

F.IF.B.6: Calculate and Interpret Rate of Change

EQUATIONS AND INEQUALITIES

F.IF.B.6: Calculate and Interpret Average Rate of Change

B. Interpret functions that arise in applications in terms of the context.

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

[Selected problem set\(s\)](#)

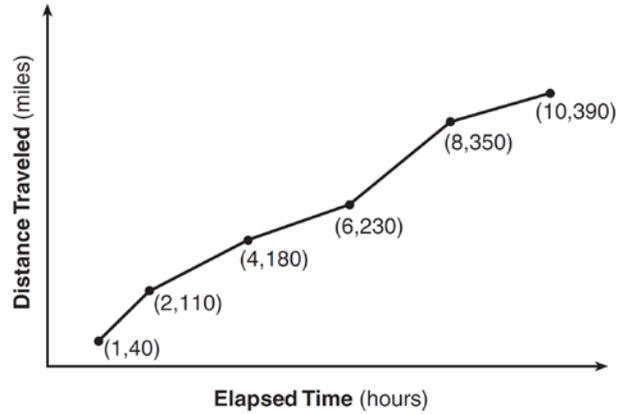
- facilitate a summary and share out of student work

Homework – Write the Math Assignment

BIG IDEA

Average Rate of Change is the *slope* of the straight line that connects the end points of the interval over which it is measured.

Example: The graph below shows how far a family travelled on a trip.



The slopes of the different line segments indicate they travelled at different speeds during the trip. Speed is a rate of change. Instantaneous speed is how fast they were travelling at any point in time, and is measured by the car's speedometer. To compute their average speed, or (average rate of change) for the entire trip, use the end points of the interval over which they travelled. They started at the origin (0,0) and they ended their trip ten hours and 390 miles later, expressed on the graph as (10,390). Their average rate of change can be calculated using the slope formula, as follows:

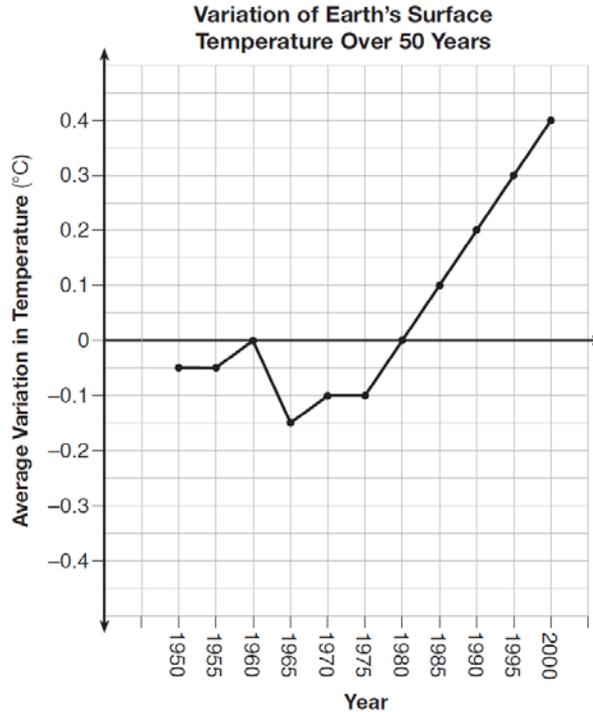
$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{average rate of change} = \frac{390 - 0}{10 - 0}$$

$$\text{average speed} = 39$$

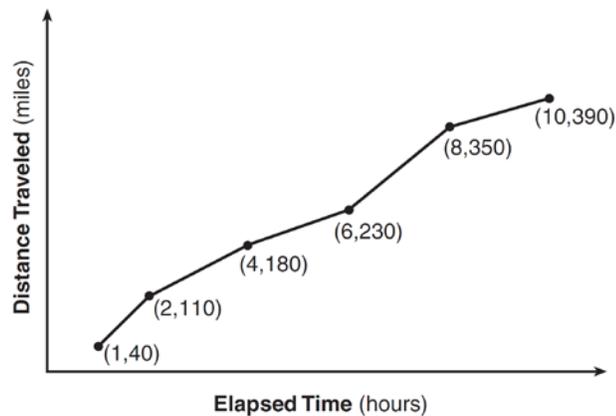
REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. The graph below shows the variation in the average temperature of Earth's surface from 1950-2000, according to one source.



During which years did the temperature variation change the most per unit time? Explain how you determined your answer.

2. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.



During which interval was their average speed the greatest?

- a. the first hour to the second hour
- b. the second hour to the fourth hour
- c. the sixth hour to the eighth hour
- d. the eighth hour to the tenth hour

3. Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the nearest tenth?

