

F.LE.A.3: Compare Families of Functions

FUNCTIONS

F.LE.A.3: Compare Families of Functions

A. Linear, Quadratic, & Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity

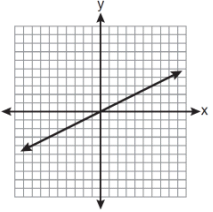
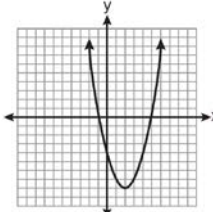
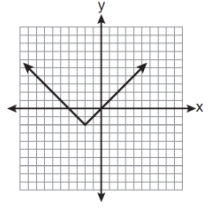
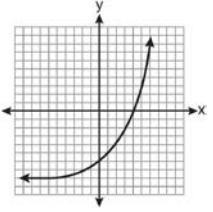
[Selected problem set\(s\)](#)

- facilitate a summary and share out of student work
- Homework – Write the Math Assignment**

Big Idea

A quantity increasing *exponentially* will eventually exceed a quantity increasing *linearly* or *quadratically*. In other words, given a big enough values of x , the exponential growth of $f(x)$ will always be greater than the linear or quadratic growth of $f(x)$.

Use a graphing calculator and different views of functions to compare linear, quadratic, and exponential models and solve problems.

 <p><u>Linear Functions</u> ...look like straight lines that are <i>not</i> vertical. The rate of change is a constant.</p>	 <p><u>Quadratic Functions</u> ...look like parabolas that open up or down.</p>	 <p><u>Absolute Value Functions</u> ...look like v-shapes.</p>	 <p><u>Exponential Functions</u> ...look like one-sided curves. The rate of change is exponential.</p>
---	--	---	--

REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. If $f(x) = 3^x$ and $g(x) = 2x + 5$, at which value of x is $f(x) < g(x)$?
 - a. -1
 - b. 2
 - c. -3
 - d. 4

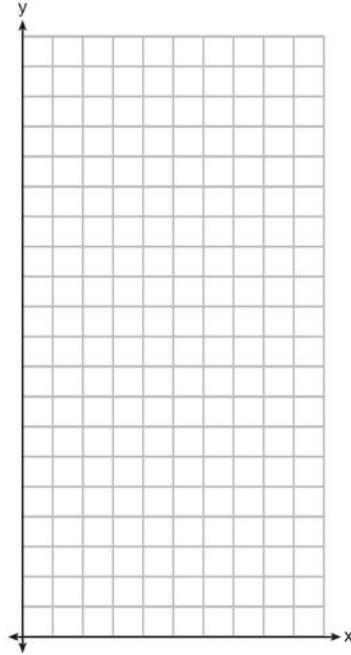
2. Alicia has invented a new app for smart phones that two companies are interested in purchasing for a 2-year contract. Company A is offering her \$10,000 for the first month and will increase the amount each month by \$5000. Company B is offering \$500 for the first month and will double their payment each month from the previous month. Monthly payments are made at the end of each month. For which monthly payment will company B 's payment first exceed company A 's payment?
 - a. 6
 - b. 7
 - c. 8
 - d. 9

3. What is the largest integer, x , for which the value of $f(x) = 5x^4 + 30x^2 + 9$ will be greater than the value of $g(x) = 3^x$?
 - a. 7
 - b. 8
 - c. 9
 - d. 10

4. As x increases beyond 25, which function will have the largest value?
 - a. $f(x) = 1.5^x$
 - b. $g(x) = 1.5x + 3$
 - c. $h(x) = 1.5x^2$
 - d. $k(x) = 1.5x^3 + 1.5x^2$

5. Graph $f(x) = x^2$ and $g(x) = 2^x$ for $x \geq 0$ on the set of axes below.

Lesson Plan

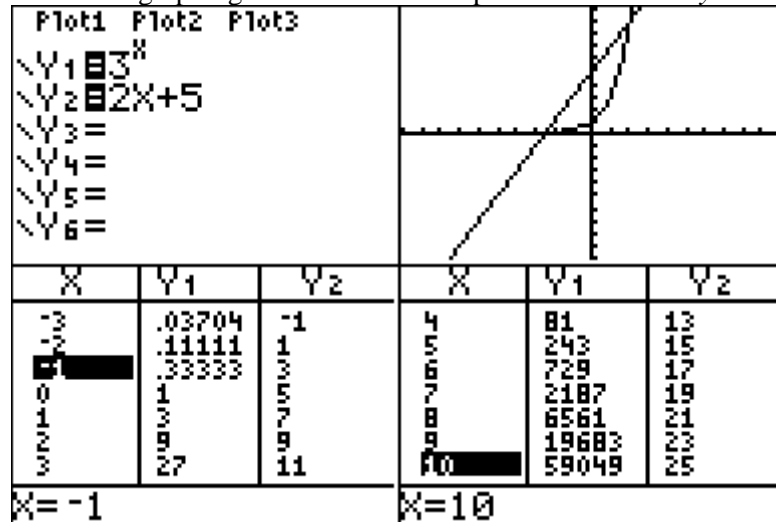


State which function, $f(x)$ or $g(x)$, has a greater value when $x = 20$. Justify your reasoning.

