K – Polynomials, Lesson 4, Factoring the Difference of Perfect Squares (r. 2018)

**POLYNOMIALS**

Factoring the Difference of Perfect Squares

<table>
<thead>
<tr>
<th>Common Core Standard</th>
<th>Next Generation Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A-SSE.2</strong> Use the structure of an expression to identify ways to rewrite it. For example, see ( x^4 - y^4 ) as ((x^2)^2 - (y^2)^2), thus recognizing it as a difference of squares that can be factored as ((x^2 - y^2)(x^2 + y^2)).</td>
<td><strong>AI-A.SSE.2</strong> Recognize and use the structure of an expression to identify ways to rewrite it. (Shared standard with Algebra II) e.g., ( x^2 - x^2 - x = x(x^2 - x - 1) ) ( 53^2 - 47^2 = (53 + 47)(53 - 47) ) ( 16x^2 - 36 = (4x)^2 - (6)^2 = (4x + 6)(4x - 6) = 4(2x + 3)(2x - 3) ) or ( 16x^2 - 36 = 4(4x^2 - 9) = 4(2x + 3)(2x - 3) ) (-2x^2 + 8x + 10 = -2(x^2 - 4x - 5) = -2(x - 5)(x + 1) ) ( x^4 + 6x^2 - 7 = (x^2 + 7)(x^2 - 1) = (x^2 + 7)(x + 1)(x - 1) ) Note: Algebra I expressions are limited to numerical and polynomial expressions in one variable. Use factoring techniques such as factoring out a greatest common factor, factoring the difference of two perfect squares, factoring trinomials of the form ( ax^2 + bx + c ) with a lead coefficient of 1, or a combination of methods to factor completely. Factoring will not involve factoring by grouping and factoring the sum and difference of cubes.</td>
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**LEARNING OBJECTIVES**

Students will be able to:

1) factor the difference of perfect squares.

**Overview of Lesson**

**Teacher Centered Introduction**

- Overview of Lesson
  - activate students’ prior knowledge
  - vocabulary
  - learning objective(s)
  - big ideas: direct instruction
  - modeling

**Student Centered Activities**

- guided practice
  - Teacher: anticipates, monitors, selects, sequences, and connects student work
  - developing essential skills
  - Regents exam questions
  - formative assessment assignment (exit slip, explain the math, or journal entry)

**VOCABULARY**

Completely factor
Perfect square binomial

Square of a number
Square root of a number
BIG IDEA

General Rule

\[(a^2 - b^2) = (a + b)(a - b)\]

Examples

\[x^2 - 4 = (x + 2)(x - 2)\]

\[x^4 - 9 = (x^2 + 3)(x^2 - 3)\]

DEVELOPING ESSENTIAL SKILLS

1. The expression \(x^2 - 16\) is equivalent to
   a. \((x + 2)(x - 8)\)
   b. \((x - 2)(x + 8)\)
   c. \((x + 4)(x - 4)\)
   d. \((x + 8)(x - 8)\)

2. Factored, the expression \(16x^2 - 25y^2\) is equivalent to
   a. \((4x - 5y)(4x + 5y)\)
   b. \((4x - 5y)(4x - 5y)\)
   c. \((8x - 5y)(8x + 5y)\)
   d. \((8x - 5y)(8x - 5y)\)

3. The expression \(9x^2 - 100\) is equivalent to
   a. \((3x - 10)(3x + 10)\)
   b. \((3x - 10)(3x + 10)\)
   c. \((3x - 100)(3x - 1)\)
   d. \((9x - 100)(x + 1)\)

4. Factor completely: \(4x^3 - 36x\)

5. Which expression is equivalent to \(9x^2 - 16\)?
   a. \((3x + 4)(3x - 4)\)
   b. \((3x - 4)(3x - 4)\)
   c. \((3x + 8)(3x - 8)\)
   d. \((3x - 8)(3x - 8)\)

