Lesson Plan

F.BF.A.1: Model Explicit and Recursive Processes

FUNCTIONS

F.BF.A.l: Model Explicit and Recursive Processes

A. Build a function that models a relationship between two quantities.
1. Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

An **explicit formula** is one where you do not need to know the value of the term in front of the term that you are seeking.
A **recursive formula** requires you to know the value of another term, usually the preceding term, to find the value of a specific term.

BIG IDEA

There are four views of a function: 1) the function rule; 2) the table of values; 3) the graph; and 4) the narrative or "context" view. Sometimes, it is easier to understand a function using different views. For example, the same function can be modeled in foour different views. Understanding one view can help to find the other views.

Modeling a Sample Function

Context View: The inside of a freeze is kept at a constant temperature of 15 degrees farenheit. When a quart of liquid water is placed in the freezer, its farenheit temperature drops by one-half every 20 minutes until it turn into ice and reaches a constant temperature of 15 degrees.

Table View: These tables shows what the temperatures of two different quarts of water would be after *m* minutes in the freezer.

Initial Temperature = 80 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	80	40	20	15	15

Initial Temperature = 120 degrees

Minutes in Freezer (*m*)	0	20	40	60	80
Temperature *f(m)*	120	60	30	15	15

Function Rule View

The narrative view and the table views suggest that the temperature drops exponentially at first, then stays at a constant temperature of 15 degrees.

* The exponential parts of the functions can be modeled using the formula for exponential decay, as follows:

$$f(m) = I(1 - \frac{1}{2})^{\frac{m}{20}}$$

$$f(m) = I\left(\frac{1}{2}\right)^{\frac{m}{20}}$$

$f(m)$ represents the temperature of the water after *m* minutes in the freezer.

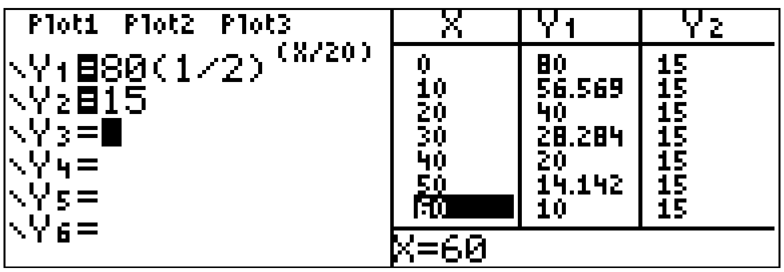
I represents the initial temperature of the water.

$(\frac{1}{2})$ represents the exponential rate of decay.

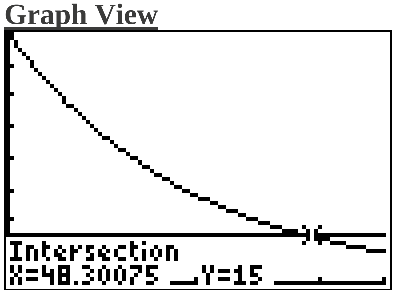
$\frac{m}{20}$ represents time.

The **range** of the function would be limited to $212 \geq f(m) \geq 15$

Check: Input the system of equations in a graphing calculator for a quart of water with an initial temperature of 80 degrees fareheit.



Graph View



DIMS - Does It Make Sense? Yes, all four views of the function show that the water cools down quickly at first, then more slowly, then reaches a final temperature of 15 degrees. The graph view shows that it would take about 48 minutes for a quart of liquid water with an initial temperature of 80 degrees to reach a frozen temperature of 15 degrees.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Alex is selling tickets to a school play. An adult ticket costs $6.50 and a student ticket costs $4.00. Alex sells *x* adult tickets and 12 student tickets. Write a function, $f(x)$, to represent how much money Alex collected from selling tickets.

2. Each day Toni records the height of a plant for her science lab. Her data are shown in the table below.

Day (n)	1	2	3	4	5
Height (cm)	3.0	4.5	6.0	7.5	9.0

The plant continues to grow at a constant daily rate. Write an equation to represent $h(n)$, the height of the plant on the *n*th day.

