

F.IF.C.8: Identify Characteristics of Quadratics

POLYNOMIALS AND QUADRATICS

F.IF.C.8: Identify Characteristics of Parabolas by Completing the Square

C. Analyze functions using different representations.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

[Selected problem set\(s\)](#)

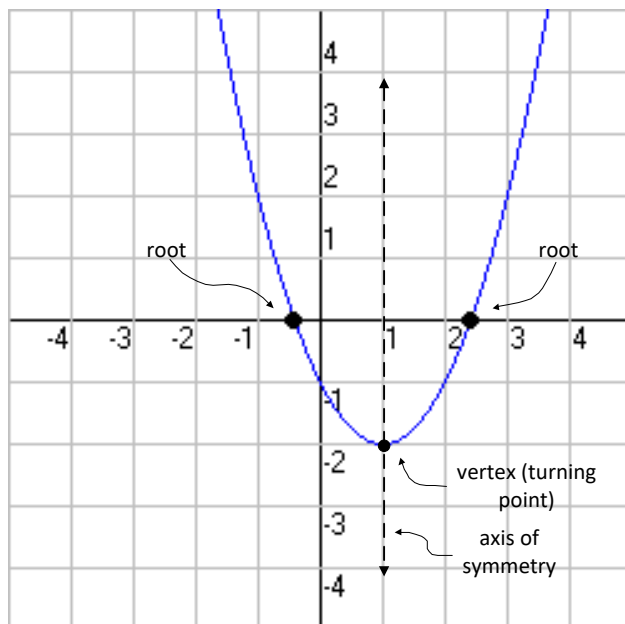
- facilitate a summary and share out of student work

Homework – Write the Math Assignment

BIG IDEAS

Completing the Square is an efficient method to find the zeros, vertex, and axis of symmetry of a parabola.

The graph of a quadratic equation is called a **parabola**.



- The vertex form of a quadratic function is given by $a(x - h)^2 + k = 0$, where (h, k) is the vertex of the parabola and $x = h$ is the axis of symmetry.
- The x-value of the point where a parabola touches the x-axis is called:
 - Root
 - Zero
 - Solution
 - X-axis intercept
- Completing the square can be used to find the zeros of a quadratic.

EXEMPLAR OF FINDING THE CHARACTERISTICS OF A PARABOLA	
Find the axis of symmetry, vertex, and zeros of $4x^2 - 12x = 7$	
STEP 1 $4x^2 - 12x = 7$ $x^2 - 3x = \frac{7}{4}$	
STEP 2 $x^2 - 3x = \frac{7}{4}$ $x^2 - 3x + \left(-\frac{3}{2}\right)^2 = \frac{7}{4} + \frac{9}{4}$ $x^2 - 3x + \left(-\frac{3}{2}\right)^2 = \frac{16}{4}$	
STEP 3 $\left(x - \frac{3}{2}\right)^2 = \frac{16}{4}$	
STEP 4a (solving for zeros) $\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{16}{4}}$ $x - \frac{3}{2} = \pm 2$ $x = \frac{3}{2} \pm 2$ $x = \left\{-\frac{1}{2} \text{ and } 3\frac{1}{2}\right\}$ Solutions are $-\frac{1}{2}$ and $3\frac{1}{2}$	STEP 4b (solving for axis of symmetry & extreme) $4\left(x - \frac{3}{2}\right)^2 = 4\left(\frac{16}{4}\right)$ $4\left(x - \frac{3}{2}\right)^2 = 16$ $4\left(x - \frac{3}{2}\right)^2 - 16 = 0$ axis of symmetry is $+\frac{3}{2}$ vertex is $\left(\frac{3}{2}, -16\right)$

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1.
 - a) Given the function $f(x) = -x^2 + 8x + 9$, state whether the vertex represents a maximum or minimum point for the function. Explain your answer.
 - b) Rewrite $f(x)$ in vertex form by completing the square.

