F.IF.B.5: Use Sensible Domains and Ranges

FUNCTIONS
F.IF.B.5: Use Sensible Domains and Ranges

B. Interpret functions that arise in applications in terms of the context.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function y (linear, exponential and quadratic).

Overview of Lesson
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
	Selected problem set(s)
- facilitate a summary and share out of student work
	Homework – Write the Math Assignment

Vocabulary

The **domain of x** and the **range of y**.

The coordinate plane consists of two perpendicular number lines, which are commonly referred to as the x-axis and the y-axis. Each number line represents **the set of real numbers**.

The Set of Real Numbers
§ **Counting numbers** {1,2,3,...}
§ **Whole numbers** {0,1,2,3,...}
§ **Integers** are whole numbers and their opposites {...-3, -2, -1, 0, 1, 2, 3, ...}.
§ **Rational numbers** (all number that can be expressed as a ratio of two integers)
	Rational begins with the word ratio. A ratio is a comparison of two numbers using division.
		§ A ratio can be expressed as a fraction.
		§ All fractions are rational numbers.
		§ All repeating or terminating decimals.
§ **Irrational numbers** (all numbers that cannot be expressed as ratios of integers)
		§ Never ending, never repeating decimals, such as π, e, and the square roots of all prime numbers.

Big Ideas

A functions maps an element of the **domain** onto one and only one element of the **range**.

Many functions make sense only when a subset of all the **Real Numbers** are used as inputs. This subset of the **Real Numbers** that makes sense is known as the **domain** of the function.
Example: If a vendor makes $2.00 profit on each sandwich sold, total profits might be modeled by the function $P(s) = 2s$, where $P(s)$ represents total profits and s represents the number of sandwiches sold. It would not make sense to use the entire set of real numbers as inputs for this function.
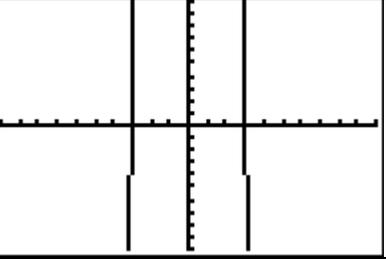
- It **would not make sense** to say that the vendor sold -3 sandwiches or to use any other negative numbers.

- It **would not make sense** to say the vendor sold π, sandwiches, e sandwiches, or $\sqrt{7}$ sandwiches.

- It **would make sense** to say that the vendor sold 0, 1, 2, or any whole number of sandwiches.

Thus, the **domain** of $P(s) = 2s$ can be restricted to a subset of the Real Number system, which can be described as either the set of **whole numbers** or by listing the set {0,1,2,3,...}. The **range** of a function can also be limited to a well-defined subset of the Real Numbers on the y-axis.

Domains and **ranges** can be either **continuous** or **discrete**.

NOTE: The **window function on a graphing calculator** allows us to set specific **continuous** intervals for the domain and range of the graph of a function.

These screenshots show **inappropriate domain and range** settings for the first Regents Problem in this lesson.	WINDOW Xmin=-10 Xmax=10 Xscl=1 Ymin=-10 Ymax=10 Yscl=1 ↓Xres=1	
These screenshots show **proper domain and range** settings for the first Regents Problem in this lesson.	WINDOW Xmin=0 Xmax=3 Xscl=1 Ymin=0 Ymax=144 Yscl=1 ↓Xres=1	
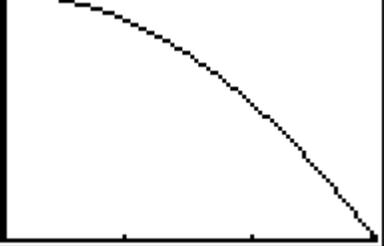
REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. The function $h(t) = -16t^2 + 144$ represents the height, $h(t)$, in feet, of an object from the ground at t seconds after it is dropped. A realistic domain for this function is
    a. $-3 \le t \le 3$
    c. $0 \le h(t) \le 144$
    b. $0 \le t \le 3$
    d. all real numbers

