Lesson Plan

N.Q.A.1 Use Units to Solve Problems

NUMBERS, OPERATIONS, AND PROPERTIES

N.Q.A.l: Use Units to Solve Problems

A. Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Overview of Lesson

- activate prior knowledge and review learning objectives (see above)
- explain vocabulary and/or big ideas associated with the lesson
- connect assessment practices with curriculum
- model an assessment problem and solution strategy
- facilitate guided discussion of student activity
- facilitate guided practice of student activity
Selected problem set(s)
- facilitate a summary and share out of student work
Homework – Write the Math Assignment

Vocabulary

A **scale** is a ratio of the $\frac{\text{measurement of a model}}{\text{measurement of the real thing}}$.

Example. A toy car is 1 foot long. The real car it represents is 20 feet long. The scale of the model is:

$$\frac{\text{measurement of toy car}}{\text{measurement of real car}} = \frac{1 \text{ feet}}{20 \text{ feet}} = \frac{1}{20}$$

The word **scale** also refers to what you mark on the axes of a graph. The marks you make on a graph are called **scale** intervals, and the distance between each mark must be equal (represent the same number of units).

Conversions are sometimes necessary when working with units. A **conversion** occurs when you change the units of a scale.

Example: Suppose you are working with units that are expressed in feet, but want your answer to be in units expressed as inches.

Feet may be **converted** to inches by using the ratio of $\frac{12 \text{ inches}}{1 \text{ foot}}$.

Twenty feet may be **converted** to inches by using proportions (equivalent ratios), as follows:

$$\frac{\text{inches}}{\text{feet}} \left| \frac{12}{1} = \frac{x}{20} \right.$$

Using cross multiplication, we can solve for *x*.

$$20 \times 12 = 1 \times x$$

$$240 = x$$

The are 240 inches in 20 feet.

Per usually means “for each” when used with units.

miles **per** hour means miles for *each* hour and can be expressed as the ratio $\frac{\text{miles}}{1 \text{ hour}}$

miles **per** gallon means miles for *each* gallon and can be expressed as the ratio $\frac{\text{miles}}{1 \text{ gallon}}$

Conversions Chart Used in Regents Algebra 1 (Common Core) Exams

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Big Idea #1
Units can cancel!

Cancellation of Units: Cancellation can be used with units.
Examples:

$$\frac{1\ yard}{3\ \cancel{feet}} \times \frac{27\ \cancel{feet}}{1} = \frac{1\ yard \times 27}{3} = \frac{27\ yards}{3}$$

To find the number of seconds in a year, use cancellation of units.

$$\frac{60\text{ seconds}}{1\ \cancel{\text{minute}}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}} \times \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \times \frac{365\ \cancel{\text{days}}}{1\text{ year}}$$

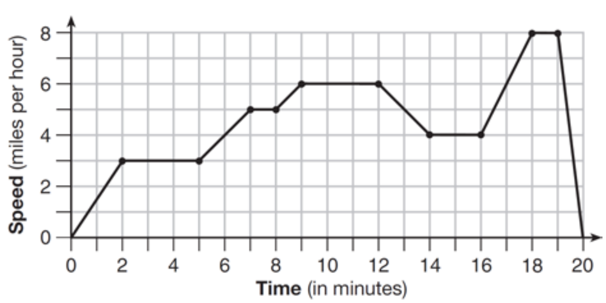
$$= \frac{60\text{ seconds} \times 60 \times 24 \times 365}{1 \times 1 \times 1 \times 1\text{ year}}$$

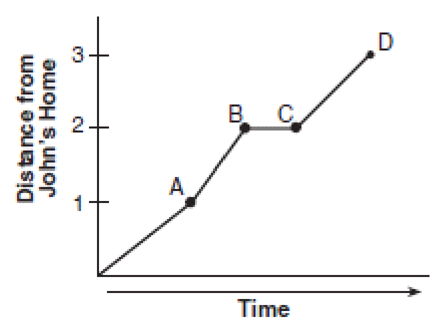
$$= \frac{30{,}536{,}000\text{ seconds}}{1\text{ year}}$$

Big Idea #2
Knowing the units is important when interpreting graphs!

A graph is one view of the relationship between two variables. The variables are measured in specific units, which are very important to understanding the meaning of the graph.

Example: The two graphs below are from different Regents problems. The units for the *x*-axis are the same, but the units for the y-axis are different. The different units for the *y*-axes require different interpretations of the two graphs.





