A.SSE.B.3b: Transform Quadratics by Completing the Square

POLYNOMIALS AND QUADRATICS
A.SSE.B.3b: Transform Quadratics by Completing the Square

B. Write expressions in equivalent forms to solve problems.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
b. Complete the square in a quadratic expression to reveal the max and min value of the function it defines.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
  Selected problem set(s)
- facilitate a summary and share out of student work
  Homework – Write the Math Assignment

BIG IDEA: Completing the Square

TO FIND THE ZEROS AND/OR EXTREMES OF A QUADRATIC	
Start with any quadratic equation of the general form $ax^2 + bx + c = 0$	
STEP 1 **Isolate all terms with** x^2 **and** x **on one side of the equation. If** $a \neq 1$**, divide every term in the equation by** a **to get one expression in the form of** $x^2 + bx$	
STEP 2 **Complete the Square by adding** $\left(\frac{b}{2}\right)^2$ **to both sides of the equation.**	
STEP 3 **Factor the side containing** $x^2 + bx + \left(\frac{b}{2}\right)^2$ **into a binomial expression of the form** $\left(x + \frac{b}{2}\right)^2$	
STEP 4a **(solving for roots and zeros only)** **Take the square root of both sides of the equation and simplify,**	**STEP 4b** **(solving for maxima and minima only)** **Multiply both sides of the equation by** a**.** **Move all terms to left side of equation.** **Solve the factor in parenthesis for axis of symmety and x-value of the vertex..** **The number not in parentheses is the y-value of the vertes.**

Example of Completing the Square		
PROCEDURE:	**EXAMPLE A**	**EXAMPLE B**
Start with any quadratic equation of the general form $ax^2+bx+c=n$	$x^2+2x+3=4$	$5x^2+2x+3=4$
STEP 1) **Isolate all terms with** x^2 **and** x **on one side of the equation.** **If** $a\neq 1$**, divide every term in the equation by** a **to get one expression in the form of** x^2+bx	$x^2+2x=1$	$5x^2+2x=1$ $\frac{5x^2}{5}+\frac{2x}{5}=\frac{1}{5}$ $x^2+\frac{2}{5}x=\frac{1}{5}$
STEP 2) **Complete the Square by adding** $\left(\frac{b}{2}\right)^2$ **to both sides of the equation.**	$b=2,\ \frac{b}{2}=\frac{2}{2}=1,\ \left(\frac{b}{2}\right)^2=(1)^2$ $x^2+2x+(1)^2=1+(1)^2$ $x^2+2x+(1)^2=2$	$b=\frac{2}{5},\ \frac{b}{2}=\frac{1}{5},\ \left(\frac{b}{2}\right)^2=\left(\frac{1}{5}\right)^2$ $x^2+\frac{2}{5}x+\left(\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$
STEP 3) **Factor the side containing** $x^2+bx+\left(\frac{b}{2}\right)^2$ **into a binomial expression of the form** $\left(x+\frac{b}{2}\right)^2$	$(x+1)^2=2$	$\left(x+\frac{1}{5}\right)^2=\frac{1}{5}+\left(\frac{1}{5}\right)^2$ $\left(x+\frac{1}{5}\right)^2=\frac{5}{25}+\frac{1}{25}$ $\left(x+\frac{1}{5}\right)^2=\frac{6}{25}$
NOTE **Steps 1, 2, and 3 are identical, whether you are finding the vertex or the roots. Steps 4a and 4b on the next page diverge.**		

Example of Completing the Square (Continued)		
STEP 4a) *(To Find the Roots)* **Take the square roots of both sides of the equation and simplify.**	$(x+1)^2 = 2$ $\sqrt{(x+1)^2} = \sqrt{2}$ $x+1 = \pm\sqrt{2}$ $x = -1 \pm \sqrt{2}$	$\sqrt{\left(x+\frac{1}{5}\right)^2} = \sqrt{\frac{6}{25}}$ $x+\frac{1}{5} = \pm\frac{\sqrt{6}}{5}$ $x = -\frac{1}{5} \pm \frac{\sqrt{6}}{5} = \boxed{\frac{1\pm\sqrt{6}}{5}}$
STEP 4b *(To find the vertex)* **Multiply both sides of the equation by *a*. Move all terms to left side of equation. Solve the factor in parenthesis for axis of symmety and x-value of the vertex. The number not in parentheses is the y-value of the vertes.**	$1(x+1)^2 = 1(2)$ $\quad (x+1)^2 = 2$ $(x+1)^2 = 2$ $(x+1)^2 - 2 = 0$ vertex form. -1 is the axis of symmetry -2 is the y value of the vertex The vertex is at $(-1,-2)$	$5\left(x+\frac{1}{5}\right)^2 = 5\left(\frac{6}{25}\right)$ $5\left(x+\frac{1}{5}\right)^2 = \frac{6}{5}$ $5\left(x+\frac{1}{5}\right)^2 - \frac{6}{5} = 0$ vertex form. $-\frac{1}{5}$ is the axis of symmetry $-\frac{6}{5}$ is the y value of the vertex The vertex is at $\left(-\frac{1}{5},-\frac{6}{5}\right)$

