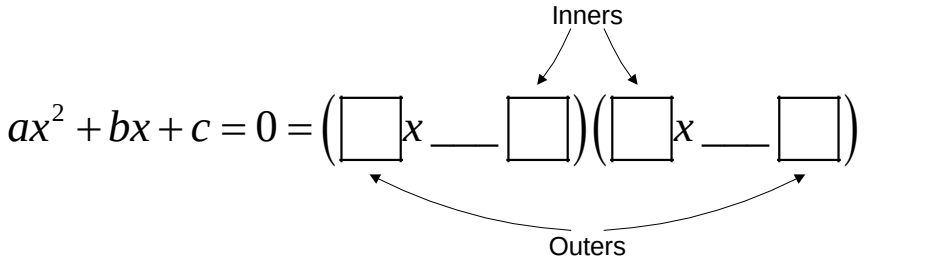
$$x^2 - 4 = (x+2)(x-2)$$

$$x^4 - 9 = \left(x^2+3\right)\left(x^2-3\right)$$

Factoring Trinomials.

Given a trinomial in the form $ax^2+bx+c=0$ whose discriminant equals zero or a perfect square, it may be factored as follows:





STEP 3. The product of the outer numbers plus the product of the inner numbers must sum to b.

Modeling:

$$x^2-5x+6=(x-2)(x-3)$$
$$2x^2-8x+6=(2x-2)(x-3)$$
$$4x^2-10x+6=(2x-2)(2x-3)$$

Turning Factors into Roots, Solutions, and Zeros. Students frequently do not understand why each factor of a binomial or trinomial can be set to equal zero, thus leading to the roots of the equation. Recall that the standard form of a quadratic equation is $ax^2+bx+c=0$ and only the left side of the equation is factored. Thus, the left side of the equation equals zero.

For all numbers	$a\cdot 0=0$		
and if	$a\cdot b=0$	$b\neq 0$	(NOTE: substitute any two factors)
Therefore	$a=0$		

Lesson Plan

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Factor the expression $x^4 + 6x^2 - 7$ completely.

2. When factored completely, $x^3 - 13x^2 - 30x$ is
   a. $x(x+3)(x-10)$
   b. $x(x-3)(x-10)$
   c. $x(x+2)(x-15)$
   d. $x(x-2)(x+15)$

3. When factored completely, the expression $p^4 - 81$ is equivalent to
   a. $(p^2+9)(p^2-9)$
   b. $(p^2-9)(p^2-9)$
   c. $(p^2+9)(p+3)(p-3)$
   d. $(p+3)(p-3)(p+3)(p-3)$

4. If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as
   a. $(x-3y)(x+3y)$
   b. $(x^2-3y)(x^2+3y)$
   c. $(x^2-3y)(x^2-3y)$
   d. $(x^4+y)(x-9y)$

5. Which expression is equivalent to $36x^2 - 100$?
   a. $4(3x - 5)(3x - 5)$
   b. $4(3x + 5)(3x - 5)$
   c. $2(9x - 25)(9x - 25)$
   d. $2(9x + 5)(9x - 25)$

6. Four expressions are shown below.

   I $2(2x^2 - 2x - 60)$
   II $4(x^2 - x - 30)$
   III $4(x + 6)(x - 5)$
   IV $4x(x - 1) - 120$

   The expression $4x^2 - 4x - 120$ is equivalent to
   a. I and II, only
   b. II and IV, only
   c. I, II, and IV
   d. II, III, and IV

A.SSE.A.2: Factor Polynomials
Answer Section

1. ANS:
$(x^2+7)(x+1)(x-1)$

Strategy: Factor the trinomial, then factor the perfect square.

STEP 1. Factor the trinomial $x^4 + 6x^2 - 7$.

$x^4 + 6x^2 - 7$

$(x^2 + \_\_\_)(x^2 - \_\_\_)$

The factors of 7 are 1 and 7.

$(x^2+7)(x^2-1)$

STEP 2. Factor the perfect square.

$(x^2+7)(x^2-1)$

$(x^2+7)(x+1)(x-1)$

PTS: 2 REF: 061431ai NAT: A.SSE.A.2 TOP: Factoring Polynomials

2. ANS: C

$x^3 - 13x^2 - 30x$

$x(x^2 - 13x - 30)$

$x(x+2)(x-15)$

PTS: 2 REF: 011612ai NAT: A.SSE.A.2 TOP: Factoring Polynomials

3. ANS: C

Strategy: Use difference of perfect squares.

STEP 1. Factor $p^4 - 81$

$p^4 - 81$

$(p^2+9)(p^2-9)$

STEP 2. Factor $p^2 - 9$

$(p^2+9)(p^2-9)$

$(p^2+9)(p+3)(p-3)$

PTS: 2 REF: 011522ai NAT: A.SSE.A.2 TOP: Factoring Polynomials

4. ANS: B

Strategy: Use the distributive property to work backwards from the answer choices.

a.	c.

$(x-3y)(x+3y)$ $x^2+3xy-3xy-9y^2$ x^2-9y^2 (wrong)	$(x^2-3y)(x^2-3y)$ $x^4-3x^2y-3x^2y+9y^2$ $x^4-6x^2y+9y^2$ (wrong)
b. $(x^2-3y)(x^2+3y)$ $x^4+3x^2y-3x^2y-9y^2$ x^4-9y^2 (correct)	d. $(x^4+y)(x-9y)$ $x^5-9x^4y+xy-9y^2$ (wrong)

PTS: 2 REF: 061503ai NAT: A.SSE.A.2 TOP: Factoring Polynomials

5. ANS: B

Strategy 1.

Recognize that the expression $36x^2-100$ is a difference of perfect squares. Therefore,

$$36x^2-100 \quad .$$

$$(6x+10)(6x-10)$$

Since this is not an answer choice, continue factoring, as follows:

$$(6x+10)(6x-10)$$

$$\big(2(3x+5)\big)\big(2(3x-5)\big)$$

$$4(3x+5)(3x-5)$$

Strategy 2.

Examine the answer choices, which begin with factors 4 and 2. Extract these factors first, as follows:

Start by extracting a 4	Start by extracting a 2
$36x^2-100$ $4\left(9x^2-25\right)$ $4(3x+5)(3x-5)$	$36x^2-100$ $2\left(18x^2-50\right)$ $(2)(2)\left(9x^2-25\right)$ $(2)(2)(3x+5)(3x-5)$ $4(3x+5)(3x-5)$

PTS: 2 REF: 081608ai NAT: A.SSE.A.2

6. ANS: C

Strategy: Use the distributive property to expand each expression, then match the expanded expressions to the answer choices.

I	III
$2(2x^2-2x-60)$ $4x^2-4x-120$ *yes*	

	$4(x + 6)(x - 5)$ $(4x + 24)(x - 5)$ $4x^2 - 20x + 24x - 120$ $4x^2 + 4x - 120$ *no*
II $4(x^2 - x - 30)$ $4x^2 - 4x - 120$ *yes*	IV $4x(x - 1) - 120$ $4x^2 - 4x - 120$ *yes*

Answer choice *c* is correct.

PTS: 2 REF: 081509ai NAT: A.SSE.A.2 TOP: Factoring Polynomials

Lesson Plan

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

Lesson Plan

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is $350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.