F.LE.A.3: Compare Families of Functions
Answer Section

1. ANS: A

Strategy: Input both functions in a graphing calculator and compares the values of y for various values of x .



The table of values shows:
 When $x = -1$, $f(x) < g(x)$
 When $x = 2$, $f(x) = g(x)$
 When $x = -3$, $f(x) > g(x)$
 When $x = 4$, $f(x) > g(x)$

PTS: 2 REF: 061515ai NAT: F.LE.A.3 TOP: Families of Functions

2. ANS: C

Strategy: Build a table of values for the integer values of the domain $6 \leq x \leq 9$ to compare both offers.

x	$A = 5000x + 10000$	$B = 500(2)^{x-1}$
6	40,000	16,000
7	45,000	32,000
8	50,000	64,000
9	55,000	128,000

Offer B is greater than offer A when $x = 8$.

PTS: 2 REF: 081518ai NAT: F.LE.A.3
 TOP: Comparing Linear and Exponential Functions

3. ANS: C

Step 1. Understand that the problem asks you to select the largest value of x where the value of $f(x)$ will be greater than the value of $g(x)$.

Step 2. Strategy. Input both functions in a graphing calculator and explore the table of values.

Step 3. Execution of Strategy.

Lesson Plan

Plot1 Plot2 Plot3	X	Y1	Y2
$\sqrt{Y_1} = 5X^4 + 30X^2 + 9$	7	13484	2187
$\sqrt{Y_2} = 3^X$	8	22409	6561
$\sqrt{Y_3} =$	9	35244	19683
$\sqrt{Y_4} =$	10	53009	59049
$\sqrt{Y_5} =$	11	76844	177147
$\sqrt{Y_6} =$	12	108009	531441
	13	147884	1.59E6
	X=13		

The table shows that $f(x)$ is greater than $g(x)$ when $x = 7$, $x = 8$, and $x = 9$, but not when $x = 10$. The largest integer for which $f(x)$ is greater than $g(x)$ is 9.

Step 4. Does it make sense? Yes. $f(x) = 5x^4 + 30x^2 + 9$ is a quadratic function and $g(x) = 3^x$ is an exponential function. Exponential growth eventually outpaces quadratic growth.

PTS: 2 REF: 061621ai NAT: F.LE.A.3 TOP: Families of Functions

4. ANS: A

Strategy: Input all functions in a graphing calculator, then inspect the table of values for $x = 26$, which is beyond 25, as required by the problem..

Let $f(x) = y_1$

$g(x) = y_2$

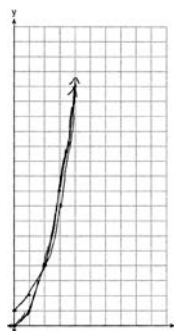
$h(x) = y_3$

$k(x) = y_4$

Plot1 Plot2 Plot3	X	Y1	Y2	X	Y3	Y4
$\sqrt{Y_1} = 1.5^x$	20	3325.3	33	20	600	12600
$\sqrt{Y_2} = 1.5x + 3$	21	4987.9	34.5	21	661.5	14553
$\sqrt{Y_3} = 1.5x^2$	22	7481.8	36	22	726	16698
$\sqrt{Y_4} = 1.5x^3 + 1.5x^2$	23	11223	37.5	23	793.5	19044
	24	16834	39	24	864	21600
	25	25251	40.5	25	937.5	24375
	26	37877	42	26	1014	27378
	X=26			Y4=27378		

PTS: 2 REF: 081618ai NAT: F.LE.A.3

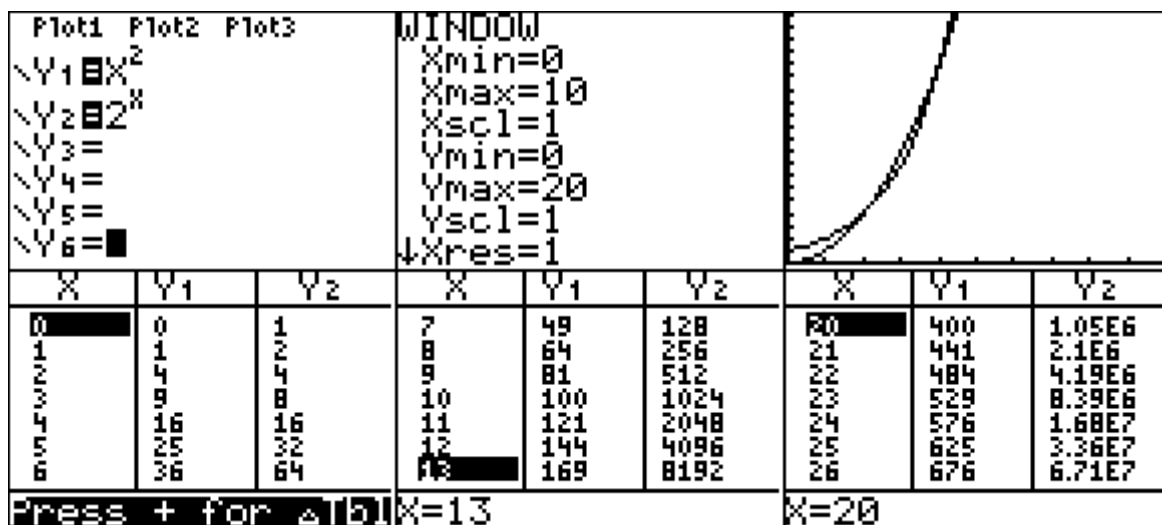
5. ANS:



