Lesson Plan

F.IF.C.9: Four Views of a Function

FUNCTIONS
F.IF.C.9: Four Views of a Function

A. Analyze functions using different representations.

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Overview of Lesson

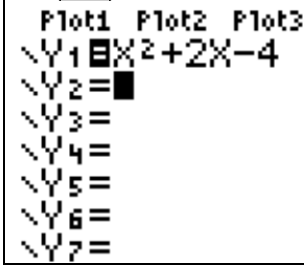
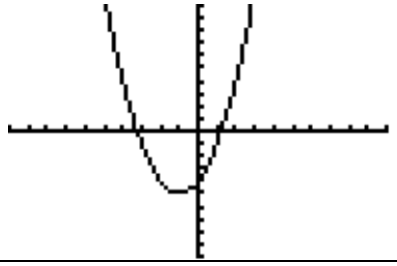
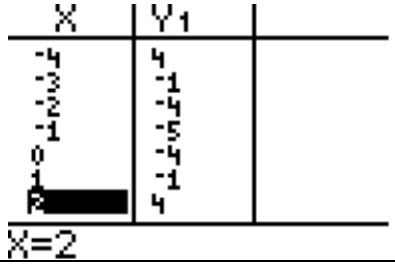
- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

BIG IDEAS:

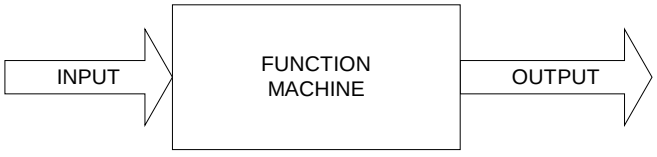
A function can be represented mathematically through four inter-related views. These are:

#1 a function rule (equation)
#2 a table of values
#3 a graph.
#4 context (words)

The TI-83+ graphing calculator allows you to input the function rule and access the graph and table of values, as shown below:

Y= Function Rule View	GRAPH View	2*nd* TABLE View
		

Definition of a Function: a function takes the input value of an independent variable and pairs it with one and only one output value of a dependent variable.



Function: A function is a relation that assigns exactly one value of the dependent variable to each value of the independent variable. A function is always a relation.

Example: y=2x

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Function Rules show the relationship between dependent and independent variables in the form of an equation with two variables.

- The **independent** variable is the **input** of the function and is typically denoted by the x-variable.
- The **dependent** variable is the **output** of the function and is typically denoted by the y-variable.

When inputting function rules in a TI 83+ graphing calculator, the y-value (dependent variable) must be isolated as the left expression of the equation.

Tables of Values show the relationship between dependent and independent variables in the form of a table with columns and rows:

- The **independent** variable is the **input** of the function and is typically shown in the left column of a vertical table or the top row of a horizontal table.
- The **dependent** variable is the **output** of the function and is typically shown in the right column of a vertical table or the bottom row of a horizontal table.

Graphs show the relationship between dependent and independent variables in the form of line or curve on a coordinate plane:

- The value of **independent** variable is the **input** of the function and is typically shown on the **x-axis** (horizontal axis) of the coordinate plane.
- The value of the **dependent** variable is the **output** of the function and is typically shown on the **y-axis** (vertical axis) of the coordinate plane.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

and

x	–3	–2	–1	0	1	2	3	4	5
n(x)	–7	0	5	8	9	8	5	0	–7

Which statement about these functions is true?

a. Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.

b. The *y*-intercept of $g(x)$ is greater than the *y*-intercept for $n(x)$.

c. The function $g(x)$ has a greater maximum value than $n(x)$.

d. The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

2. Which quadratic function has the largest maximum?

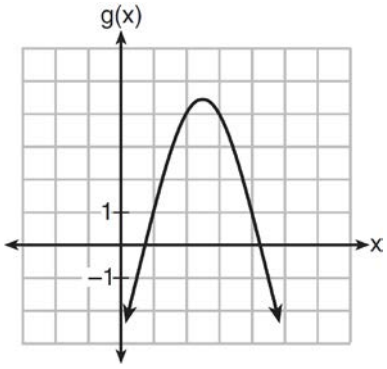
a. $h(x) = (3 - x)(2 + x)$

b.

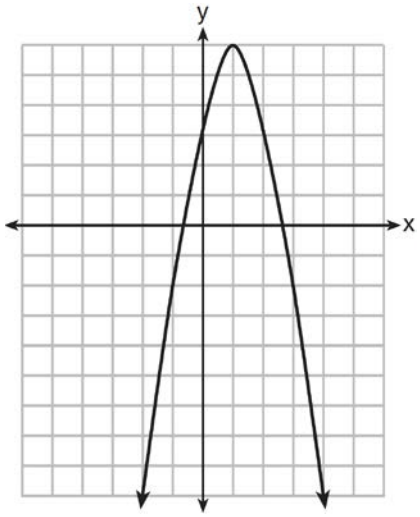
x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

c. $k(x) = -5x^2 - 12x + 4$

d.



