Lesson Plan

F.IF.A.2: Use Function Notation

FUNCTIONS
F.IF.A.2: Use Function Notation

A. Understand the concept of a function and use function notation.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

The **domain of x** and the **range of y**.

The **domain** of a function is the interval covered by the function on the x-axis.
The **range** of a function is the interval covered by the function on the y-axis.

BIG IDEA #1 - Function Notation

In a function, the dependent y variable (on the y-axis of the graph) is paired with a a specific value of the independent x variable (on the x-axis of the graph). In function notation, $f(x)$ is used instead of the letter y. When graphing using function notation, the label of the y-axis is changed to show the f(x) axis. There are three primary advantages to using function notation:

1. The use of function notation indicates that the relationship is a function, and
2. The use of function notation simplifies evaluation of the value of f(x) for specific values of x.
3. The use of function notation allows greater flexibility and specificity in naming variables.

Examples:

The equation	...can be written using function notation as...			
$y = 3x^2 + 4$	$f(x) = 3x^2 + 4$	$x \xrightarrow{f} 3x^2 + 4$	$f : x \mapsto 3x^2 + 4$	$f = \{(x,y) \mid 3x^2 + 4\}$
$y = 2x$	$f(x) = 2x$	$x \xrightarrow{f} 2x$	$f : x \mapsto 2x$	$f = \{(x,y) \mid 2x\}$

Note that the y variable can be replaced with many forms in function notation. The letters f and x are often replaced with other letter, so you might see something like $g(h) = 3x^2 + 4$. In this example, $g(h)$ still represents the value of y, the dependent variable.

To evaluate a function, substitute the indicated number of expression for the variable.

Example:

Given the function rule $f(x) = 3x^2 + 4$, find the value of $f(5)$ as follows:

$$f(x) = 3x^2 + 4$$
$$f(5) = 3(5)^2 + 4$$
$$f(5) = 3(25) + 4$$
$$f(5) = 75 + 4$$
$$f(5) = 79$$

BIG IDEA #2 - Functions Have Domains and Ranges

The coordinate plane consists of two perpendicular number lines, which are commonly referred to as the x-axis and the y-axis. Each number line represents **the set of real numbers**.

A functions maps an element of the **domain** onto one and only one element of the **range**.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. The equation to determine the weekly earnings of an employee at The Hamburger Shack is given by $w(x)$, where *x* is the number of hours worked.

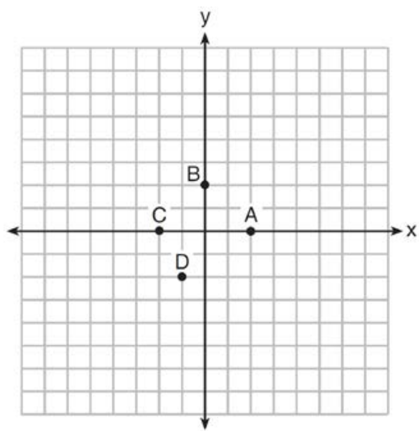
$$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$$

Determine the difference in salary, *in dollars,* for an employee who works 52 hours versus one who works 38 hours. Determine the number of hours an employee must work in order to earn $445. Explain how you arrived at this answer.

2. If $f(x) = \frac{1}{3}x + 9$, which statement is always true?

a. $f(x) < 0$
b. $f(x) > 0$
c. If $x < 0$, then $f(x) < 0$.
d. If $x > 0$, then $f(x) > 0$.

3. The graph of $y = f(x)$ is shown below.



Which point could be used to find $f(2)$?

a. *A*
b. *B*
c. *C*
d. *D*

4. If $f(x) = \dfrac{\sqrt{2x+3}}{6x-5}$, then $f\left(\dfrac{1}{2}\right) =$

a. 1
b. -2
c. -1
d. $-\dfrac{13}{3}$

5. Given that $f(x) = 2x + 1$, find $g(x)$ if $g(x) = 2[f(x)]^2 - 1$.

6. The value in dollars, $v(x)$, of a certain car after *x* years is represented by the equation $v(x) = 25,000(0.86)^x$. To the *nearest dollar*, how much more is the car worth after 2 years than after 3 years?