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Which equation has the same solutions as $2x^2 + x - 3 = 0$
   a. $(2x-1)(x+3) = 0$
   b. $(2x+1)(x-3) = 0$
   c. $(2x-3)(x+1) = 0$
   d. $(2x+3)(x-1) = 0$

2. The zeros of the function $f(x) = 2x^2 - 4x - 6$ are
   a. 3 and -1
   b. 3 and 1
   c. -3 and 1
   d. -3 and -1

3. If Lylah completes the square for $f(x) = x^2 - 12x + 7$ in order to find the minimum, she must write $f(x)$ in the general form $f(x) = (x - a)^2 + b$. What is the value of *a* for $f(x)$?

a. 6
b. -6
c. 12
d. -12

4. In the function $f(x) = (x - 2)^2 + 4$, the minimum value occurs when *x* is

a. -2
b. 2
c. -4
d. 4

5. The function $f(x) = 3x^2 + 12x + 11$ can be written in vertex form as

a. $f(x) = (3x + 6)^2 - 25$
b. $f(x) = 3(x + 6)^2 - 25$
c. $f(x) = 3(x + 2)^2 - 1$
d. $f(x) = 3(x + 2)^2 - 25$

6. Janice is asked to solve $0 = 64x^2 + 16x - 3$. She begins the problem by writing the following steps:

Line 1 $0 = 64x^2 + 16x - 3$
Line 2 $0 = B^2 + 2B - 3$
Line 3 $0 = (B + 3)(B - 1)$

Use Janice's procedure to solve the equation for x.

Explain the method Janice used to solve the quadratic equation.

7. Keith determines the zeros of the function $f(x)$ to be -6 and 5. What could be Keith's function?

a. $f(x) = (x+5)(x+6)$
b. $f(x) = (x+5)(x-6)$
c. $f(x) = (x-5)(x+6)$
d. $f(x) = (x-5)(x-6)$

8. A sunflower is 3 inches tall at week 0 and grows 2 inches each week. Which function(s) shown below can be used to determine the height, $f(n)$, of the sunflower in *n* weeks?

I. $f(n) = 2n + 3$
II. $f(n) = 2n + 3(n-1)$
III. $f(n) = f(n-1) + 2$ where $f(0) = 3$

a. I and II
b. II, only
c. III, only
d. I and III

A.SSE.B.3b: Transform Quadratics by Completing the Square
Answer Section

1. ANS: D

Strategy 1: Factor by grouping.

$$2x^2 + x - 3 = 0$$
$$|ac| = 6$$
Factors of 6 are
1 and 6
2 and 3 (use these)
$$2x^2 + 3x - 2x - 3 = 0$$
$$\left(2x^2 + 3x\right) - (2x + 3) = 0$$
$$x(2x - 3) - 1(2x + 3) = 0$$
$$(x - 1)(2x + 3) = 0$$

Answer choice *d* is correct

Strategy 2: Work backwards by using the disrtibutive property to expand all answer choices and match the expanded trinomials to the function $2x^2 + x - 3 = 0$.

a. $(2x - 1)(x + 3) = 0$ $2x^2 + 6x - x - 3$ $2x^2 + 5x - 3$ (Wrong Choice)	c. $(2x - 3)(x + 1) = 0$ $2x^2 + 2x - 3x - 3$ $2x^2 - x - 3$ (Wrong Choice)
b. $(2x + 1)(x - 3) = 0$ $2x^2 - 6x + x - 3 = 0$ $2x^2 - 5x - 3 = 0$ (Wrong Choice)	d. $(2x + 3)(x - 1) = 0$ $2x^2 - 2x + 3x - 3 = 0$ $2x^2 + x - 3 = 0$ (Correct Choice)

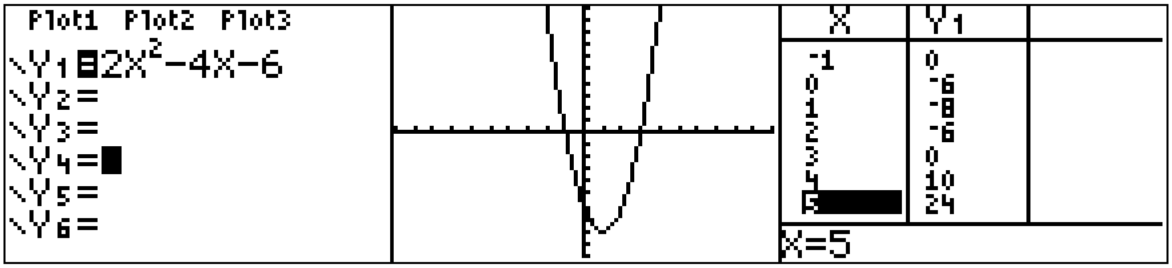
PTS: 2 REF: 011503ai NAT: A.SSE.B.3 TOP: Solving Quadratics