$g(x)$ has a greater value: $2^{20} > 20^2$

Strategy: Input both functions in a graphing calculator, use the table of values to create the paper graph, and to compare the values of y for various values of x .

Lesson Plan



The table of values shows that when $x = 20$, $g(x) > f(x)$.

DIMS? Does It Make Sense? Yes. $2^{20} > 20^2$

PTS: 4 REF: 081533ai NAT: F.LE.A.3

TOP: Comparing Quadratic and Exponential Functions

Homework - Write the Math Assignment

START Write your name, date, topic of lesson, and class on your paper.
 NAME: Mohammed Chen
 DATE: December 18, 2015
 LESSON: Missing Number in the Average
 CLASS: Z

PART 1a. Copy **the problem** from the lesson and underline/highlight key words.
 PART 1b. State your understanding of **what the problem is asking**.
 PART 1c. **Answer** the problem.
 PART 1d. Explanation of **strategy** with all work shown.

PART 2a. Create **a new problem** that addresses the same math idea.
 PART 2b. State your understanding of **what the new problem is asking**.
 PART 2c. **Answer** the new problem.
 PART 2d. Explanation of **strategy** used in solving the new problem with all work shown.

Clearly label each of the eight parts.

Grading Rubric

Each homework writing assignment is graded using a four point rubric, as follows:

Part 1. The Original Problem	Up to 2 points will be awarded for: a) correctly restating the original problem; b) explicitly stating what the original problem is asking; c) answering the original problem correctly; and d) explaining the math.
Part 2. My New Problem	Up to 2 points will be awarded for: a) creating a new problem similar to the original problem; b) explicitly stating what the new problem is asking; c) answering the new problem correctly; and d) explaining the math.

This assignment/activity is designed to incorporate elements of [Polya's four step universal algorithm](#) for problem solving with the idea that writing is thinking. Polya's four steps for solving any problem are:

1. Read and understand the problem.
2. Develop a strategy for solving the problem.
3. Execute the strategy.
4. Check the answer for reasonableness.

EXEMPLAR OF A WRITING THE MATH ASSIGNMENT

Part 1a. The Problem

TOP Electronics is a small business with five employees. The mean (average) weekly salary for the five employees is \$360. If the weekly salaries of four of the employees are \$340, \$340, \$345, and \$425, what is the salary of the fifth employee?

Part 1b. What is the problem asking?

Find the salary of the fifth employee.

Part 1c. Answer

The salary of the fifth employee is \$350 per week.

Part 1d. Explanation of Strategy

The arithmetic mean or average can be represented algebraically as:

$$\bar{X} = \frac{x_1 + x_2 + \dots + x_n}{n}$$

I put information from the problem into the formula. The problem says there are 5 employees, so $n = 5$. The problem also gives the mean (average) salary and the salaries of 4 of the employees. These numbers can be substituted into the formula as follows:

$$360 = \frac{340 + 340 + 345 + 425 + x_5}{5}$$

$$1800 = 340 + 340 + 345 + 425 + x_5$$

$$1800 = 1450 + x_5$$

$$1800 - 1450 = x_5$$

$$350 = x_5$$

$$\text{Check: } 360 = \frac{340 + 340 + 345 + 425 + 350}{5} = \frac{1800}{5} = 360$$

Part 2a. A New Problem

Joseph took five math exams this grading period and his average score on all of the exams is 88. He remembers that he received test scores of 78, 87, 94, and 96 on four of the examinations, but he has lost one examination and cannot remember what he scored on it. What was Joseph's score on the missing exam?

Part 2b. What is the new problem asking?

Find Joseph's score on the missing exam.

Part 2c. Answer to New Problem

Joseph received a score of 85 on the missing examination.

Part 2d. Explanation of Strategy

I substitute information from the problem into the formula for the arithmetic mean, as follows:

$$88 = \frac{78 + 87 + 94 + 96 + x_5}{5}$$

$$440 = 355 + x_5$$

$$85 = x_5$$

$$88 = \frac{78 + 87 + 94 + 96 + 85}{5} = \frac{440}{5} = 88$$

The answer makes sense.