Lesson Plan

F.LE.A.1: Model Families of Functions

FUNCTIONS

F.LE.A.1: Model Families of Functions

A. Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

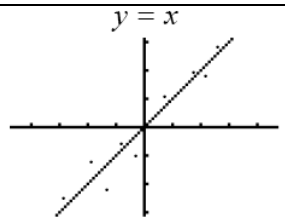
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

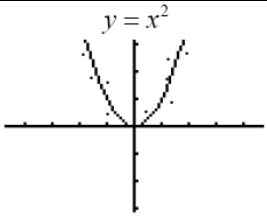
Families of Functions



If the graph is a straight line, the function is in the family of **linear functions.**

All **first degree functions** are linear functions, except those lines that are vertical.

All linear functions can be expressed as $y = mx + b$, where m is a constant defined slope and b is the y-intercept.



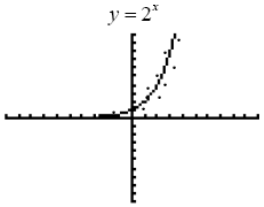
If the graph is a parabola, the function is in the family of **quadratic functions**.

All **quadratic functions** have an exponent of 2 or can be factored into a single factor with an exponent of 2.

Examples:

$x^2 + 6x + 9 = (x + 3)^2$

$x^{16} + 6x^8 + 9 = \left(x^8 + 3\right)^2$



If the graph is a curve that approached a horizontal limit on one end and gets steeper on the other end, the function is in the family of **exponential functions**.

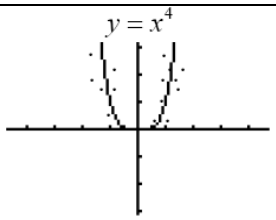
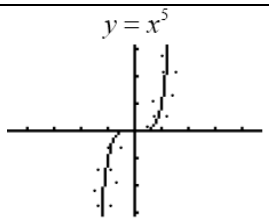
An **exponential function** is a function that contains a variable for an exponent.

Example: $y = 2^x$

Exponential growth and decay can be modeled using the general formula

$A = P(1 + r)^t$

Lesson Plan

 NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an ***odd number***, take the form of parabolas. The larger the value of *n*, the wider the flat part at the bottom/top.	 NOTE: All functions in the form of $y = ax^n$, where $a \neq 0$ and $n > 1$ and n is an ***even number***, take the form of hyperbolas. These are not quadratic functions.

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. The function, $t(x)$, is shown in the table below.

x	t(x)
–3	10
–1	7.5
1	5
3	2.5
5	0

Determine whether $t(x)$ is linear or exponential. Explain your answer.

2. Which situation could be modeled by using a linear function?
   a. a bank account balance that grows at a rate of 5% per year, compounded annually
   b. a population of bacteria that doubles every 4.5 hours
   c. the cost of cell phone service that charges a base amount plus 20 cents per minute
   d. the concentration of medicine in a person's body that decays by a factor of one-third every hour

3. Grisham is considering the three situations below.
   I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
   II. The value of a car depreciates at a rate of 15% per year after it is purchased.
   III. The amount of bacteria in a culture triples every two days during an experiment.
   Which of the statements describes a situation with an equal difference over an equal interval?
   a. I, only
   b. II, only
   c. I and III
   d. II and III

4. The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

   Which type of function best models the given data?
   a. linear function with a negative rate of change
   b. linear function with a positive rate of change
   c. exponential decay function
   d. exponential growth function

5. The tables below show the values of four different functions for given values of x.

x	f(x)
1	12
2	19
3	26
4	33

x	g(x)
1	–1
2	1
3	5
4	13

x	h(x)
1	9
2	12
3	17
4	24

x	k(x)
1	–2
2	4
3	14
4	28

Which table represents a linear function?·

a. $f(x)$
b. $g(x)$
c. $h(x)$
d. $k(x)$

6. Rachel and Marc were given the information shown below about the bacteria growing in a Petri dish in their biology class.

Number of Hours, x	1	2	3	4	5	6	7	8	9	10
Number of Bacteria, B(x)	220	280	350	440	550	690	860	1070	1340	1680

Rachel wants to model this information with a linear function. Marc wants to use an exponential function. Which model is the better choice? Explain why you chose this model.