6. If Ann correctly factors an expression that is the difference of two perfect squares, her factors could be
   a. \((2x + y)(2x - y)\)
   b. \((2x + 3y)(2x - 3y)\)
   c. \((x - 4)(x - 4)\)
   d. \((2y - 5)(y - 5)\)

7. Which expression is equivalent to \(121 - x^2\)?
   a. \((x - 11)(x + 11)\)
   b. \((x + 11)(x - 11)\)
   c. \((11 - x)(11 + x)\)
   d. \((11 - x)(11 - x)\)

8. When \(a^2 - 4a\) is factored completely, the result is
   a. \((a - 2)(a + 2)\)
   b. \(a(a - 2)(a + 2)\)
   c. \(a^2(a - 4)\)
   d. \(a(a - 2)^2\)

9. The expression \(x^2 - 36y^2\) is equivalent to
   a. \((x - 6y)(x + 6y)\)
   b. \((x + 18y)(x - 18y)\)
   c. \((x + 6y)(x - 6y)\)
   d. \((x + 18y)(x + 18y)\)

10. Which expression represents \(36x^2 - 100y^6\) factored completely?
    a. \(2(9x + 25y^3)(9x - 25y^3)\)
    b. \(4(3x + 5y^3)(3x - 5y^3)\)
    c. \((6x + 10y^3)(6x - 10y^3)\)
    d. \((18x + 50y^3)(18x - 50y^3)\)

11. Which expression is equivalent to \(64 - x^2\)?
    a. \((8 - x)(8 + x)\)
    b. \((8 - x)(8 - x)\)
    c. \((x - 8)(x - 8)\)
    d. \((x - 8)(x + 8)\)

12. The expression \(9a^2 - 64b^2\) is equivalent to
    a. \((9a - 8b)(a + 8b)\)
    b. \((9a - 8b)(a - 8b)\)
    c. \((3a - 8b)(3a + 8b)\)
    d. \((3a - 8b)(3a - 8b)\)

13. The expression \(100a^2 - 1\) is equivalent to
14. When \( 9x^2 - 100 \) is factored, it is equivalent to \((3x - b)(3x + b)\). What is a value for \(b\)?
   a. 50  
   b. 10  
   c. 3  
   d. 100

15. Which expression is equivalent to \( 81 - 16x^2 \)?
   a. \((9 - 8x)(9 + 8x)\)  
   b. \((9 - 8x)(9 + 2x)\)  
   c. \((9 - 4x)(9 + 4x)\)  
   d. \((9 - 4x)(9 - 4x)\)

16. One of the factors of \(4x^2 - 9\) is
   a. \((x + 3)\)  
   b. \((2x + 3)\)  
   c. \((4x - 3)\)  
   d. \((x - 3)\)

17. Factor completely: \(3x^2 - 27\)
   a. \(3(x - 3)^2\)  
   b. \(3(x^2 - 27)\)  
   c. \(3(x + 3)(x - 3)\)  
   d. \((3x + 3)(x - 9)\)

18. Written in simplest factored form, the binomial \(2x^2 - 50\) can be expressed as
   a. \(2(x - 5)(x + 5)\)  
   b. \(2(x - 5)(x + 5)\)
   c. \((x - 5)(x + 5)\)  
   d. \(2x(x - 50)\)

19. Expressed in factored form, the binomial \(4a^2 - 9b^2\) is equivalent to
   a. \((2a - 3b)(2a + 3b)\)  
   b. \((2a + 3b)(2a - 3b)\)
   c. \((4a - 3b)(a + 3b)\)  
   d. \((2a - 9b)(2a + b)\)

20. What is a common factor of \(x^2 - 9\) and \(x^2 - 5x + 6\)?
   a. \(x + 3\)  
   b. \(x - 3\)  
   c. \(x - 2\)  
   d. \(x^2\)

**Answers**

1. ANS: C  
2. ANS: A  
3. ANS: B  
4. ANS:   
   \[4x(x + 3)(x - 3)\]  
   \[4x^3 - 36x = 4x(x^2 - 9) = 4x(x + 3)(x - 3)\]