REGENTS PROBLEMS TYPICAL OF THIS STANDARD

1. Peyton is a sprinter who can run the 40-yard dash in 4.5 seconds. He converts his speed into miles per hour, as shown below.

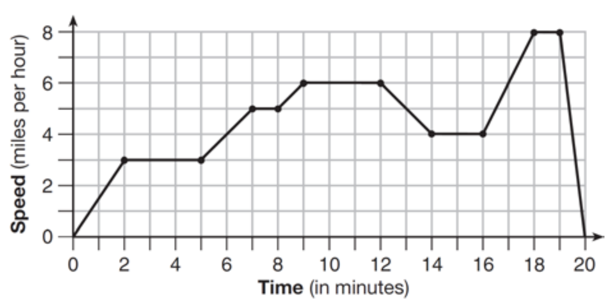
$$\frac{40\text{ yd}}{4.5\text{ sec}} \cdot \frac{3\text{ ft}}{1\text{ yd}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$$

   Which ratio is *incorrectly* written to convert his speed?

   a. $\frac{3\text{ ft}}{1\text{ yd}}$
   b. $\frac{5280\text{ ft}}{1\text{ mi}}$
   c. $\frac{60\text{ sec}}{1\text{ min}}$
   d. $\frac{60\text{ min}}{1\text{ hr}}$

2. Dan took 12.5 seconds to run the 100-meter dash. He calculated the time to be approximately
   a. 0.2083 minute
   b. 750 minutes
   c. 0.2083 hour
   d. 0.52083 hour

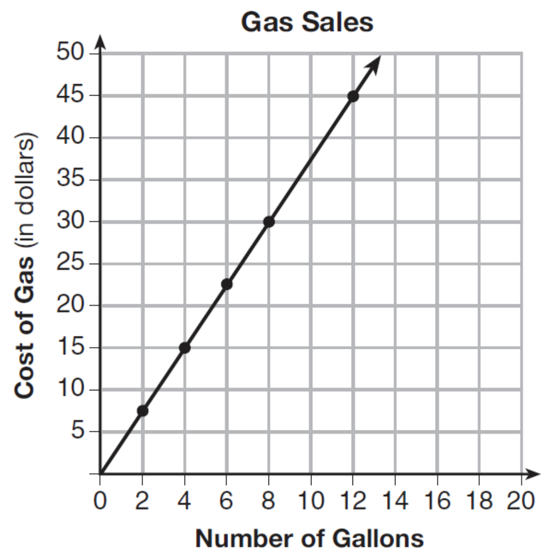
3. The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.



   Which statement best describes what the jogger was doing during the $9-12$ minute interval of her jog?
   a. She was standing still.
   b. She was increasing her speed.
   c. She was decreasing her speed
   d. She was jogging at a constant rate.

4. Patricia is trying to compare the average rainfall of New York to that of Arizona. A comparison between these two states for the months of July through September would be best measured in
   a. feet per hour
   b. inches per hour
   c. inches per month
   d. feet per month

5. The graph below was created by an employee at a gas station.



Which statement can be justified by using the graph?

a. If 10 gallons of gas was purchased, $35 was paid.

b. For every gallon of gas purchased, $3.75 was paid.

c. For every 2 gallons of gas purchased, $5.00 was paid.

d. If zero gallons of gas were purchased, zero miles were driven.

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Answer Section

1. ANS: B
Strategy: Work through each step of the problem and ask the DIMS question. Does It Make Sense.

STEP 1. $\frac{40 \text{ yards}}{4.5 \text{ seconds}} \times \frac{3 \text{ feet}}{1 \text{ yard}} = \frac{120 \text{ feet}}{4.5 \text{ seconds}}$ This makes sense. The yard units cancel and Peyton's speed becomes measured in feet per second instead of yards per second. We take the ratio of $\frac{120 \text{ feet}}{4.5 \text{ seconds}}$ to the next step in our analysis.

STEP 2. $\frac{120 \text{ feet}}{4.5 \text{ seconds}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = \frac{120 \times 5280 \text{ feet}^2}{4.5 \text{ second miles}}$. This does not make sense. The speed of a runner would not be measured in feet^2 per second miles. The problem is that the numerator and denominator are switched. It should be $\frac{1 \text{ mile}}{5280 \text{ feet}}$. When the numerator and denominator are changed, the problem becomes $\frac{120 \text{ feet}}{4.5 \text{ seconds}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} = \frac{120 \text{ miles}}{23{,}760 \text{ seconds}}$. The feet units cancel and our measurement of Peyton's speed has distance over time, which makes sense. Answer choice b is selected to show that this ratio is *incorrectly* written.

