F.IF.A.3: Define Sequences as Functions

FUNCTIONS
F.IF.A.3: Define Sequences as Recursive Functions

A. Understand the concept of a function and use function notation.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

An **explicit formula** is one where you do not need to know the value of the term in front of the term that you are seeking. For example, if you want to know the 55th term in a series, an explicit formula could be used without knowing the value of the 54th term.
Example: The sequence 3, 11, 19, 27, ... begins with 3, and 8 is added each time to form the pattern. The sequence can be shown in a table as follows:

Term # (n)	1	2	3	4
$f(n)$	3	11	19	27

Explicit formulas for the sequence 3, 11, 19, 27, ... can be written as:
$f(n) = 8n - 5$ or $f(n) = 3 + 8(n - 1)$
Using these **explicit formulas**, we can find the following values for any term, and we do not need to know the value of any other term, as shown below:

$f(n) = 8n - 5$	$f(n) = 3 + 8(n - 1)$
$f(1) = 8(1) - 5 = 3$	$f(1) = 3 + 8(1 - 1) = 3 + 0 = 3$
$f(2) = 8(2) - 5 = 16 - 5 = 11$	$f(2) = 3 + 8(2 - 1) = 3 + 8 = 11$
$f(3) = 8(3) - 5 = 24 - 5 = 19$	$f(3) = 3 + 8(3 - 1) = 3 + 16 = 19$
$f(4) = 8(4) - 5 = 32 - 5 = 27$	$f(4) = 3 + 8(4 - 1) = 3 + 24 = 27$
$f(5) = 8(5) - 5 = 40 - 5 = 35$	$f(5) = 3 + 8(5 - 1) = 3 + 32 = 35$
$f(10) = 8(10) - 5 = 80 - 5 = 75$	$f(10) = 3 + 8(10 - 1) = 3 + 72 = 75$
$f(100) = 8(100) - 5 = 800 - 5 = 795$	$f(100) = 3 + 8(100 - 1) = 3 + 792 = 795$

Recursive formulas requires you to know the value of another term, usually the preceding term, to find the value of a specific term.

Example: Using the same sequence 3, 11, 19, 27, ... as above, a **recursive formula** for the sequence 3, 11, 19, 27, ... can be written as:

$$f(n+1) = f(n) + 8$$

This **recursive formula** tells us that the value of any term in the sequence is equal to the value of the term before it plus 8. A recursive formula must usually be anchored to a specific term in the sequence (usually the first term), so the recursive formula for the sequence 3, 11, 19, 27, ... could be anchored with the statement

$$f(1) = 3$$

Using this **recursive formula**, we can reconstruct the sequence as follows:

$$f(1) = 3$$

$$f(2) = f(1) + 8 = 3 + 8 = 11$$

$$f(3) = f(2) + 8 = 11 + 8 = 19$$

$$f(4) = f(3) + 8 = 19 + 8 = 27$$

$$f(5) = f(4) + 8 = 27 + 8 = 35$$

$$f(10) = f(9) + 8 = ? + 8 = ?$$

Observe that the recursive formula $f(n+1) = f(n) + 8$ includes two different values of the dependent variable, which in this example are $f(n)$ and $f(n+1)$, and we can only reconstruct our original sequence using this **recursive formula** if we know the term immediately preceding the term we are seeking.

Two Kinds of Sequences

arithmetic sequence *(A2T)* A set of numbers in which the common difference between each term and the preceding term is constant.

Example: In the **arithmetic sequence** 2, 5, 8, 11, 14, … the common difference between each term and the preceding term is 3. A table of values for this sequence is:

Term # (*n*)	1	2	3	4	5
$f(n)$	2	5	8	11	14

An **explicit formula** for this sequence is $f(n) = 3n - 1$

A **recursive formula** for this sequence is: $f(n+1) = f(n) + 3, \quad f(1) = 2$

geometric sequence *(A2T)* A set of terms in which each term is formed by multiplying the preceding term by a common nonzero constant.

Example: In the geometric sequence 2, 4, 8, 16, 32... the common ratio is 2. Each term is 2 times the preceding term. A table of values for this sequence is:

Term (*n*)	1	2	3	4	5
$f(n)$	2	4	8	16	32

An **explicit formula** for this sequence is $f(n) = 2^n$

A **recursive formula** for this sequence is: $f(n+1) = 2f(n), \quad f(1) = 2$

REGENTS PROBLEM TYPICAL OF THIS STANDARD

1. If $f(1) = 3$ and $f(n) = -2f(n-1) + 1$, then $f(5) =$

a. -5	c. 21
b. 11	d. 43

Lesson Plan

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Answer Section

1. ANS: D
Strategy: Use the recursive formula: $f(1) = 3$ and $f(n) = -2f(n-1) + 1$ to find each term in the sequence.
$f(1) = 3$
$f(n) = -2f(n-1) + 1$
$f(2) = -2f(2-1) + 1 = -2f(1) + 1 = -2(3) + 1 = -6 + 1 = -5$
$f(3) = -2f(3-1) + 1 = -2f(2) + 1 = -2(-5) + 1 = 10 + 1 = 11$
$f(4) = -2f(4-1) + 1 = -2f(3) + 1 = -2(11) + 1 = -22 + 1 = -21$
$f(5) = -2f(5-1) + 1 = -2f(4) + 1 = -2(-21) + 1 = 42 + 1 = 43$

PTS: 2 REF: 081424ai NAT: F.IF.A.3 TOP: Sequences

Homework - Write the Math Assignment

START	Write your name, date, topic of lesson, and class on your paper.	
	NAME:	Mohammed Chen
	DATE:	December 18, 2015
	LESSON:	Missing Number in the Average
	CLASS:	Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. **The Problem**

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. **What is the problem asking?**

Find the salary of the fifth employee.

Part 1c. **Answer**

The salary of the fifth employee is $350 per week.

Part 1d. **Explanation of Strategy**

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. **A New Problem**

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b **What is the new problem asking?**

Find Joseph's score on the missing exam.

Part 2c. **Answer to New Problem**

Joseph received a score of 85 on the missing examination.

Part 2d. **Explanation of Strategy**

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.