

A.CED.A.1: Exponential Growth

- In the equation $y = 0.5(1.21)^x$, y represents the number of snowboarders in millions and x represents the number of years since 1988. Find the year in which the number of snowboarders will be 10 million for the first time. (Only an algebraic solution will be accepted.)
- Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$, where:
 G = final number of bacteria
 A = initial number of bacteria
 t = time (in hours)
In approximately how many hours will 4 bacteria first increase to 2,500 bacteria? Round your answer to the *nearest hour*.
- The number of houses in Central Village, New York, grows every year according to the function $H(t) = 540(1.039)^t$, where H represents the number of houses, and t represents the number of years since January 1995. A civil engineering firm has suggested that a new, larger well must be built by the village to supply its water when the number of houses exceeds 1,000. During which year will this first happen?
- Currently, the population of the metropolitan Waterville area is 62,700 and is increasing at an annual rate of 3.25%. This situation can be modeled by the equation $P(t) = 62,700(1.0325)^t$, where $P(t)$ represents the total population and t represents the number of years from now. Find the population of the Waterville area, to the *nearest hundred*, seven years from now. Determine how many years, to the *nearest tenth*, it will take for the original population to reach 100,000.
[Only an algebraic solution can receive full credit.]

- Seth's parents gave him \$5000 to invest for his 16th birthday. He is considering two investment options. Option A will pay him 4.5% interest compounded annually. Option B will pay him 4.6% compounded quarterly. Write a function of option A and option B that calculates the value of each account after n years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option B will earn than option A to the *nearest cent*. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option B to double Seth's initial investment.
- Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment

P = amount borrowed

r = annual interest rate

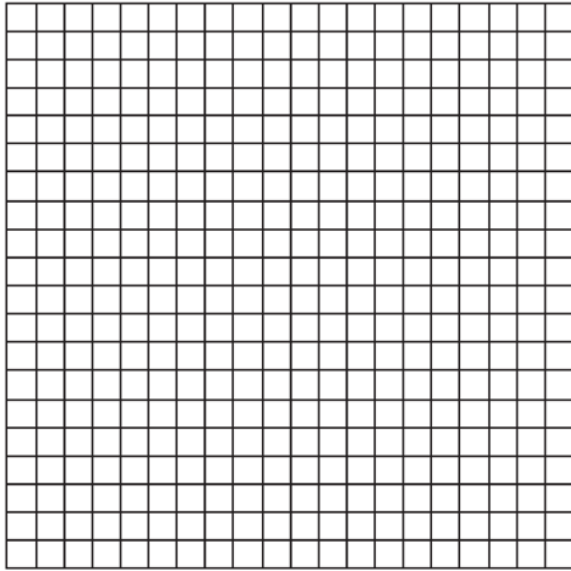
n = number of monthly payments

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than \$720.

- 7 Kristen invests \$5,000 in a bank. The bank pays 6% interest compounded monthly. To the *nearest tenth of a year*, how long must she leave the money in the bank for it to double? (Use the formula

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

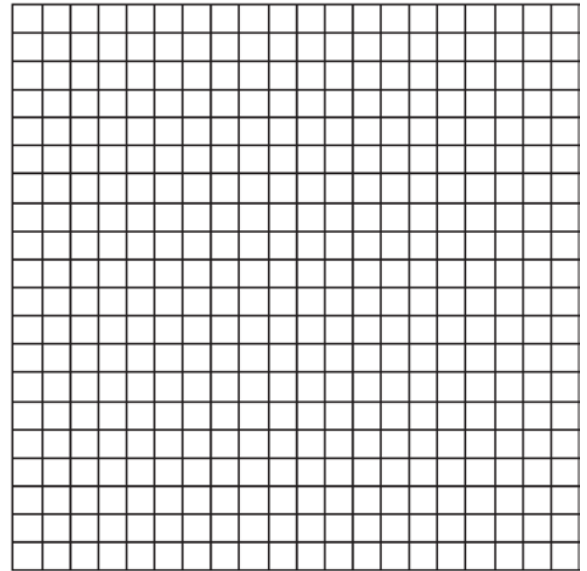
, where A is the amount accrued, P is the principal, r is the interest rate, $n = 12$, and t is the length of time, in years.) [The use of the grid is optional.]