F.LE.A.1: Model Families of Functions
Answer Section

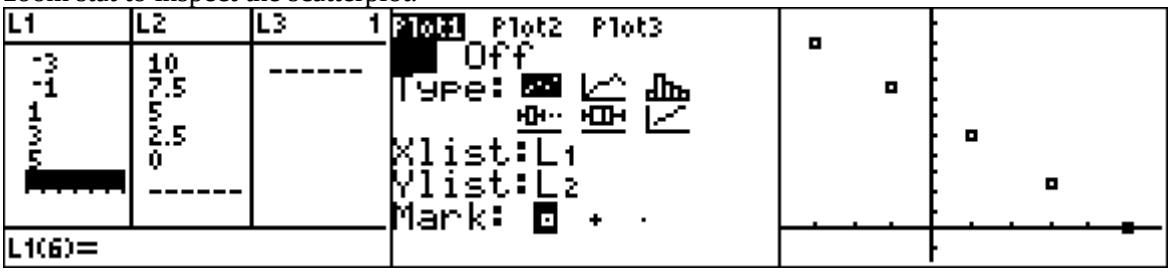
1. ANS:

Strategy #1. Calculate the change in x and the change in y for each ordered pair in the table. If the ratio of $\frac{\Delta y}{\Delta x}$ is constant, the function is linear.

Change in x	x	t(x)	Change in y
	-3	10	
+2<	-1	7.5	>-2.5
+2<	1	5	>-2.5
+2<	3	2.5	>-2.5
+2<	5	0	>-2.5

This table shows a linear function, because the ratio of $\frac{\Delta y}{\Delta x}$ can always be expressed as $\frac{-2.5}{2}$.

Strategy #2. Input values from the table into the stats editor of a graphing calculator, turn stats plot on, then use zoom stat to inspect the scatterplot.



The scatterplot shows a linear relationship.

PTS: 2 REF: 011625ai NAT: F.LE.A.1 TOP: Families of Functions

2. ANS: C

Strategy: Eliminate wrong answers.

a) Eliminate answer choice *a* because it describes exponential growth of money in a bank account.
b) Eliminate answer choice *b* because is describes exponential growth of bacteria.
c) Choose answer choice *c* because it can be modeled using the slope intercept formula as follows:

$$y = mx + b$$

$$\text{cost of cell phone service} = \$0.20 \times \text{number of minutes plus the base cost}$$

d) Eliminate answer choice *d* because it describes exponential decay of medicine in the body.

PTS: 2 REF: 081412ai NAT: F.LE.A.1 TOP: Families of Functions

3. ANS: A

Interpreting the Question: Equal differences over equal intervals suggests a constant rate of change, which would be a linear relationship.

Strategy: Model each situation with a function rule, then select the linear functions.

I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.

This can be modeled with the **linear** function $h = 3.5d$, where h represents the height of the sunflower and d represents the number of days. Since this function is linear, it represents a situation with an equal difference over an equal interval.

II. The value of a car depreciates at a rate of 15% per year after it is purchased.

This can be modeled with the **exponential decay** function $V = P(1-.15)^t$, where V represents the value of the car, P represents its price when purchased, -.15 represents the annual depreciation rate, and t represents the number of years after purchase. This is an exponential decay function, so it does not represent a situation with an equal difference over an equal interval.

III. The amount of bacteria in a culture triples every two days during an experiment.

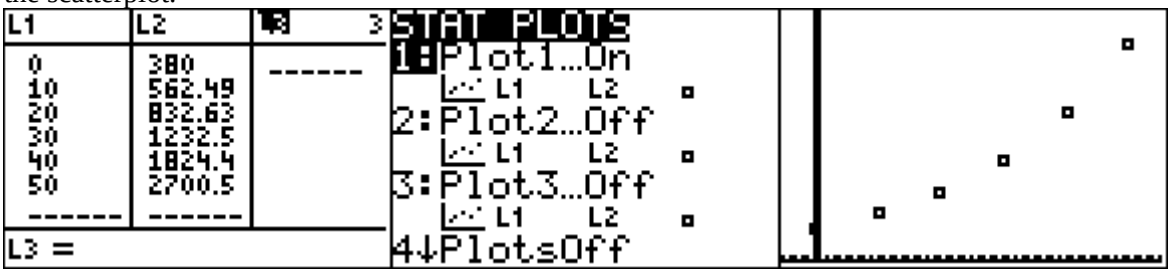
This can be modeled with the **exponential growth** function $A = B(3)^{\frac{d}{2}}$, where A represents the amount of bacteria, B represents starting amount of bacteria, 3 represents the growth rate, and $\frac{d}{2}$ represents the number of growth cyles. This is an exponential growth function, so it does not represent a situation with an equal difference over an equal interval.