5. ANS: A  
6. ANS: B  
7. ANS: C  
8. ANS: B  
9. ANS: C  
10. ANS: B  
11. ANS: B  
12. ANS: C  
13. ANS: A  
14. ANS: B  
15. ANS: C  
16. ANS: B  
17. ANS: C  
18. ANS: B  
19. ANS: B  
20. ANS: B
A.SSE.A.2: Difference of Perfect Squares

348) When factored completely, the expression \( p^4 - 81 \) is equivalent to

1) \((p^2 + 9)(p^2 - 9)\)
2) \((p^2 - 9)(p^2 + 9)\)
3) \((p + 3)(p - 3)\)
4) \((p + 3)(p - 3)(p + 3)(p - 3)\)

349) If the area of a rectangle is expressed as \( x^4 - 9y^2 \), then the product of the length and the width of the rectangle could be expressed as

1) \((x - 3y)(x + 3y)\)
2) \((x^2 - 3y)(x^2 + 3y)\)
3) \((x^2 - 3y)(x^2 - 3y)\)
4) \((x + y)(x - 9y)\)

350) The expression \( x^4 - 16 \) is equivalent to

1) \((x^2 + 8)(x^2 - 8)\)
2) \((x^2 - 8)(x^2 + 8)\)
3) \((x + 4)(x - 4)\)
4) \((x + 4)(x - 4)\)

351) Which expression is equivalent to \( 36x^2 - 100 \)?

1) \(4(3x - 5)(3x + 5)\)
2) \(4(3x + 5)(3x - 5)\)
3) \(2(9x - 25)(9x + 25)\)
4) \(2(9x + 5)(9x - 25)\)

352) Which expression is equivalent to \( 16x^2 - 36 \)?

1) \(4(2x - 3)(2x + 3)\)
2) \(4(2x + 3)(2x - 3)\)
3) \((4x - 6)(4x + 6)\)
4) \((4x + 6)(4x + 6)\)

353) Which expression is equivalent to \( 16x^4 - 64 \)?

1) \((4x^2 - 8)^2\)
2) \((8x^2 - 32)^2\)
3) \((4x^2 + 8)(4x^2 - 8)\)
4) \((8x^2 + 32)(8x^2 - 32)\)

354) The expression \( 49x^2 - 36 \) is equivalent to

1) \((7x - 6)^2\)
2) \((24.5x - 18)^2\)
3) \((7x - 6)(7x + 6)\)
4) \((24.5x - 18)(24.5x + 18)\)

355) Which expression is equivalent to \( y^4 - 100 \)?

1) \((y^2 - 10)^2\)
2) \((y^2 - 50)^2\)
3) \((y^2 + 10)(y^2 - 10)\)
4) \((y^2 + 50)(y^2 - 50)\)
348) ANS: 3
Strategy: Use difference of perfect squares.

STEP 1. Factor $p^4 - 81$

$$p^4 - 81 = \left(p^2 + 9\right) \left(p^2 - 9\right)$$

STEP 2. Factor $p^2 - 9$

$$\left(p^2 + 9\right) \left(p^2 - 9\right) = \left(p^2 + 9\right)(p + 3)(p - 3)$$

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

349) ANS: 2
Strategy: Use the distributive property to work backwards from the answer choices.