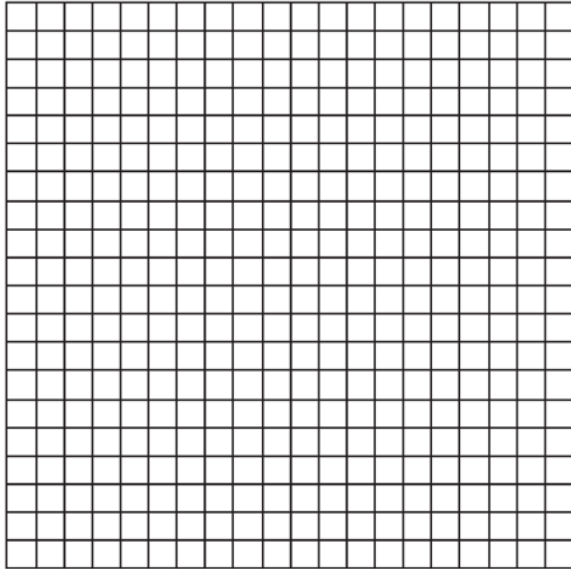
- 8 An amount of P dollars is deposited in an account paying an annual interest rate r (as a decimal) compounded n times per year. After t years, the amount of money in the account, in dollars, is

given by the equation $A = P \left(1 + \frac{r}{n} \right)^{nt}$. Rachel

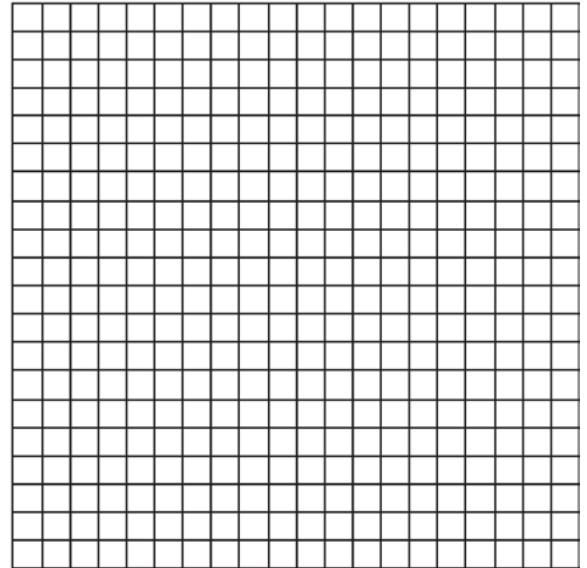
deposited \$1,000 at 2.8% annual interest, compounded monthly. In how many years, to the *nearest tenth of a year*, will she have \$2,500 in the account? [The use of the grid is optional.]



- 9 Since January 1980, the population of the city of Brownville has grown according to the mathematical model $y = 720,500(1.022)^x$, where x is the number of years since January 1980. Explain what the numbers 720,500 and 1.022 represent in this model. If this trend continues, use this model to predict the year during which the population of Brownville will reach 1,548,800. [The use of the grid is optional.]



- 10 After an oven is turned on, its temperature, T , is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where m represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit. How many minutes does it take for the oven's temperature to reach 300°F ? Round your answer to the *nearest minute*. [The use of the grid is optional.]



A.CED.A.1: Exponential Growth Answer Section

1 ANS:

$$10 = 0.5(1.21)^x$$

$$20 = 1.21^x$$

$$\log 20 = \log 1.21^x$$

$$2004. \log 20 = x \log 1.21$$

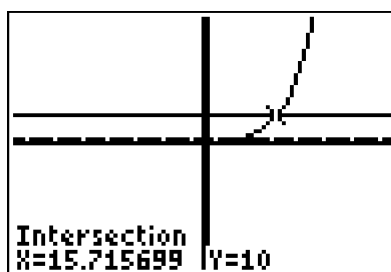
$$x = \frac{\log 20}{\log 1.21}$$

$$x \approx 15.7$$

```

Plot1 Plot2 Plot3
Y1=10
Y2=0.5(1.21)^X
Y3=
Y4=
Y5=
Y6=
Y7=

```



REF: fall9930b

2 ANS:

$$2,500 = 4(2.7)^{0.584t}$$

$$625 = (2.7)^{0.584t}$$

$$\log 625 = \log 2.7^{0.584t}$$

$$11 \log 625 = 0.584t \cdot \log 2.7$$

$$\frac{\log 625}{0.584 \cdot \log 2.7} = t$$

$$t \approx 11.1$$

```

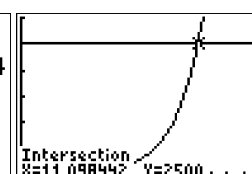
WINDOW
Xmin=0
Xmax=15
Xscl=1
Ymin=0
Ymax=3000
Yscl=500
Xres=1