3. Let f be the function represented by the graph below.



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

4. The graph representing a function is shown below.

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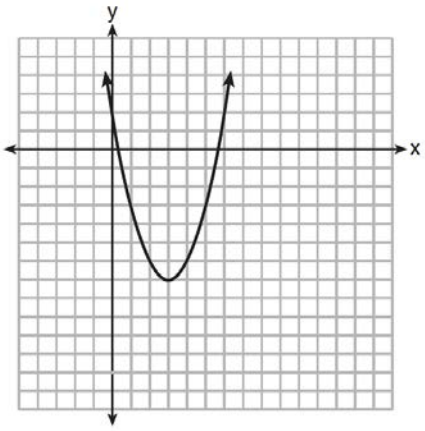


Which function has a minimum that is *less* than the one shown in the graph?

a. $y = x^2 - 6x + 7$

b. $y = |x + 3| - 6$

c. $y = x^2 - 2x - 10$

d. $y = |x - 8| + 2$

F.IF.C.9: Four Views of a Function
Answer Section

1. ANS: D

Strategy: Each answer choice must be evaluated using a different strategy.

a. Use the slope formula to find the rate of change for

$$m_{g(x)} = \frac{[g(1)] - [g(-1)]}{[1] - [-1]} = \frac{4-6}{2} = \frac{-2}{2} = -1$$

$$m_{n(x)} = \frac{[n(1)] - [n(-1)]}{[1] - [-1]} = \frac{9-5}{2} = \frac{4}{2} = 2$$

Statement a is false. The average rate of change for $n(x)$ is *more* than that for $g(x)$.

b. Compare the y-intercepts for both functions. The y-intercepts occur when $x = 0$.

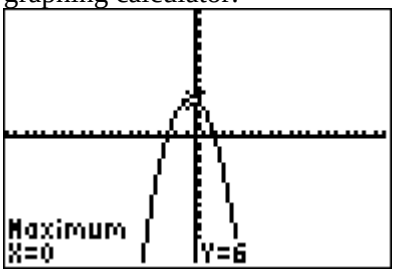
The y-intercept for g(x) = 6. $g(0) = -0^2 - 0 + 6 = 6$

The y-intercept for n(x) = 8 from the table.

Statement b is false. The *y*-intercept of $g(x)$ is *less* than the *y*-intercept for $n(x)$.

c. Compare the maxima of both functions.

The maxima of $g(x) = -x^2 - x + 6$ is 6. This can be found manually or with a graphing calculator.

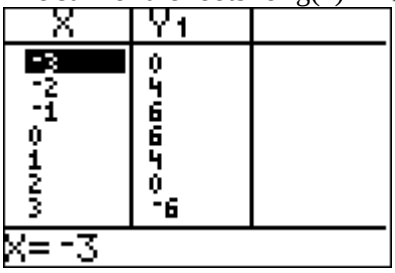


The maxima of n(x) = 9, which can be seen in the table.

Statement c is false. The function $g(x)$ has a *smaller* maximum value than $n(x)$.

d. Compare the sum of the roots for both functions.

The sum of the roots for $g(x) = -3 + 2 = -1$ from a graphing calculator.



The sum of the roots for $n(x) = -2 + 4 = 2$ from the table.

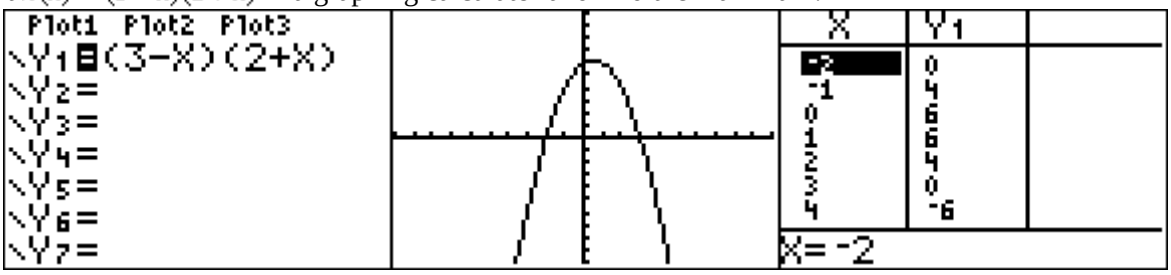
Statement d is true. The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

PTS: 2 REF: 081521ai NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

2. ANS: C

Strategy: Each answer choice needs to be evaluated for the largest maximum using a different strategy..