3. Krystal was given $3000 when she turned 2 years old. Her parents invested it at a 2% interest rate compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Krystal had in the account when she turned 18?

a. $3000(1+0.02)^{16}$
b. $3000(1-0.02)^{16}$
c. $3000(1+0.02)^{18}$
d. $3000(1-0.02)^{18}$

4. Caitlin has a movie rental card worth $175. After she rents the first movie, the card's value is $172.25. After she rents the second movie, its value is $169.50. After she rents the third movie, the card is worth $166.75. Assuming the pattern continues, write an equation to define $A(n)$, the amount of money on the rental card after *n* rentals. Caitlin rents a movie every Friday night. How many weeks in a row can she afford to rent a movie, using her rental card only? Explain how you arrived at your answer.

5. In 2013, the United States Postal Service charged $0.46 to mail a letter weighing up to 1 oz. and $0.20 per ounce for each additional ounce. Which function would determine the cost, in dollars, $c(z)$, of mailing a letter weighing *z* ounces where *z* is an integer greater than 1?
   a. $c(z) = 0.46z + 0.20$
   b. $c(z) = 0.20z + 0.46$
   c. $c(z) = 0.46(z - 1) + 0.20$
   d. $c(z) = 0.20(z - 1) + 0.46$

6. Jackson is starting an exercise program. The first day he will spend 30 minutes on a treadmill. He will increase his time on the treadmill by 2 minutes each day. Write an equation for $T(d)$, the time, in minutes, on the treadmill on day *d*. Find $T(6)$, the minutes he will spend on the treadmill on day 6.

F.BF.A.1: Model Explicit and Recursive Processes
Answer Section

1. ANS:
$f(x) = 6.50x + 4(12)$

Strategy: Translate the words into math.

$6.50 per adult ticket plus $4.00 per student ticket equals total money collected.
$6.50 times x plus $4.00 times 12 students equals total money collected

$$\$6.50x + 4(12) = f(x)$$

PTS: 2 REF: 061526ai NAT: F.BF.A.1 TOP: Modeling Linear Equations

2. ANS:
$y = 1.5x + 1.5$

Strategy 1: The problem states that the plant grows at a constant daily rate, so the rate of change is constant. Use the slope-intercept form of a line, $y = mx + b$, and data from the table to identify the slope and y-intercept.

STEP 1: Extend the table to show the y-intercept, as follows:

Day (n)	0	1	2	3	4	5
Height (cm)	1.5	3	4.5	6	7.5	9

The y-intercept is 1.5, so we can write $y = mx + 1.5$.

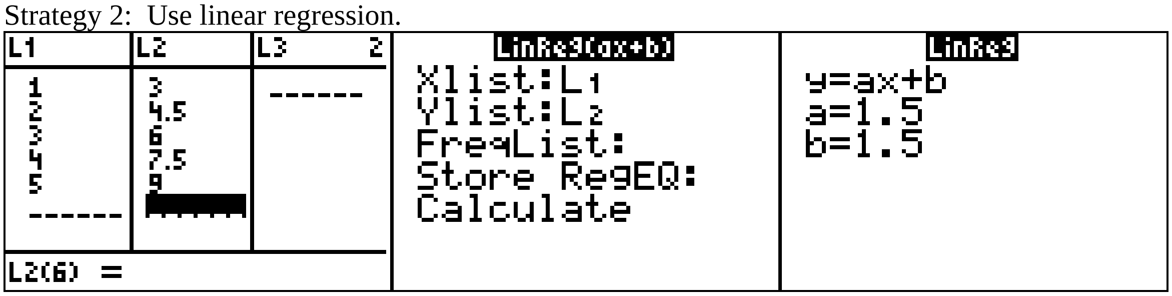
STEP 2. Use the slope formula and any two pairs of data to find the slope. In the following calculation, the points (1,3) and (5,9) are used.

$$y = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-3}{5-1} = \frac{6}{4} = \frac{3}{2} = 1.5$$

The slope is 1.5, so we can write $y = 1.5x + 1.5$

DIMS?: See below.