```

```

Plot1 Plot2 Plot3
Y1=2500
Y2=4(2.7)^(.584X)
Y3=
Y4=
Y5=
Y6=

```



REF: 060224b

3 ANS:

$$1000 = 540(1.039)^t$$

$$\frac{1000}{540} = 1.039^t$$

$$\log \frac{50}{27} = \log 1.039^t$$

$$2011. \log \frac{50}{27} = t \log 1.039$$

$$t = \frac{\log \frac{50}{27}}{\log 1.039}$$

$$x \approx 16.1$$

```

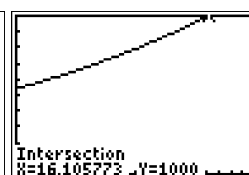
WINDOW
Xmin=0
Xmax=20
Xscl=1
Ymin=0
Ymax=1000
Yscl=100
Xres=1

```

```

Plot1 Plot2 Plot3
Y1=1000
Y2=540(1.039)^X
Y3=
Y4=
Y5=
Y6=

```



REF: 010828b

4 ANS:
78,400, 14.6

REF: 011031b

5 ANS:

$$A = 5000(1.045)^n \quad 5000\left(1 + \frac{.046}{4}\right)^{4(6)} - 5000(1.045)^6 \approx 6578.87 - 6511.30 \approx 67.57 \quad 10000 = 5000\left(1 + \frac{.046}{4}\right)^{4n}$$

$$B = 5000\left(1 + \frac{.046}{4}\right)^{4n}$$

$$2 = 1.0115^{4n}$$

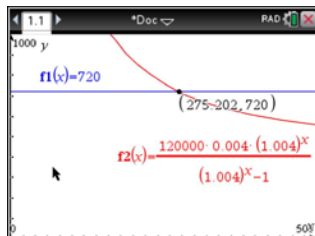
$$\log 2 = 4n \cdot \log 1.0115$$

$$n = \frac{\log 2}{4 \log 1.0115}$$

$$n \approx 15.2$$

REF: 081637aii

6 ANS:



$$720 = \frac{120000\left(\frac{.048}{12}\right)\left(1 + \frac{.048}{12}\right)^n}{\left(1 + \frac{.048}{12}\right)^n - 1} \quad \frac{275.2}{12} \approx 23 \text{ years}$$

$$720(1.004)^n - 720 = 480(1.004)^n$$

$$240(1.004)^n = 720$$

$$1.004^n = 3$$

$$n \log 1.004 = \log 3$$

$$n \approx 275.2 \text{ months}$$

REF: spr1509aii

7 ANS:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$10000 = 5000\left(1 + \frac{.06}{12}\right)^{12t}$$

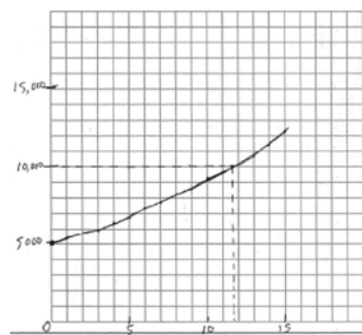
$$2 = 1.005^{12t}$$

$$11.6. \quad \log 2 = \log 1.005^{12t}$$

$$\log 2 = 12t \log 1.005$$

$$t = \frac{\log 2}{12 \log 1.005}$$

$$t \approx 11.6$$



```

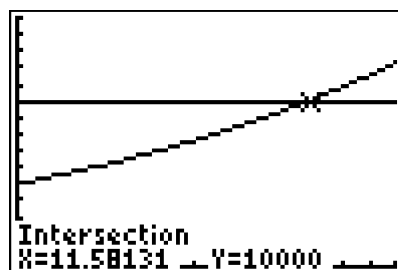
WINDOW
Xmin=0
Xmax=15
Xscl=1
Ymin=0
Ymax=15000
Yscl=1000
Xres=1

```

```

Plot1 Plot2 Plot3
\Y1=5000(1+.06/1
2)^(12X)
\Y2=10000
\Y3=
\Y4=
\Y5=
\Y6=