a. 2589
b. 6510
c. 15,901
d. 18,490

7. The range of the function defined as $y = 5^x$ is
a. $y < 0$
b. $y > 0$
c. $y \leq 0$
d. $y \geq 0$

8. The range of the function $f(x) = x^2 + 2x - 8$ is all real numbers
a. less than or equal to -9
b. greater than or equal to -9
c. less than or equal to -1
d. greater than or equal to -1

9. Let f be a function such that $f(x) = 2x - 4$ is defined on the domain $2 \leq x \leq 6$. The range of this function is
a. $0 \leq y \leq 8$
b. $0 \leq y < \infty$
c. $2 \leq y \leq 6$
d. $-\infty < y < \infty$

F.IF.A.2: Use Function Notation
Answer Section

1. ANS:
a) The difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours, is \$200.
b) An employee must work 43 hours in order to earn \$445. See work below.

Strategy: Part a: Use the piecewise function to first determine the salaries of 1) an employee who works 52 hours, and 2) an employee who works 38 hours. Then, find the difference of the two salaries.

Working 38 Hours	Working 52 Hours
$x = 38$	$x = 52$
$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$	$w(x) = \begin{cases} 10x, & 0 \le x \le 40 \\ 15(x-40)+400, & x > 40 \end{cases}$
$w(38) = \begin{cases} 10(38), & 0 \le x \le 40 \\ \text{not applicable}, & x > 40 \end{cases}$	$w(52) = \begin{cases} \text{not applicable}, & 0 \le x \le 40 \\ 15(52-40)+400, & x > 40 \end{cases}$
$w(38) = \{10(38), \quad 0 \le x \le 40$	$w(52) = \{15(52-40)+400, \quad x > 40$
$w(38) = 380$	$w(52) = \{15(12)+400, \quad x > 40$
	$w(52) = \{180+400, \quad x > 40$
	$w(52) = 580$

The difference between the values of $w(38)$ and $w(52)$ is \$200.

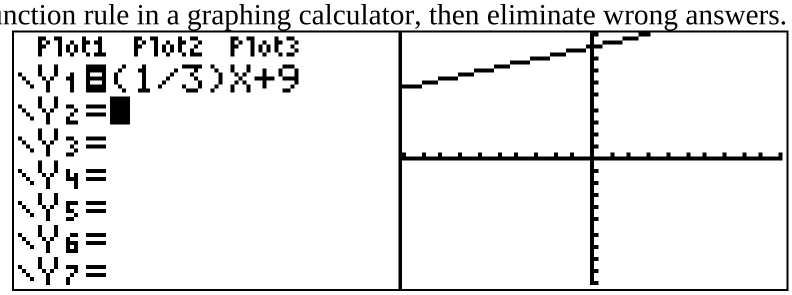
Strategy: Part b: The employee must work more than 40 hours, and compensation for hours worked in excess of 40 hours is found in the second formula and is equal to \$15 per hour. The compensation worked in excess of 40 hours is $\$445 - \$400 = \$45$, so

$$\frac{45 \text{ dollars}}{15 \text{ dollars/hour}} = 3 \text{ hours}$$

The employee must work a total of 43 hours. The employee receives \$400 for the first 40 hours and \$45 for the 3 hours in excess of 40 hours.

PTS: 4 REF: 061534ai NAT: F.IF.A.2 TOP: Functional Notation

2. ANS: D
Strategy: Inspect the function rule in a graphing calculator, then eliminate wrong answers.



X	Y1	
-3	8	
-2	8.3333	
-1	8.6667	
0	9	
1	9.3333	
2	9.6667	
3	10	
X=0		

X	Y1	
-30	-1	
-29	-.6667	
-28	-.3333	
-27	0	
-26	.33333	
-25	.66667	
-24	1	
X= -27		

Answer choice *a* can be eliminated because the table clearly shows $f(x)$ values greater than zero.
Answer choice *b* can be eliminated because the table clearly shows $f(x)$ values less than zero.
Answer choice *c* can be eliminated because if x is greater than -27, then $f(x) > 0$.
Choose answer choice *d* because the graph and table clearly show that all values of $f(x)$ are positive when values of x are positive.

PTS: 2 REF: 061417ai NAT: F.IF.A.2 TOP: Domain and Range

3. ANS: A

Strategy: Understand that the meaning of $f(2)$ is the value of y when $x = 2$, then eliminate wrong answers.