<table>
<thead>
<tr>
<th>a.  $(x - 3y)(x + 3y)$</th>
<th>c.  $(x^2 - 3y)(x^2 + 3y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2 + 3xy - 3xy - 9y^2$</td>
<td>$x^4 - 3x^2y - 3x^2y + 9y^2$</td>
</tr>
<tr>
<td>$x^2 - 9y^2$</td>
<td>$x^4 - 6x^2y + 9y^2$</td>
</tr>
<tr>
<td>(wrong)</td>
<td>(wrong)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b.  $(x^2 - 3y)(x^2 + 3y)$</th>
<th>d.  $(x^4 + y)(x - 9y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^4 + 3x^3y - 3x^2y - 9y^2$</td>
<td>$x^5 - 9x^4y + xy - 9y^2$</td>
</tr>
<tr>
<td>$x^4 - 9y^2$</td>
<td>(wrong)</td>
</tr>
<tr>
<td>(correct)</td>
<td></td>
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</tbody>
</table>

PTS: 2 NAT: A.SSE.A.2 TOP: Factoring Polynomials

350) ANS: 3
Step 1. Understand the problem as a “difference of perfect squares”, because the terms $x^4$ and 16 are both perfect squares and the operation is subtraction.

Step 2. Strategy: Use the pattern $a^2 - b^2 = (a + b)(a - b)$ to separate $x^4 - 16$ into two binomials.

Step 3. Execution of Strategy

- The square root of $x^4$ is $x^2$.
- The square of 16 is 4.
- $x^4 - 16 = (x^2 + 4)(x^2 - 4)$

Step 4. Does it make sense? Yes. You can show that $(x^2 + 4)(x^2 - 4) = x^4 - 16$ using the distributive property, as follows:
PTS: 2  NAT: A.SSE.A.2  TOP: Factoring the Difference of Perfect Squares

351) ANS: 2

Strategy 1.
Recognize that the expression \( 36x^2 - 10c \) is a difference of perfect squares. Therefore,
\[
36x^2 - 100.
\]
\[
(6x + 10)(6x - 10).
\]
Since this is not an answer choice, continue factoring, as follows:
\[
(6x + 10)(6x - 10)
\]
\[
(2(3x + 5))(2(3x - 5)).
\]
\[
4(3x + 5)(3x - 5).
\]

Strategy 2.
Examine the answer choices, which begin with factors 4 and 2. Extract these factors first, as follows:

<table>
<thead>
<tr>
<th>Start by extracting a 4</th>
<th>Start by extracting a 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 36x^2 - 100 )</td>
<td>( 36x^2 - 100 )</td>
</tr>
<tr>
<td>( 4(9x^2 - 25) )</td>
<td>( 2(18x^2 - 50) )</td>
</tr>
<tr>
<td>( 4(3x + 5)(3x - 5) )</td>
<td>( (2)(2)(9x^2 - 25) )</td>
</tr>
<tr>
<td></td>
<td>( (2)(2)(3x + 5)(3x - 5) )</td>
</tr>
<tr>
<td></td>
<td>( 4(3x + 5)(3x - 5) )</td>
</tr>
</tbody>
</table>

352) ANS: 2

Strategy 1: Factor
\[
16x^2 - 36
\]
\[
4(4x^2 - 9)
\]
\[
4(2x + 3)(2x - 3).
\]

Strategy 2: Recognize that \( 16x^2 - 36 \) appears to be a difference of perfect squares. Recall that \( a^2 - b^2 = (a + b)(a - b) \). Eliminate any answers that do not take the form of \( (a + b)(a - b) \), which leaves only one choice:
\[
4(2x + 3)(2x - 3).
\]
Check:
Note that the expression \(16x^4 - 64\) is the difference of perfect squares.
\[
a^2 - b^2 = (a + b)(a - b)
\]
\[
16x^4 - 64 = (4x^2 + 8)(4x^2 - 8)
\]

Note that 49\(x^2\) and 36 are both perfect squares. Therefore, 49\(x^2 - 36\) is the difference of perfect squares.
\[
a^2 - b^2 = (a + b)(a - b)
\]
\[
49x^2 - 36 = (7x + 6)(7x - 6)
\]

\(y^4 - 100\) is a difference of perfect squares. All polynomials in the form of \(a^2 - b^2\) can be factored into \((a + b)(a - b)\).
\[
y^4 - 100
\]
\[
\left(y^2 + 10\right)\left(y^2 - 10\right)
\]