2. ANS: A

Strategy #1: Solve by factoring:

$$f(x) = 2x^2 - 4x - 6$$
$$0 = 2x^2 - 4x - 6$$
$$0 = 2(x^2 - 2x - 3)$$
$$0 = 2(x - 3)(x + 1)$$
$$x = 3 \ \textit{and} \ x = -1$$

Strategy #2: Solve by inputing equation into graphing calculator, the use the graph and table views to identify the zeros of the function.



The graph and table views show the zeros to be at -1 and 3.

PTS: 2 REF: 011609ai NAT: A.SSE.B.3 TOP: Solving Quadratics
KEY: zeros of polynomials

3. ANS: A

Strategy: Transform $f(x) = x^2 - 12x + 7$ into the form of $f(x) = (x - a)^2 + b$ and find the value of a.

$$x^2 - 12x + 7 = f(x)$$

$$x^2 - 12x + 7 = 0$$

$$x^2 - 12x = -7$$

$$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = -7 + \left(\frac{-12}{2}\right)^2$$

$$x^2 - 12x + (-6)^2 = -7 + (-6)^2$$

$$(x - 6)^2 = -7 + 36$$

$$(x - 6)^2 = +29$$

$$(x - 6)^2 - 29 = 0$$

$$f(x) = (x - 6)^2 - 29$$

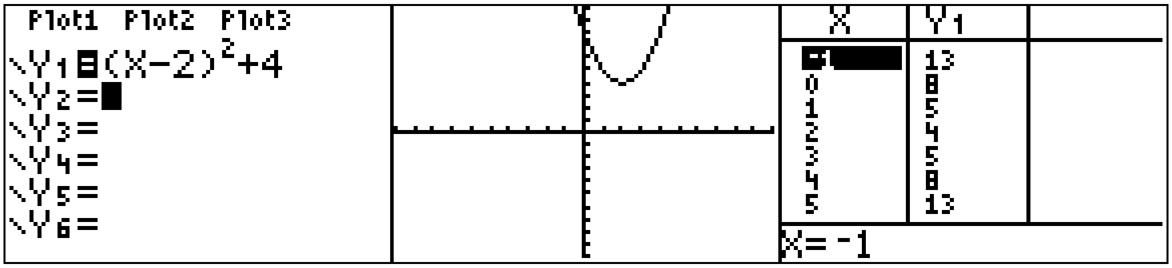
If $-a = -6$, then $a = 6$.

PTS: 2 REF: 081520ai NAT: A.SSE.B.3 TOP: Solving Quadratics
KEY: completing the square

4. ANS: B

Strategy #1. Recognize that the function $f(x) = (x - 2)^2 + 4$ is expressed in vertex form, and that the vertex is located at $(2, 4)$. Accordingly, the minimum value of $f(x)$ occurs when $x = 2$.

Strategy #2: Input the function rule in a graphing calculator, then examine the graph and tabler views to determine the vertex. The problem wants to know the x value of the when $f(x)$ is at its minimum.



The minimum value of $f(x) = 4$ when x is equal to 2.

Strategy #3: Substitute each value of x into the equation and determine the minimum value of $f(x)$.

$f(x) = (x - 2)^2 + 4$

$f(-2) = (-2 - 2)^2 + 4$

$f(-2) = (-4)^2 + 4$

$f(-2) = 16 + 4$

$f(-2) = 20$

$f(2) = (2 - 2)^2 + 4$

$f(2) = (0)^2 + 4$

$f(2) = 4$

$f(-4) = (-4 - 2)^2 + 4$

$f(-4) = (-6)^2 + 4$

$f(-4) = 36 + 4$

$f(-4) = 40$

$f(4) = (4 - 2)^2 + 4$

$f(4) = (2)^2 + 4$

$f(4) = 4 + 4$

$f(4) = 8$

PTS: 2 REF: 011601ai NAT: A.SSE.B.3 TOP: Vertex Form of a Quadratic
NOT: NYSED classifies this as A.SSE.3