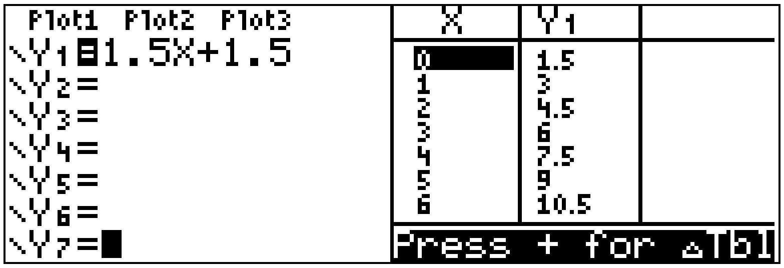
Strategy 2: Use linear regression.



The equation is $y = 1.5x + 1.5$

DIMS? Does It Make Sense? Yes. The equation can be used to reproduce the table view, as follows:

Lesson Plan



PTS: 2 REF: 081525ai NAT: F.BF.A.1 TOP: Modeling Linear Functions

3. ANS: A

Strategy 1: Use the formula for exponential growth to model the problem.

The formula for exponential **growth** is $y = a(1+r)^t$.

The formula for exponential **decay** is $y = a(1-r)^t$.

y = **final amount** after measuring growth/decay
a = **initial amount** before measuring growth/decay
r = growth/decay **rate** (usually a percent)
t = **number of time intervals** that have passed

The problem asks for the *right side expression for exponential growth*.
The problem states that $3,000 is the **initial amount**.
The problem states that the **growth factor** is 2%, which is written as .02 and added to 1.
The problem states that interest is compounded annually from age 2 through age 18, so the number of time intervals is 16 years.
The final expression for the right side of the exponential growth equation is written as $3000(1+0.02)^{16}$.

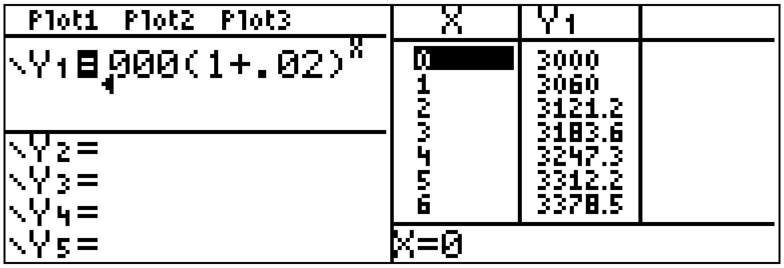
Strategy 2. Build a model and eliminate wrong answers.
Model the words using a table of values to see the pattern.

Krystal's Age	# Times Compounding	Amount
2	0	3000
3	1	3060
4	2	3121.2
5	3	3183.624
...	...	...
18	16	?

It is clear from the table that the number of times interest compounds is 2 less than Krystal's age. Eliminate answer choices *c* and *d*, because both show exponents of 18, which is Krystal's age, not the number of times the interest will compound.

The choices now are a and b. The table shows that the amounts are increasing, which is exponential growth, not exponential decay. Eliminate choice *b* because it shows exponential decay.

Check by putting choice a in a graphing calculator using x as the exponent.



Answer choice *a* creates the same table of values, and the amount of money on Krystal's 18th birthday will be $3000(1+0.02)^{16}$ dollars.

PTS: 2 REF: 011504ai NAT: F.BF.A.1 TOP: Modeling Exponential Equations

4. ANS:
63 weeks

Strategy: Model the problem with a linear function.

$A(n) = \$175 - \$2.75n$
Each movie rental costs \$2.75
Let *n* represent the number of rentals.
Let $A(n)$ represent the amount of money on the rental card after *n* rentals.
Caitlin can rent a movie for 63 weeks in a row.

Explanation:
Caitlin has \$175.
Each movie rental costs \$2.75
\$175 divided by \$2.75 equals 63.6, so \$2.75 times 63.6 equals \$175.
Caitlin has enough money to rent 63 videos. After 63 weeks, Caitlin will not have enough money to rent another movie.