STEP 3. Though we have solved the problem, we can continue our step by step analysis by taking the ratio of $\frac{120 \text{ miles}}{23{,}760 \text{ seconds}}$ to the next step in our analysis. The problem now becomes $\frac{120 \text{ miles}}{23{,}760 \text{ seconds}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = \frac{120 \times 60 \text{ miles}}{23{,}760 \times 1 \text{minutes}} = \frac{72{,}000 \text{ miles}}{23{,}760 \text{ minutes}}$. This makes sense. The seconds units cancel and we again have distance over miles. We take the ratio $\frac{72{,}000 \text{ miles}}{23{,}760 \text{ minutes}}$ to the next step.

STEP 4. $\frac{72{,}000 \text{ miles}}{23{,}760 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{72{,}000 \times 60 \text{ miles}}{23{,}760 \times 1 \text{ hours}} = \frac{432{,}000 \text{ miles}}{23{,}760 \text{ hours}} = 18\frac{2}{11}$ miles per hour. This makes sense. Peyton is a fast sprinter.

PTS: 2 REF: 011502ai NAT: N.Q.A.1 TOP: Conversions

2. ANS: A
Step 1. Read both the question and the answers. Understand that the problem is asking you to convert seconds into either minutes or hours. The 100 meters is constant, so it is not important to the problem of converting time into minutes or hours.
Step 2. Create two proportions using the conversion rates of 1) 60 seconds per minute; and 2) 3600 seconds per hour, to express 12.5 seconds in minutes and hours.
Step 3. Execute the strategy.

12.5 second equals how many minutes?	12.5 second equals how many hours?

$\frac{\text{seconds}}{\text{minutes}}$ $\frac{12.5}{x}=\frac{60}{1}$ $12.5=60x$ $\frac{12.5}{60}=x$ $.208\overline{3}\text{ minutes}=x$	$\frac{\text{seconds}}{\text{hours}}$ $\frac{12.5}{x}=\frac{3600}{1}$ $12.5=3600x$ $\frac{12.5}{3600}=x$ $.00347\overline{2}\text{ hours}=x$

The correct choice is a), 12.5 seconds equals 0.2083 minutes.
4. Does it make sense? Yes. It is obvious that 12.5 seconds does not equal 750 minutes (choice b) and it is also obvious that 12.5 seconds is not more than half an hour (choice d). The only choice that is less than a minute is choice a), and 12.5 seconds is definitely less than a minute.

PTS: 2 REF: 061608ai NAT: N.Q.A.1 TOP: Conversions
KEY: dimensional analysis

3. ANS: D
Strategy: Pay close attention to the labels on the x-axis and the y-axis, then eliminate wrong answers. NOTE: A horizontal line (no slope) means that speed is not changing.

Answer a can be eliminated because she would have a speed of 0 if she were standing still. She was only standing still at the start and end of her jog.
Answer b can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer c can be eliminated because the speed does not change during the $9-12$ minute interval of her jog.
Answer d is the correct choice because a horizontal line (no slope) means that speed is not changing.

PTS: 2 REF: 061502ai NAT: N.Q.A.1 TOP: Relating Graphs to Events

4. ANS: C
Rainfall is typically measured in inches rather than feet. Since Patricia is trying to compare average rainfall between New York and Arizona for the months of July through September, the second unit would be months rather than hours. Inches per month is the best choice.

PTS: 2 REF: 081609ai NAT: N.Q.A.1

5. ANS: B
Strategy #1: Use the slope of the line to determine the cost per gallon of gas. Select any two points that are on intersections of vertical and horizontal gridlines, then substitute them into the slope formula to determine the rate of change, which is the cost per gallon of gas.

$$\textit{Select }(8,30)\textit{ and }(4,15)$$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{30-15}{8-4}=\frac{15}{4}=\$3.75$$

or

$$\textit{Select }(12,45)\textit{ and }(8,30)$$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{45-30}{12-8}=\frac{15}{4}=\$3.75$$

For every gallon of gas purchased. $3.75 was paid.

Strategy #2. Eliminate wrong answers.
Choice (a) is wrong because the chart shows that 10 gallongs of gas costs $37.50, not $35.00.

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Choice (b) is correct.
Choice (c) is wrong because the chart shows that 2 gallons of gas cost $7.50, not $5.00.
Choice (d) is wrong because the chart says nothing about the number of miles driven.

PTS: 2 REF: 011602ai NAT: N.Q.A.1 TOP: Graphing Linear Functions