```



REF: 080832b

8 ANS:

$$2500 = 1000\left(1 + \frac{.028}{12}\right)^{12t}$$

$$\frac{5}{2} = \left(1 + \frac{7}{3000}\right)^{12t}$$

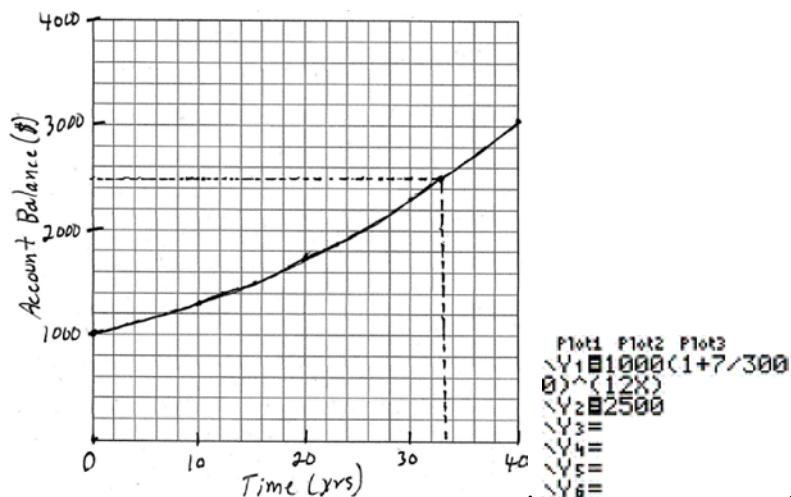
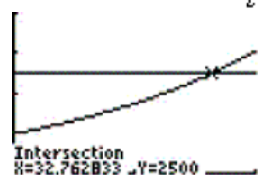
$$\log \frac{5}{2} = \log\left(1 + \frac{7}{3000}\right)^{12t}$$

32.8.

$$\log \frac{5}{2} = 12t \cdot \log\left(1 + \frac{7}{3000}\right)$$

$$\frac{\log \frac{5}{2}}{\log\left(1 + \frac{7}{3000}\right)} = 12t$$

$$t \approx 32.8$$



REF: 080428b

9 ANS:

720,500 is the population in 1980, 1.022 represents a growth rate of 2.2%, 2015.

$$1,548,800 = 720,500(1.022)^x$$

$$\frac{1,548,800}{720,500} = 1.022^x$$

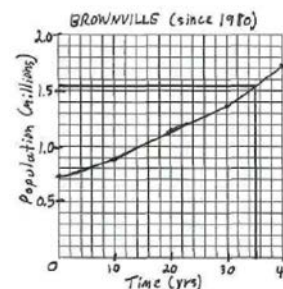
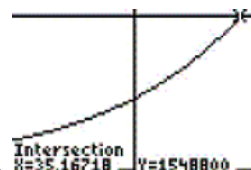
$$\log \frac{1,548,800}{720,500} = \log 1.022^x$$

$$\log \frac{1,548,800}{720,500} = x \log 1.022$$

$$x = \frac{\log \frac{1,548,800}{720,500}}{\log 1.022}$$

$$x \approx 35$$

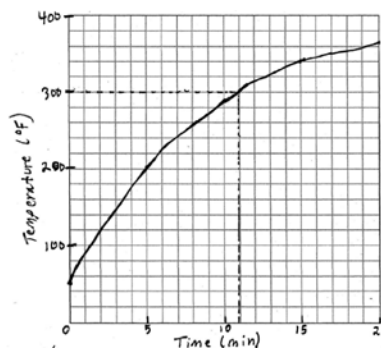
Plot1 Plot2 Plot3
Y1=1548800
Y2=720500(1.022)^X
Y3=
Y4=
Y5=
Y6=



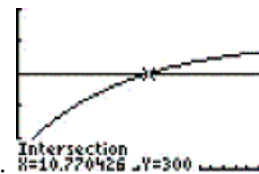
REF: 010728b

10 ANS:

$$\begin{aligned}
 300 &= 400 - 350(3.2)^{-0.1m} \\
 -100 &= -350(3.2)^{-0.1m} \\
 \frac{2}{7} &= (3.2)^{-0.1m} \\
 11. \quad \log \frac{2}{7} &= \log 3.2^{-0.1m} \\
 \log \frac{2}{7} &= -0.1m \cdot \log 3.2 \\
 \frac{\log \frac{2}{7}}{\log 3.2} &= -0.1m \\
 m &\approx 11
 \end{aligned}$$



Plot1 Plot2 Plot3
 Y1 = 400 - 350(3.2)
 Y2 = 300
 Y3 =
 Y4 =
 Y5 =
 Y6 =



REF: 080632b