2. Which equation and ordered pair represent the correct vertex form and vertex for $j(x) = x^2 - 12x + 7$?
 - a. $j(x) = (x - 6)^2 + 43$, $(6, 43)$
 - b. $j(x) = (x - 6)^2 + 43$, $(-6, 43)$
 - c. $j(x) = (x - 6)^2 - 29$, $(6, -29)$
 - d. $j(x) = (x - 6)^2 - 29$, $(-6, -29)$

F.IF.C.8: Identify Characteristics of Quadratics
Answer Section

1. ANS:
 a) The vertex represents a maximum since $a < 0$.
 b) $f(x) = -(x - 4)^2 + 25$

$$\left. \begin{aligned} f(x) &= -x^2 + 8x + 9 \\ -x^2 + 8x + 9 &= 0 \end{aligned} \right\} \text{(set } f(x) \text{ to 0)}$$

$$\left. \begin{aligned} -x^2 + 8x &= -9 \\ \frac{-x^2}{-1} + \frac{8x}{-1} &= \frac{-9}{-1} \\ x^2 - 8x &= 9 \end{aligned} \right\} \text{(isolate both variables with 1 as coefficient of leading variable)}$$

$$\left. \begin{aligned} x^2 - 8x + (-4)^2 &= 9 + (-4)^2 \\ (x - 4)^2 &= 9 + 16 \\ (x - 4)^2 &= 25 \end{aligned} \right\} \text{(complete the square)}$$

$$\left. \begin{aligned} -1(x - 4)^2 &= -1(25) \\ -1(x - 4)^2 + 25 &= 0 \end{aligned} \right\} \text{(multiply by a)}$$

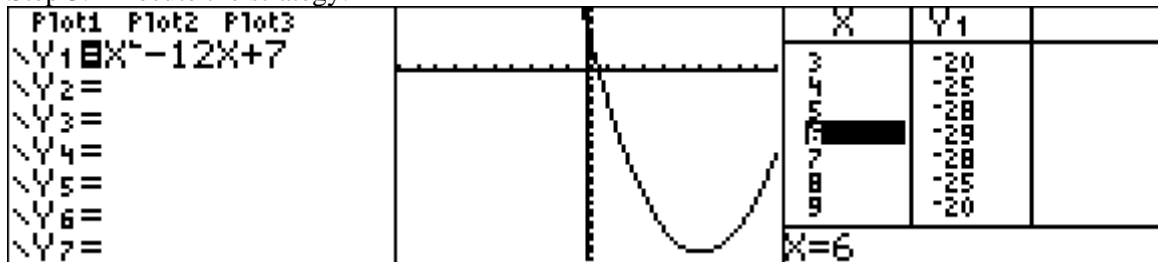
The vertex is at (4,25), but this information is not required by the problem.

PTS: 4 REF: 011536ai NAT: F.IF.C.8 TOP: Graphing Quadratic Functions

2. ANS: C
 Step 1. Understand from the answer choices that the problem wants us to choose the answer that is equivalent to $j(x) = x^2 - 12x + 7$.

Step 2. Strategy: Input $j(x) = x^2 - 12x + 7$ in a graphing calculator and inspect the table and graph views of the function, then eliminate wrong answers.

Step 3. Execute the strategy.



Choice c) is correct because it is the only answer choice that shows the vertex at (6, -29).

Lesson Plan

Step 4. Does it make sense? Yes. You can see that $j(x) = x^2 - 12x + 7$ and $j(x) = (x - 6)^2 - 29$, $(6, -29)$ are the same function by inputting both in a graphing calculator.

Plot1 Plot2 Plot3	X	Y1	Y2
\Y1 = X ² - 12X + 7	2	-20	-20
\Y2 = (X - 6) ² - 29	4	-25	-25
\Y3 =	5	-28	-28
\Y4 =	6	-29	-29
\Y5 =	7	-28	-28
\Y6 =	8	-25	-25
	9	-20	-20
Press + for Δ b			

PTS: 2

REF: 061616ai

NAT: F.IF.C.8

TOP: Vertex Form of a Quadratic

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.

PART 1b. State your understanding of **what the problem is asking**.

PART 1c. **Answer** the problem.

PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.

PART 2b. State your understanding of **what the new problem is asking**.

PART 2c. **Answer** the new problem.

PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.