The only choice that represents a situation with an equal difference over an equal interval is the first situation.

PTS: 2 REF: 011623ai NAT: F.LE.A.1 TOP: Families of Functions

4. ANS: D

Strategy: Input the table into the stats editor of a graphing calculator, then plot the points and examine the shape of the scatterplot.



The data in this table creates a scatterplot that appears to model an exponential growth function.

DIMS? Does It Make Sense? Yes. Savings accounts are excellent exemplars of exponential growth.

PTS: 2 REF: 061406ai NAT: F.LE.A.1 TOP: Modeling Exponential Equations

5. ANS: A

Step 1. Notice that in each of the tables, the values of the indepedendent variable (x) are 1, 2, 3, and 4, while the dependent variables are different. The question asks which table represents a linear function and, bu definition, a linear function must have a constant rate of change.

Step 2. Use the slope formula and data from each table to determine which table represents a constant rate of change.

Step 3. Execute the strategy.

f(x) rate of change $= \frac{f(x)_2 - f(x)_1}{x_2 - x_1}$. Every time x increases by 1, f(x) increases by 7. This is a constant rate of change, so f(x) is a linear function.

g(x) rate of change $= \frac{g(x)_2 - g(x)_1}{x_2 - x_1}$. Every time x increases by 1, g(x) increases by a different amount.

This is not a constant rate of change, so g(x) is not a linear function.

h(x) rate of change = $\frac{h(x)_2 - h(x)_1}{x_2 - x_1}$. Every time x increases by 1, h(x) increases by a different amount.

This is not a constant rate of change, so h(x) is not a linear function.

k(x) rate of change = $\frac{k(x)_2 - k(x)_1}{x_2 - x_1}$. Every time x increases by 1, k(x) increases by a different amount.

This is not a constant rate of change, so k(x) is not a linear function.

Step 4. Does it make sense? Yes. Only one table shows a constant rate of change.

PTS: 2 REF: 061606ai NAT: F.LE.A.1 TOP: Families of Functions

6. ANS:

Exponential, because the function does not grow at a constant rate.

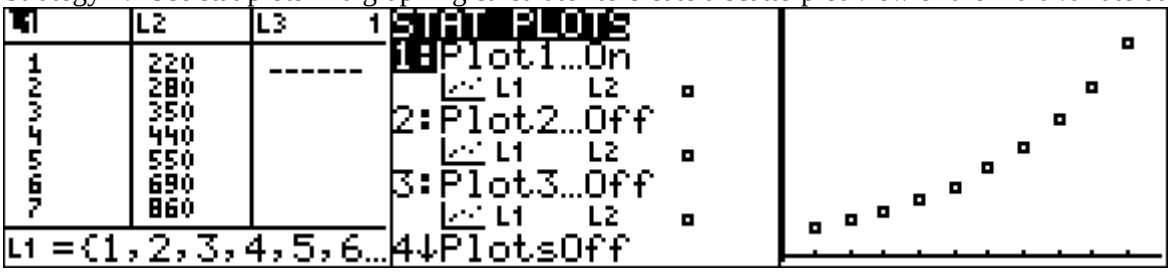
Strategy 1.

Compare the rates of change for different pairs of data using the slope formula.

Rate of change between (1, 220) and (5, 550): $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{550 - 220}{5 - 1} = \frac{330}{4} = 82.5$

Rate of change between (6, 690) and (10, 1680): $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1680 - 690}{10 - 6} = \frac{990}{4} = 247.5$

Strategy 2: Use stat plots in a graphing calculator to create a scatterplot view of the multivariate data.



The graph view of the data clearly shows that the data is not linear.

PTS: 2 REF: 081527ai NAT: S.ID.B.6a
TOP: Comparing Linear and Exponential Functions

Lesson Plan

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.

NAME: Mohammed Chen
DATE: December 18, 2015
LESSON: Missing Number in the Average
CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
PART 1b. State your understanding of **what the problem is asking**.
PART 1c. **Answer** the problem.
PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
PART 2b. State your understanding of **what the new problem is asking**.
PART 2c. **Answer** the new problem.
PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	**Up to 2** points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	**Up to 2** points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of Polya's four step universal algorithm for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

Lesson Plan

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is $360. If the weekly salaries of four of the employees are $340, $340, $345, and $425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is $350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\overline{X} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

Check: $360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.