5. ANS: C

Strategy: Complete the square to transform $f(x) = 3x^2 + 12x + 11$ from standard form to vertex form, as follows:

$$f(x) = 3x^2 + 12x + 11$$

$$3x^2 + 12x + 11 = f(x)$$

$$3x^2 + 12x + 11 = 0$$

$$3x^2 + 12x = -11$$

$$\frac{3x^2}{3} + \frac{12x}{3} = \frac{-11}{3}$$

$$x^2 + 4x = \frac{-11}{3}$$

$$x^2 + 4x + (2)^2 = \frac{-11}{3} + (2)^2$$

$$(x + 2)^2 = \frac{-11}{3} + 4$$

$$(x + 2)^2 = \frac{1}{3}$$

$$3(x + 2)^2 = 3\left(\frac{1}{3}\right)$$

$$3(x + 2)^2 = 1$$

$$3(x + 2)^2 - 1 = 0$$

$$3(x + 2)^2 - 1 = f(x)$$

$$f(x) = 3(x + 2)^2 - 1$$

PTS: 2 REF: 081621ai NAT: A.SSE.B.3 TOP: Families of Functions

6. ANS:

Use Janice’s procedure to solve for X.

Line 4 $B = -3$ and $B = 1$

Line 5 Therefore:

$8x = -3$ and $8x = 1$

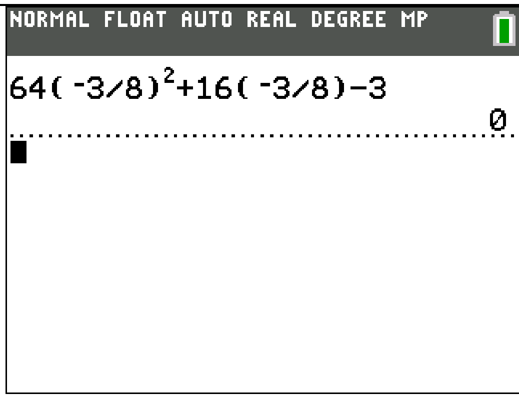
$x = -\frac{3}{8}$ $x = \frac{1}{8}$

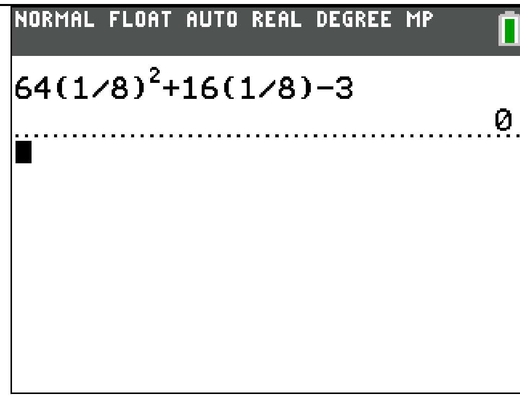
Explain the method Janice used to solve the quadratic formula.

Janice made the problem easier by substituting B for *8x*, then solving for B. After solving for B, she reversed her substitution and solved for *x*.

Check:

$x = -\frac{3}{8}$ $0 = 64x^2 + 16x - 3$	$x = \frac{1}{8}$ $0 = 64x^2 + 16x - 3$





PTS: 4 REF: 081636ai NAT: A.CED.A.4

7. ANS: C

Strategy: Convert the zeros to factors.

If the zeros of $f(x)$ are -6 and 5, then the factors of $f(x)$ are $(x+6)$ and $(x-5)$.
Therefore, the function can be written as $f(x) = (x+6)(x-5)$.
The correct answer choice is *c*.

PTS: 2 REF: 061412ai NAT: A.APR.B.3 TOP: Solving Quadratics

8. ANS: D

Strategy: If sunflower's height is modelled using a table, then the three formulas can be tested to see which one(s) produce results that agree with the table.

Weeks (n)	Height $f(n)$	$f(n) = 2n + 3$	$f(n) = 2n + 3(n-1)$	$f(n) = f(n-1) + 2$ where $f(0) = 3$
0	3	$f(0) = 2(0) + 3 = 3$	$f(0) = 2(0) + 3(0-1) = -3$	$f(0) = 3$
1	5	$f(1) = 2(1) + 3 = 5$		$f(1) = f(0) + 2 = 3 + 2 = 5$
2	7	$f(2) = 2(2) + 3 = 7$		$f(2) = f(1) + 2 = 5 + 2 = 7$
3	9	$f(3) = 2(3) + 3 = 9$		$f(3) = f(2) + 2 = 7 + 2 = 9$

Formula I, $f(n) = 2n + 3$, is an explicit formula that *agrees* with the table.
Formula II is an explicit formula that *does not agree* with the table.
Formula III, $f(n) = f(n-1) + 2$ where $f(0) = 3$, is a recursive formula that agrees with the table.

PTS: 2 REF: 061421ai NAT: F.IF.A.3 TOP: Sequences

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is \$350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.