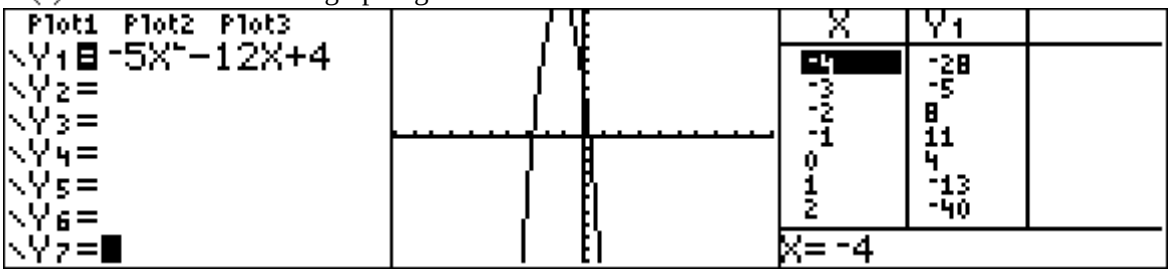
a) Input $h(x) = (3 - x)(2 + x)$ in a graphing calculator and find the maximum.



The maximum for answer choice *a* is a little more than 6.

b) The table shows that the maximum is a little more than 9.

c) Input $k(x) = -5x^2 - 12x + 4$ in a graphing calculator and find the maximum.



The table of values shows that the maximum is 11 or more.

d) The graph shows that the maximum is a little more than 4.

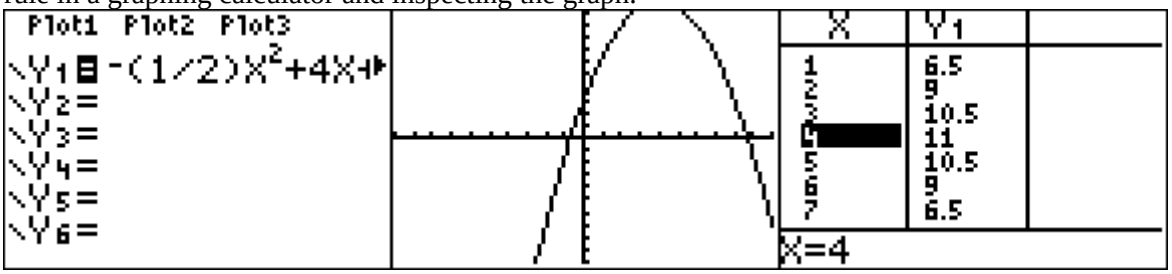
Answer choice *c* is the best choice.

PTS: 2 REF: 061514ai NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

3. ANS:

Function g has the larger maximum value. The maximum of function *g* is 11. The maximum of function f is 6.

Strategy: Determine the maximum for *f* from the graph. Determine the maximum for g by inputting the function rule in a graphing calculator and inspecting the graph.



The table of values shows the maximum for *g* is 11.

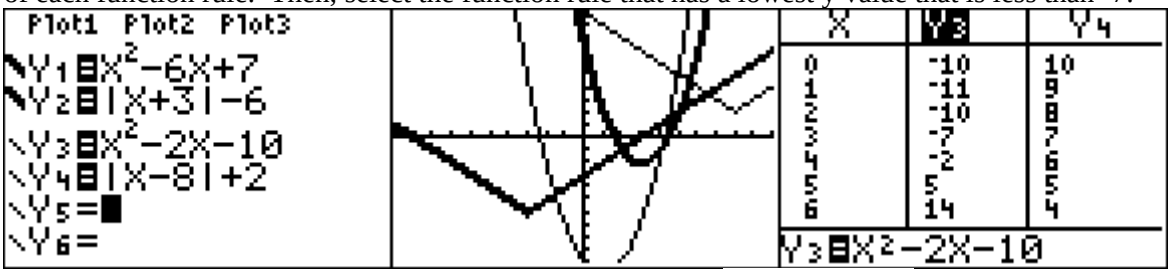
Another way of finding the maximum for g is to use the axis of symmetry formula and the function rule, as follows: $x = \frac{-b}{2a} = \frac{-4}{2\left(-\frac{1}{2}\right)} = \frac{-4}{-1} = 4$

$$y = -\frac{1}{2}(4)^2 + 4(4) + 3 = -8 + 16 + 3 = 11$$

PTS: 2 REF: 081429ai NAT: F.IF.C.9 TOP: Graphing Quadratic Functions

4. ANS: C

Strategy: The graph shows a parabola with a vertex at (3, -7), so the minima is at -7. Identify the lowest y-value of each function rule. Then, select the function rule that has a lowest y value that is less than -7.



The graph view of the four functions shows that the function $y = x^2 - 2x - 10$ has a y-value less than -7.

PTS: 2 REF: 011622ai NAT: F.IF.C.9 TOP: Comparing Functions

Lesson Plan

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

Lesson Plan

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is $350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.