- a. 4.3
- b. 4.5
- c. 5.4
- d. 6.0

4. The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

- a. 1898-1971
- b. 1971-1985
- c. 1985-2006
- d. 2006-2012

5. The table below shows the average diameter of a pupil in a person’s eye as he or she grows older.

Age (years)	Average Pupil Diameter (mm)
20	4.7
30	4.3
40	3.9
50	3.5
60	3.1
70	2.7
80	2.3

What is the average rate of change, in millimeters per year, of a person’s pupil diameter from age 20 to age 80?

- a. 2.4
- b. 0.04
- c. -2.4
- d. -0.04

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Answer Section

1. ANS:

During 1960-1965, because the graph has the steepest slope during these years.

PTS: 2 REF: 011628ai NAT: F.IF.B.6 TOP: Rate of Change

2. ANS: A

Strategy: Equate speed with rate of change. $speed = \frac{\Delta y}{\Delta x} = \frac{rise}{run} = slope = rate\ of\ change$

Make a visual estimate of the steepest line segment on the graph, then use the slope formula to calculate the exact rates of change.

STEP 1. The line segment from (1, 40) to (2, 110) appears to be the steepest line segment in the graph. The line segment from (6, 230) to (8, 350) also seems very steep.

STEP 2. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$

The line segment from (1, 40) to (2, 110) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{110 - 40}{2 - 1} = \frac{70\ miles}{1\ hour}$.

The line segment from (6, 230) to (8, 350) has $slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350 - 230}{8 - 6} = \frac{120}{2} = \frac{60\ miles}{1\ hour}$.

PTS: 2 REF: 061418ai NAT: F.IF.B.6 TOP: Rate of Change

3. ANS: C

Strategy: Use the slope formula and data from the table to calculate the exact rate of change over four enlargements.

STEP 1. Use $slope = \frac{y_2 - y_1}{x_2 - x_1}$ to compute the rate of change between (0, 15) and (4, 36.6).

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{36.6 - 15}{4 - 0} = \frac{21.6}{4} = 5.4.$$

DIMS? Does it make sense? Yes. If you start with 15 and add $5.4 + 5.4 + 5.4 + 5.4$, you end up with 36.6. There were four enlargements and the average increase of each enlargement was 5.4 square inches.

PTS: 2 REF: 061511ai NAT: F.IF.B.6 TOP: Rate of Change

4. ANS: D

Strategy: Find the average rate of change using the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(a) $\frac{6 - 1}{1971 - 1898} = \frac{5}{73} \approx .07$

(b) $\frac{14 - 6}{1985 - 1971} = \frac{8}{14} \approx .57$

$$(c) \frac{24 - 14}{2006 - 1985} = \frac{10}{21} \approx 48$$

$$(d) \frac{35 - 24}{2012 - 2006} = \frac{11}{6} \approx 1.83$$

PTS: 2 REF: 011613ai NAT: F.IF.B.6 TOP: Rate of Change

5. ANS: D

Strategy: Rate of change is the same as slope. Use the slope formula to find the rate of change between (20, 4.7) and (80, 2.3).

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.3 - 4.7}{80 - 20} = \frac{-2.4}{60} = -0.04.$$

DIMS? Does it make sense? Yes. The average pupil diameter gets smaller very very slowly. Choices a and c are way too big and choices a and b indicate that the average pupil size is getting bigger rather than smaller.

PTS: 2 REF: 081414ai NAT: F.IF.B.6 TOP: Rate of Change

6. ANS: A

Strategy: Use the formula for speed: $\text{speed} = \frac{\text{distance}}{\text{time}}$ and information from the problem to calculate average speed.

STEP 1. Calculate $d(t)$ for $t = 5$ and $t = 10$.

$$d(t) = 0.8t^2 \quad \text{and} \quad d(t) = 0.8t^2$$

$$d(5) = 0.8(5)^2 \quad d(10) = 0.8(10)^2$$

$$d(5) = 0.8(25) \quad d(10) = 0.8(100)$$

$$d(5) = 20 \quad d(10) = 80$$

The rock had fallen 20 meters after 5 seconds and 80 meters after 10 seconds.

The total distance traveled was 60 meters in 5 seconds.

STEP 2: Use the speed formula to find average speed.

$$\text{Substituting distance and time in the speed formula, } \text{speed} = \frac{\text{distance}}{\text{time}} = \frac{60 \text{ meters}}{5 \text{ seconds}} = \frac{12 \text{ meters}}{1 \text{ second}}.$$

The rock's average speed between 5 and 10 seconds after being dropped was 12 meters per second.

DIMS? Does it make sense? Yes. The speed formula makes sense and the answer is expressed in meters per second as required by the problem.

PTS: 2 REF: 011521ai NAT: F.IF.B.6 TOP: Rate of Change

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.