Choose answer choice A because represents $f(2)$ with coordinates $(2, 0)$. $f(2) = 0$.
Answer choice b is wrong because if represents $f(0)$. $f(0) = 2$
Answer choice c is wrong because if represents $f(-2)$. $f(-2) = 0$
Answer choice d is wrong because if represents $f(-1)$. $f(-1) = -2$

PTS: 2 REF: 061420ai NAT: F.IF.A.2 TOP: Functional Notation

4. ANS: C

Strategy: Substitute $\frac{1}{2}$ for x, and solve.

$$f(x) = \frac{\sqrt{2x+3}}{6x-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{2\left(\frac{1}{2}\right)+3}}{6\left(\frac{1}{2}\right)-5}$$

$$f\left(\frac{1}{2}\right) = \frac{\sqrt{4}}{-2}$$

$$f\left(\frac{1}{2}\right) = \frac{2}{-2}$$

$$f\left(\frac{1}{2}\right) = -1$$

PTS: 2 REF: 081512ai NAT: F.IF.A.2 TOP: Functional Notation

5. ANS:

Lesson Plan

Step 1. Understand this as a composition of functions problem.
Step 2. Strategy: Substitute the expression for f(x) into the equation for g(x).
Step 3. Execution of Strategy.

$$f(x) = 2x + 1 \text{ and } g(x) = 2[f(x)]^2 - 1$$

$$g(x) = 2(2x + 1)^2 - 1 \quad (answer)$$

$$g(x) = 2(4x^2 + 4x + 1) - 1 \quad (alternate\ answer)$$

$$g(x) = 8x^2 + 8x + 2 - 1 \quad (alternate\ answer)$$

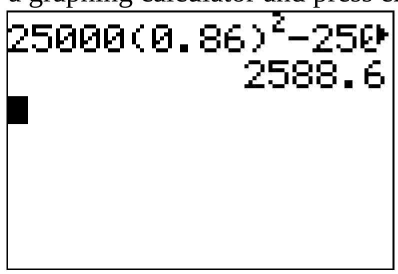
$$g(x) = 8x^2 + 8x + 1 \quad (alternate\ answer)$$

PTS: 2 REF: 061625ai NAT: F.IF.A.2 TOP: Functional Notation

6. ANS: A

Strategy #1

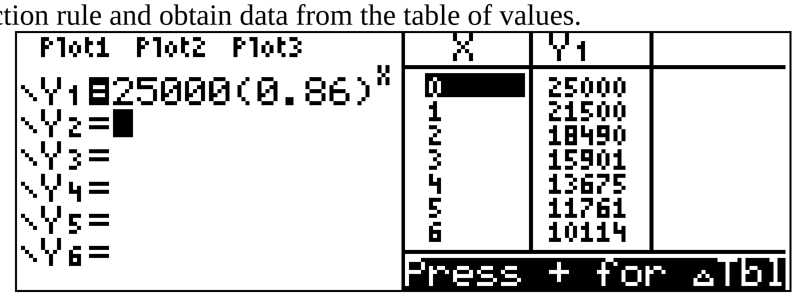
Input $25,000(0.86)^2 - 25,000(0.86)^3$ into a graphing calculator and press enter.



$$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$$

Strategy #2: Input the function rule in a graphing calculator and obtain the value of the car after 2 years and 3 years from the table of values. Then, compute the difference.

STEP 1: Input the function rule and obtain data from the table of values.



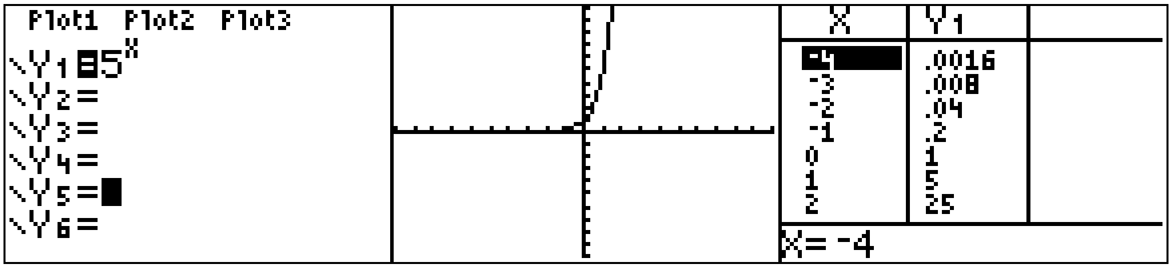
STEP 2: Compare the value of the car after 2 years and after 3 years.
The car is worth $18,490 after 2 years.
The car is worth $15,901 after 3 years.
The difference is $18490 - 15901 = 2589$

$25,000(0.86)^2 - 25,000(0.86)^3 = 18490 - 15901.40 = 2588.60$

PTS: 2 REF: 011508ai NAT: F.IF.A.2 TOP: Evaluating Exponential Expressions

7. ANS: B

Strategy: Input the function in a graphing calculator and inspect the graph and table views..