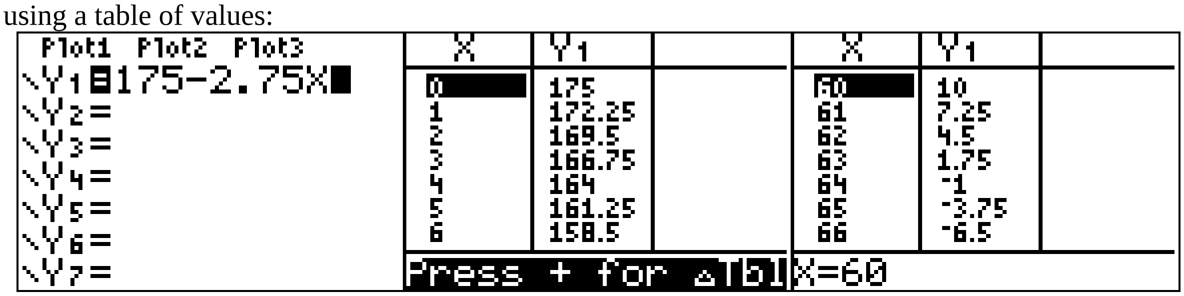
$A(63) = \$175 - \$2.75(63)$

$A(63) = \$175 - \$173.25$

$A(63) = \$1.75$

After 63 weeks, Caitlin will have \$1.75 on her rental card, which is not enough to rent another movie.

Check using a table of values:



PTS: 4 REF: 061435ai NAT: F.BF.A.1 TOP: Modeling Linear Equations

5. ANS: D
Strategy: Eliminate wrong answers.

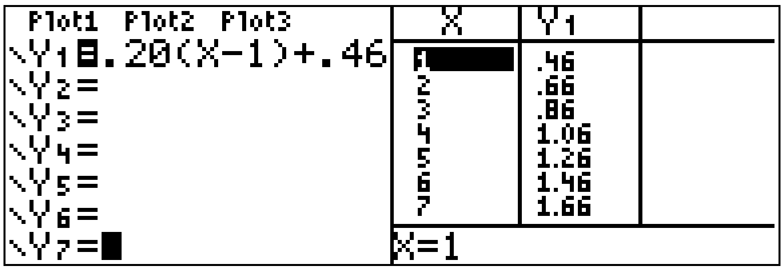
The problem states that there is a flat charge of $0.46 to mail a letter. This flat charge applies regardless of what the letter weighs. Eliminate any answer that multiplies this flat charge by the weight of the letter. Eliminate answer choices *a* and *c*.

The difference between answer choices b and d is in the terms $0.20z$ and $0.20(z-1)$, where z represents the weight of the letter in ounces. Choice b charges 20 cents for every ounce. Choice d charges 20 cents for every ounce in excess of the first ounce. Choice d is the correct answer.

DIMS? Does It Make Sense? Yes. Transform answer choice c for input into the graphing calculator.

$$c(z) = 0.20(z-1) + 0.46$$

$$Y_1 = 0.20(x-1) + 0.46$$



The table shows $0.46 to mail a letter weighing up to 1 oz. and $0.20 per ounce for each additional ounce.

PTS: 2 REF: 011523ai NAT: A.CED.A.2 TOP: Modeling Linear Equations

6. ANS:

$T(d) = 2d + 28$

Jackson will spend 40 minutes on the treadmill on day 6.

Strategy: Start with a table of values, then write an equation that models both the table view and the narrative view of the function. Then, use the equation to determine the number of minutes Jackson will spend on the treadmill on day 6.

STEP 1: Model the narrative view with a table view.

d	1	2	3	4	5	6	7	8	9
$T(d)$	30	32	34	36	38	40	42	44	46

STEP 2: Write an equation.

$$T(d) = 30 + 2(d-1)$$

$$T(d) = 30 + 2d - 2$$

$$T(d) = 28 + 2d$$

STEP 3: Use the equation to find the number of minutes Jackson will spend on the treadmill on day 6.

$$T(d) = 28 + 2d$$

$$T(6) = 28 + 2(6)$$

$$T(6) = 40$$

DIMS? Does It Make Sense? Yes. Both the equation and the table of values predict that Jackson will spend 40 minutes on the treadmill on day 6.

PTS: 2 REF: 081532ai NAT: A.CED.A.1 TOP: Modeling Linear Functions

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is \$350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.