

A.APR.D.6: Rational Expressions 3

1 Given $x \neq -2$, the expression $\frac{2x^2 + 5x + 8}{x + 2}$ is equivalent to

- 1) $2x^2 + \frac{9}{x + 2}$
- 2) $2x + \frac{7}{x + 2}$
- 3) $2x + 1 + \frac{6}{x + 2}$
- 4) $2x + 9 - \frac{10}{x + 2}$

2 The expression $\frac{x^3 + 2x^2 + x + 6}{x + 2}$ is equivalent to

- 1) $x^2 + 3$
- 2) $x^2 + 1 + \frac{4}{x + 2}$
- 3) $2x^2 + x + 6$
- 4) $2x^2 + 1 + \frac{4}{x + 2}$

3 Given $x \neq -3$, which expression is equivalent to $\frac{2x^3 + 3x^2 - 4x + 5}{x + 3}$?

- 1) $2x^3 + 9x^2 + 23x + 74$
- 2) $2x^2 - 3x + 5 - \frac{10}{x + 3}$
- 3) $2x^3 - 3x^2 + 5x - 10$
- 4) $2x^2 + 9x + 23 + \frac{74}{x + 3}$

4 Given $x \neq -3$, the expression $\frac{2x^3 + 7x^2 - 3x - 25}{x + 3}$ is equivalent to

- 1) $2x^2 + x - 6 - \frac{7}{x + 3}$
- 2) $2x^2 + 13x - 36 + \frac{83}{x + 3}$
- 3) $2x^2 + x - 13$
- 4) $x^2 + 4x - 15 + \frac{20}{x + 3}$

5 Which expression is equivalent to $\frac{2x^4 + 8x^3 - 25x^2 - 6x + 14}{x + 6}$?

- 1) $2x^3 + 4x^2 + x - 12 + \frac{86}{x + 6}$
- 2) $2x^3 - 4x^2 - x + 14$
- 3) $2x^3 - 4x^2 - x + \frac{14}{x + 6}$
- 4) $2x^3 - 4x^2 - x$

6 Which expression is equivalent to $\frac{x^3 - 2}{x - 2}$?

- 1) x^2
- 2) $x^2 + 2x + 4 + \frac{6}{x - 2}$
- 3) $x^2 - 2$
- 4) $x^2 - 2x + 4 - \frac{10}{x - 2}$

7 The expression $\frac{x^4 - 5x^2 + 4x + 14}{x + 2}$ is equivalent to

- 1) $x^3 - 2x^2 - x + 6 + \frac{2}{x + 2}$
- 2) $x^3 - 5x + 4 - \frac{14}{x + 2}$
- 3) $x^3 + 2x^2 - x + 2 + \frac{18}{x + 2}$
- 4) $x^3 + 2x^2 - 9x + 22 - \frac{30}{x + 2}$

8 The rational expression $\frac{2x^4 - 5x^2 + 3x - 2}{x - 3}$ is equivalent to

- 1) $2x^3 - 5x - 12 - \frac{38}{x - 3}$
- 2) $2x^3 + 6x^2 + 13x + 42 + \frac{124}{x - 3}$
- 3) $2x^3 - 5x + 18 - \frac{56}{x - 3}$
- 4) $2x^3 - 6x^2 + 13x - 36 + \frac{106}{x - 3}$

9 What is the quotient when $10x^3 - 3x^2 - 7x + 3$ is divided by $2x - 1$?

- 1) $5x^2 + x + 3$
- 2) $5x^2 - x + 3$
- 3) $5x^2 - x - 3$
- 4) $5x^2 + x - 3$

10 The expression $\frac{6x^3 + 17x^2 + 10x + 2}{2x + 3}$ equals

- 1) $3x^2 + 4x - 1 + \frac{5}{2x + 3}$
- 2) $6x^2 + 8x - 2 + \frac{5}{2x + 3}$
- 3) $6x^2 - x + 13 - \frac{37}{2x + 3}$
- 4) $3x^2 + 13x + \frac{49}{2} + \frac{151}{2x + 3}$

11 The expression $\frac{9x^2 - 2}{3x + 1}$ is equivalent to

- 1) $3x - 1 - \frac{1}{3x + 1}$
- 2) $3x - 1 + \frac{1}{3x + 1}$
- 3) $3x + 1 - \frac{1}{3x + 1}$
- 4) $3x + 1 + \frac{1}{3x + 1}$

12 Which expression is equivalent to $\frac{2x^3 + 2x - 7}{2x + 4}$?

- 1) $x^2 - 2x + 5 - \frac{27}{2x + 4}$
- 2) $x^2 - 1 - \frac{3}{2x + 4}$
- 3) $x^2 + 2x + 5 + \frac{13}{2x + 4}$
- 4) $x^2 + 2x - 3 + \frac{5}{2x + 4}$

13 Which expression is equivalent to $\frac{4x^3 + 9x - 5}{2x - 1}$,

where $x \neq \frac{1}{2}$?