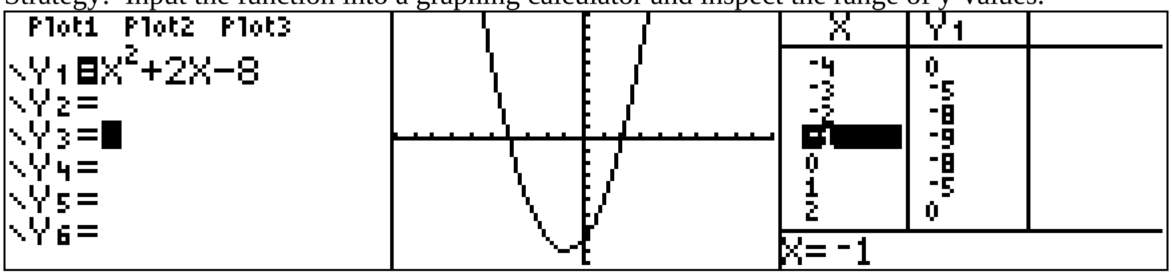
The value of y approaches zero, but never reachers zero, as the value of x decreases.

The the range of $y = 5^x$ is $y > 0$.

PTS: 2 REF: 011619ai NAT: F.IF.B.5 TOP: Domain and Range
KEY: real domain, exponential

8. ANS: B

Strategy: Input the function into a graphing calculator and inspect the range of y-values.



The graph and the table of values show that all values of f(x) are greater than or equal to -9. Choice b) is the correct answer.

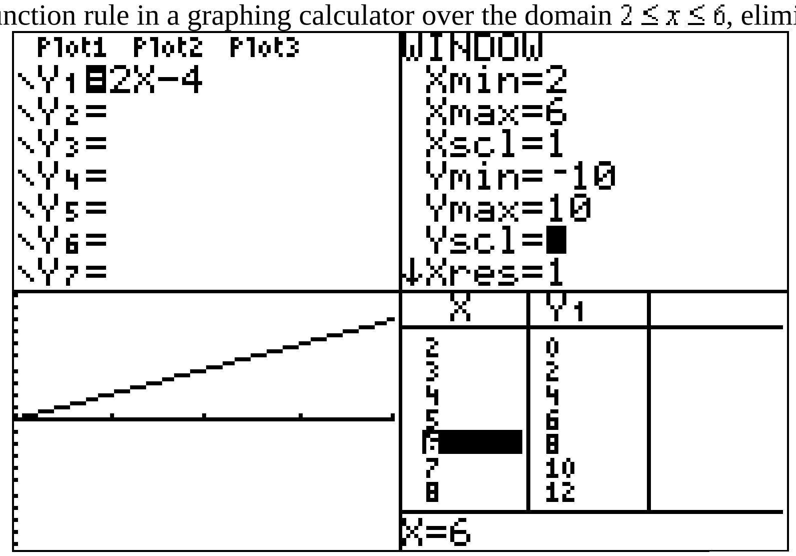
PTS: 2 REF: 061611ai NAT: F.IF.B.5 TOP: Domain and Range
KEY: real domain, quadratic

9. ANS: A

$f(2) = 0$

$f(6) = 8$

Strategy: Inspect the function rule in a graphing calculator over the domain $2 \le x \le 6$, eliminate wrong answers.



Choose answer choice a because the table of values and the graph clearly show that $f(2) = 0$ and $f(6) = 8$, and all values of y between $x = 2$ and $x = 6$ are between 0 and 8.

Eliminate answer choice *b* because infinity is clearly bigger than 8.

Lesson Plan

Eliminate answer choice *c* because these are the domain of x, not the range of y.
Eliminate answer choice *d* because negative infinity is clearly less than 0.

PTS: 2 REF: 081411ai NAT: F.IF.B.5 TOP: Domain and Range

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is \$350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.