Calculus Practice: Related Rates 2

Solve each related rate problem.

1) A hypothetical square shrinks so that the length of its sides (s) are changing at a rate of $-\frac{5}{s}$ m/min. At what rate is the area of the square changing when the sides are 2 m each?

2) A 5 ft tall person is walking towards a 19 ft tall lamppost at a rate of $\frac{3}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 12 ft from the lamppost?

3) A crowd gathers around a movie star, forming a circle. The area (A) taken up by the crowd increases at a rate of $\frac{64\pi}{A}$ ft²/sec. How fast is the radius of the crowd increasing when the radius is 2 ft?

4) A perfect cube shaped ice cube melts so that the length of its sides (s) are decreasing at a rate of $\frac{3}{s}$ mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 4 mm each?

5) A spherical snowball melts so that its radius (r) decreases at a rate of $\frac{2}{r}$ in/sec. At what rate is the volume of the snowball changing when the radius is 6 in?

6) A conical paper cup is 10 cm tall with a radius of 30 cm. The bottom of the cup is punctured so that the water level (h) goes down at a rate of $\frac{3}{h}$ cm/sec. At what rate is the volume of water in the cup changing when the water level is 8 cm?

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Solve each related rate problem.

1) A hypothetical square shrinks so that the length of its sides (s) are changing at a rate of $-\frac{5}{s}$ m/min. At what rate is the area of the square changing when the sides are 2 m each?

A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{ds}{dt} = -\frac{5}{s}$ Find: $\left.\frac{dA}{dt}\right|_{s=2}$

$$\left.\frac{dA}{dt}\right|_{s=2} = 2s \cdot \frac{ds}{dt} = -10 \text{ m}^2\text{/min}$$

2) A 5 ft tall person is walking towards a 19 ft tall lamppost at a rate of $\frac{3}{x}$ ft/sec, where x is the distance from the person to the lamppost. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 12 ft from the lamppost?

x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{19} = \frac{y}{5}$ Given rate: $\frac{dx}{dt} = -\frac{3}{x}$ Find: $\left.\frac{dy}{dt}\right|_{x=12}$

$$\left.\frac{dy}{dt}\right|_{x=12} = \frac{5}{14} \cdot \frac{dx}{dt} = -\frac{5}{56} \text{ ft/sec}$$

3) A crowd gathers around a movie star, forming a circle. The area (A) taken up by the crowd increases at a rate of $\frac{64\pi}{A}$ ft²/sec. How fast is the radius of the crowd increasing when the radius is 2 ft?

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = \frac{64\pi}{A}$ Find: $\left.\frac{dr}{dt}\right|_{r=2}$

$$\left.\frac{dr}{dt}\right|_{r=2} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{4}{\pi} \text{ ft/s}$$

4) A perfect cube shaped ice cube melts so that the length of its sides (s) are decreasing at a rate of $\frac{3}{s}$ mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 4 mm each?

V = volume of cube $\quad s$ = length of sides $\quad t$ = time

Equation: $V = s^3$ $\quad$ Given rate: $\frac{ds}{dt} = -\frac{3}{s}$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{s=4}$

$$\left.\frac{dV}{dt}\right|_{s=4} = 3s^2 \cdot \frac{ds}{dt} = -36 \text{ mm}^3\text{/sec}$$

5) A spherical snowball melts so that its radius (r) decreases at a rate of $\frac{2}{r}$ in/sec. At what rate is the volume of the snowball changing when the radius is 6 in?

V = volume of sphere $\quad r$ = radius $\quad t$ = time

Equation: $V = \frac{4}{3}\pi r^3$ $\quad$ Given rate: $\frac{dr}{dt} = -\frac{2}{r}$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{r=6}$

$$\left.\frac{dV}{dt}\right|_{r=6} = 4\pi r^2 \cdot \frac{dr}{dt} = -48\pi \text{ in}^3\text{/sec}$$

6) A conical paper cup is 10 cm tall with a radius of 30 cm. The bottom of the cup is punctured so that the water level (h) goes down at a rate of $\frac{3}{h}$ cm/sec. At what rate is the volume of water in the cup changing when the water level is 8 cm?

V = volume of material in cone $\quad h$ = height $\quad t$ = time

Equation: $V = 3\pi h^3$ $\quad$ Given rate: $\frac{dh}{dt} = -\frac{3}{h}$ $\quad$ Find: $\left.\frac{dV}{dt}\right|_{h=8}$

$$\left.\frac{dV}{dt}\right|_{h=8} = 9\pi h^2 \cdot \frac{dh}{dt} = -216\pi \text{ cm}^3\text{/sec}$$