- 1) $2x^2 + x + 5$
- 2) $2x^2 + \frac{11}{2} + \frac{1}{2(2x - 1)}$
- 3) $2x^2 - x + 5$
- 4) $2x^2 - x + 4 + \frac{1}{2x - 1}$

14 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

- 1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$
- 2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$
- 3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$
- 4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

15 Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form

$$q(x) + \frac{r(x)}{g(x)}.$$

16 Determine the quotient and remainder when $(6a^3 + 11a^2 - 4a - 9)$ is divided by $(3a - 2)$.

Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$.

17 Given $f(x) = 3x^3 - 4x^2 + 2x - 1$ and $g(x) = x - 4$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the

form $q(x) + \frac{r(x)}{g(x)}$. Is $x = 4$ a root of $f(x)$? Explain your answer.

18 Given $a(x) = x^4 + 2x^3 + 4x - 10$ and $b(x) = x + 2$, determine $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$. Is $b(x)$ a factor of $a(x)$? Explain.

19 When the function $p(x)$ is divided by $x - 1$ the quotient is $x^2 + 7 + \frac{5}{x - 1}$. State $p(x)$ in standard form.

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Answer Section

1 ANS: 3

$$\begin{array}{r}
 2x+1 \\
 x+2 \overline{) 2x^2+5x+8} \\
 \underline{2x^2+4x} \\
 x+8 \\
 \underline{x+2} \\
 6
 \end{array}$$

REF: 012007aai

2 ANS: 2

$$\begin{array}{r}
 x^2+0x+1 \\
 x+2 \overline{) x^3+2x^2+x+6} \\
 \underline{x^3+2x^2} \\
 0x^2+x \\
 \underline{0x^2+0x} \\
 x+6 \\
 \underline{x+2} \\
 4
 \end{array}$$

REF: 081611aai

3 ANS: 2

$$\begin{array}{r}
 2x^2-3x+5 \\
 x+3 \overline{) 2x^3+3x^2-4x+5} \\
 \underline{2x^3+6x^2} \\
 -3x^2-4x \\
 \underline{-3x^2-9x} \\
 5x+5 \\
 \underline{5x+15} \\
 -10
 \end{array}$$

REF: 082302aai

4 ANS: 1

$$\begin{array}{r}
 \overline{2x^2 + x - 6} \\
 x+3 \overline{) 2x^3 + 7x^2 - 3x - 25} \\
 \underline{2x^3 + 6x^2} \\
 x^2 - 3x \\
 \underline{ x^2 + 3x} \\
 -6x - 25 \\
 \underline{ -6x - 18} \\
 -7
 \end{array}$$

REF: 062203aii

5 ANS: 3

$$\begin{array}{r}
 \overline{2x^3 - 4x^2 - x + \frac{14}{x+6}} \\
 x+6 \overline{) 2x^4 + 8x^3 - 25x^2 - 6x + 14} \\
 \underline{2x^4 + 12x^3} \\
 -4x^3 - 25x^2 - 6x + 14 \\
 \underline{ -4x^3 - 24x^2} \\
 -x^2 - 6x + 14 \\
 \underline{ -x^2 - 6x} \\
 6
 \end{array}$$

REF: 081805aii

6 ANS: 2

$$\begin{array}{r}
 \overline{x^2 + 2x + 4} \\
 x-2 \overline{) x^3 - 0x^2 + 0x - 2} \\
 \underline{x^3 - 2x^2} \\
 2x^2 + 0x - 2 \\
 \underline{ 2x^2 - 4x} \\
 4x - 2 \\
 \underline{ 4x - 8} \\
 6
 \end{array}$$

REF: 082217aii

7 ANS: 1

$$\begin{array}{r}
 x^3 - 2x^2 - x + 6 \\
 x + 2 \overline{) x^4 + 0x^3 - 5x^2 + 4x + 14} \\
 \underline{x^4 + 2x^3} \\
 -2x^3 - 5x^2 \\
 \underline{-2x^3 - 4x^2} \\
 -x^2 + 4x \\
 \underline{-x^2 - 2x} \\
 6x + 14 \\
 \underline{6x + 12} \\
 2
 \end{array}$$

REF: 012305aaii

8 ANS: 2

$$\begin{array}{r}
 2x^3 + 6x^2 + 13x + 42 \\
 x - 3 \overline{) 2x^4 + 0x^3 - 5x^2 + 3x - 2} \\
 \underline{2x^4 - 6x^3} \\
 6x^3 - 5x^2 \\
 \underline{6x^3 - 18x^2} \\
 13x^2 + 3x \\
 \underline{13x^2 - 39x} \\
 42x - 2 \\
 \underline{42x - 126} \\
 124
 \end{array}$$

