

Calculus Practice 3.5A3: Differential Equations 1a

For each problem, find the particular solution of the differential equation that satisfies the initial condition.

$$1) \frac{dy}{dx} = 2x\sqrt{y}, y > 0, y(3) = \frac{9}{4}$$

$$A) 2\sqrt{y} = x^2 - 6$$

$$y = \left(\frac{x^2}{2} - 3\right)^2, x > \sqrt{6}$$

$$B) e^y = 4e^x - 1$$

$$y = \ln(4e^x - 1), x > \ln \frac{1}{4}$$

$$C) 2\sqrt{y} = x^2 - 6$$

$$y = \left(\frac{x^2}{2} - 3\right)^2, x < -\sqrt{6}$$

$$D) 2\sqrt{y} = \frac{x^2}{2} - 4$$

$$y = \left(\frac{x^2}{4} - 2\right)^2, x > 2\sqrt{2}$$

$$2) \frac{dy}{dx} = \frac{1}{\cos y}, y(-1) = 0$$

$$A) \sin y = x + 1$$

$$y = \sin^{-1}(x + 1), -2 < x < 0$$

$$B) \sin y = x - 2$$

$$y = \sin^{-1}(x - 2), 1 < x < 3$$

$$C) \sin y = x$$

$$y = \sin^{-1} x, -1 < x < 1$$

$$D) \tan y = x + 3$$

$$y = \tan^{-1}(x + 3)$$

$$3) \frac{dy}{dx} = \frac{2y^2}{x}, y(1) = 1$$

$$A) \frac{y^3}{3} = x + \frac{x^3}{3}$$

$$y = \sqrt[3]{x^3 + 3x}, x > 0$$

$$B) -\frac{1}{2y} = \ln|x| - \frac{1}{2}$$

$$y = -\frac{1}{2\ln|x| - 1}, 0 < x < \sqrt{e}$$

$$C) -\frac{1}{y} = 4x - 3$$

$$y = -\frac{1}{4x - 3}, x > \frac{3}{4}$$

$$D) -\frac{1}{y} = \ln|x| - 1$$

$$y = -\frac{1}{\ln|x| - 1}, 0 < x < e$$

$$4) \frac{dy}{dx} = \frac{1 + x^2}{y^2}, y(1) = \sqrt[3]{4}$$

$$A) \frac{y^3}{3} = 2x + \frac{x^3}{3}$$

$$y = \sqrt[3]{x^3 + 6x}, x < 0$$

$$B) \frac{y^3}{3} = x + \frac{x^3}{3}$$

$$y = \sqrt[3]{x^3 + 3x}, x > 0$$

$$C) \frac{y^3}{3} = -2x + \frac{x^3}{3}$$

$$y = \sqrt[3]{x^3 - 6x}, -\sqrt{6} < x < 0$$

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$$5) \frac{dy}{dx} = -\frac{2yx}{\ln y}, y(-1) = \frac{1}{e}$$

$$A) \ln |y-1| = -\frac{1}{x}$$

$$y = e^{-\frac{1}{x}} + 1, x > 0$$

$$B) \frac{(\ln y)^2}{2} = -\frac{x^2}{2} + 1$$

$$y = e^{-\sqrt{-x^2+2}}, -\sqrt{2} < x < \sqrt{2}$$

$$C) \frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$$

$$y = -\sqrt{-x^2+3}, -\sqrt{3} < x < \sqrt{3}$$

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$$y = e^{-\sqrt{-2x^2+3}}, -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$$

$$6) \frac{dy}{dx} = \frac{y+1}{x}, y(-2) = -3$$

$$A) \ln |y+2| = \ln |x|$$

$$y = x-2, x > 0$$

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$$7) \frac{dy}{dx} = -\frac{2x}{y}, y(-1) = -1$$

$$A) \ln |y+2| = -\frac{1}{x} + \frac{2\ln 3+1}{2}$$

$$y = 3e^{-\frac{1}{x}} - 2, x > 0$$

$$B) \frac{y^2}{2} = -x^2 + \frac{3}{2}$$

$$y = -\sqrt{-2x^2+3}, -\frac{\sqrt{6}}{2} < x < \frac{\sqrt{6}}{2}$$

$$C) \frac{y^2}{2} = -\frac{x^2}{2} + \frac{3}{2}$$

$$y = -\sqrt{-x^2+3}, -\sqrt{3} < x < \sqrt{3}$$

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$$8) \frac{dy}{dx} = \frac{3y}{x^2}, y(1) = -\frac{3}{e^3}$$

$$A) \frac{\ln |y|}{4} = -\frac{1}{x} + \frac{4+\ln 2}{4}$$

$$y = 2e^{-\frac{4}{x}}, x > 0$$

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$$C) \frac{\ln |y|}{3} = -\frac{1}{x} + \frac{3+\ln 3}{3}$$

$$y = -3e^{-\frac{3}{x}}, x > 0$$

$$D) \ln |y+1| = -\frac{1}{x} + \ln 2 + 1$$

$$y = -2e^{-\frac{1}{x}} - 1, x > 0$$

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