2. Officials in a town use a function, C, to analyze traffic patterns. $C(n)$ represents the rate of traffic through an intersection where n is the number of observed vehicles in a specified time interval. What would be the most appropriate domain for the function?
    a. $\{\ldots - 2, -1, 0, 1, 2, 3, \ldots\}$
    c. $\{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$
    b. $\{-2, -1, 0, 1, 2, 3\}$
   d. $\{0, 1, 2, 3, \ldots\}$

3. A construction company uses the function $f(p)$, where p is the number of people working on a project, to model the amount of money it spends to complete a project. A reasonable domain for this function would be
    a. positive integers
    c. both positive and negative integers
    b. positive real numbers
    d. both positive and negative real numbers

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Answer Section

1. ANS: B
Strategy: Input the function into a graphing calculator and examine it to determine a realistic range. First, transform $h(t) = -16t^2 + 144$ to $Y_1 = -16x^2 + 144$ for input.
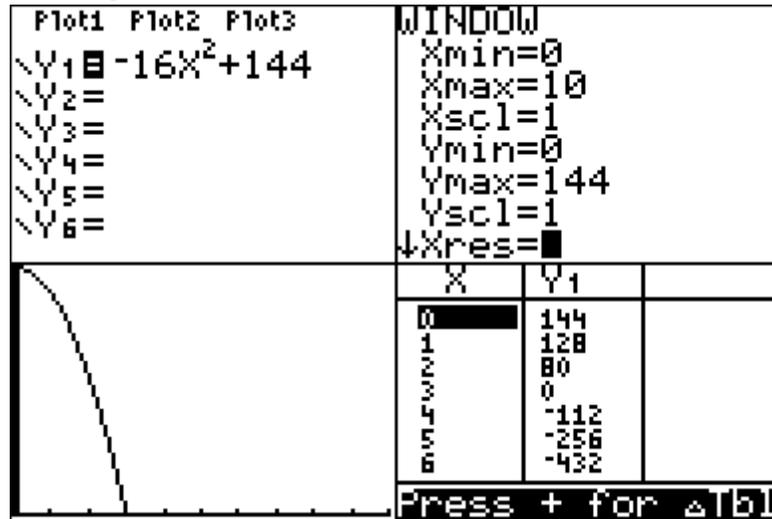
The graph and table of values show that it takes 3 seconds for the object to reach the ground. Therefore, a realistic domain for this function is $0 \le t \le 3$.
$t = 0$ represents the time when the object is dropped.
$t = 3$ represents the time when the object hits the ground.
Answer choice *b* is correct.

PTS: 2   REF: 081423ai   NAT: F.IF.B.5   TOP: Domain and Range

2. ANS: D
Strategy: Examine each answer choice and eliminate wrong answers.

Eliminate answer choices *a* and *b* because *negative numbers* of cars observed do not make sense.
Eliminate answer choice *c* because *fractional numbers* of cars observed do not make sense.
Choose answer choice *d* because it is the only choice that makes sense. The number of cars observed must be either zero or some counting number.

PTS: 2   REF: 061402ai   NAT: F.IF.B.5   TOP: Domain and Range

3. ANS: A
Strategy: Eliminate wrong answers. The number of people must be counting numbers, since it makes no sense to have a half a person or a quarter person.
The **positive integers** are 1, 2, 3, 4, ..., which makes sense.
Positive real numbers should be eliminated because positive real numbers include fractions, and fractions make no sense for the number of workers.
Both positive and negative integers should be eliminated because it makes no sense to have negative numbers of workers.
Both positive and negative real numbers should also be eliminated because it makes no sense to have negative numbers of workers.
The correct choice is **positive integers**.
PTS: 2   REF: 011615ai   NAT: F.IF.B.5   TOP: Domain and Range

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric
Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:
1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a.   The Problem
TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b.   What is the problem asking?
Find the salary of the fifth employee.

Part 1c.   Answer
The salary of the fifth employee is $350 per week.

Part 1d.   Explanation of Strategy
The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + ... + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \dfrac{340 + 340 + 345 + 425 + 350}{5} = \dfrac{1800}{5} = 360$

Part 2a.   A New Problem
Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b   What is the new problem asking?
Find Joseph's score on the missing exam.

Part 2c.   Answer to New Problem
Joseph received a score of 85 on the missing examination.

Part 2d.   Explanation of Strategy
I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.