REF: 012408aaii

9 ANS: 4

$$\begin{array}{r}
 \overline{) 10x^3 - 3x^2 - 7x + 3} \\
 \underline{10x^3 - 5x^2} \\
 2x^2 - 7x \\
 \underline{2x^2 - x} \\
 - 6x + 3 \\
 \underline{-6x + 3} \\
 0
 \end{array}$$

REF: 011809aai

10 ANS: 1

$$\begin{array}{r}
 \overline{) 6x^3 + 17x^2 + 10x + 2} \\
 \underline{6x^3 + 9x^2} \\
 8x^2 + 10x \\
 \underline{8x^2 + 12x} \\
 - 2x + 2 \\
 \underline{-2x - 3} \\
 5
 \end{array}$$

REF: fall1503aai

11 ANS: 1

$$\begin{array}{r}
 \overline{) 9x^2 + 0x - 2} \\
 \underline{9x^2 + 3x} \\
 - 3x - 2 \\
 \underline{-3x - 1} \\
 - 1
 \end{array}$$

REF: 081910aai

12 ANS: 1

$$\begin{array}{r}
 x^2 - 2x + 5 \\
 2x + 4 \overline{) 2x^3 + 0x^2 + 2x - 7} \\
 \underline{2x^3 + 4x^2} \\
 -4x^2 + 2x \\
 \underline{-4x^2 - 8x} \\
 10x - 7 \\
 \underline{10x + 20} \\
 -27
 \end{array}$$

REF: 062313aii

13 ANS: 1

$$\begin{array}{r}
 2x^2 + x + 5 \\
 2x - 1 \overline{) 4x^3 + 0x^2 + 9x - 5} \\
 \underline{4x^3 - 2x^2} \\
 2x^2 + 9x \\
 \underline{2x^2 - x} \\
 10x - 5 \\
 \underline{10x - 5} \\
 0
 \end{array}$$

REF: 081713aii

14 ANS: 2

$$\begin{array}{r}
 2x^2 - 3x + 7 \\
 2x + 3 \overline{) 4x^3 + 0x^2 + 5x + 10} \\
 \underline{4x^3 + 6x^2} \\
 -6x^2 + 5x \\
 \underline{-6x^2 - 9x} \\
 14x + 10 \\
 \underline{14x + 21} \\
 -11
 \end{array}$$

REF: 061614aii

15 ANS:

$$\begin{array}{r}
 \overline{3x+13} \\
 x-2 \overline{) 3x^2+7x-20} \quad 3x+13 + \frac{6}{x-2} \\
 \underline{3x^2-6x} \\
 13x-20 \\
 \underline{13x-26} \\
 6
 \end{array}$$

REF: 011732aii

16 ANS:

$$\begin{array}{r}
 \overline{2a^2+5a+2} \\
 3a-2 \overline{) 6a^3+11a^2-4a-9} \quad 2a^2+5a+2 - \frac{5}{3a-2} \\
 \underline{6a^3-4a^2} \\
 15a^2-4a \\
 \underline{15a^2-10a} \\
 6a-9 \\
 \underline{6a-4} \\
 -5
 \end{array}$$

REF: 061829aii

17 ANS:

$$\begin{array}{r}
 \overline{3x^2+8x+34} \\
 x-4 \overline{) 3x^3-4x^2+2x-1} \quad 3x^2+8x+34 + \frac{135}{x-4} \quad x=4 \text{ is not a root of } f(x) \text{ because } \frac{f(x)}{g(x)} \text{ has a remainder.} \\
 \underline{3x^3-12x^2} \\
 8x^2+2x \\
 \underline{8x^2-32x} \\
 34x-1 \\
 \underline{34x-136} \\
 135
 \end{array}$$

REF: 082235aii

18 ANS:

$$x+2 \overline{) \begin{array}{r} x^3 \\ x^4 + 2x^3 + 4x - 10 \end{array}} \quad x^3 + 4 - \frac{18}{x+2}. \text{ No, because there is a remainder.}$$

$$\begin{array}{r} \underline{x^4 + 2x^3} \\ 4x - 10 \\ \underline{4x + 8} \\ -18 \end{array}$$

REF: 011934aaii

19 ANS:

$$\frac{p(x)}{x-1} = x^2 + 7 + \frac{5}{x-1}$$

$$p(x) = x^3 - x^2 + 7x - 7 + 5$$

$$p(x) = x^3 - x^2 + 7x - 2$$

REF: 061930aaii