F.BF.A.1: Operations with Functions

1 The revenue, $R(x)$, from selling x units of a product is represented by the equation $R(x) = 35x$, while the total cost, $C(x)$, of making x units of the product is represented by the equation $C(x) = 20x + 500$. The total profit, $P(x)$, is represented by the equation $P(x) = R(x) - C(x)$. For the values of $R(x)$ and $C(x)$ given above, what is $P(x)$?
1) $15x$
2) $15x + 500$
3) $15x - 500$
4) $10x + 100$

2 A company produces x units of a product per month, where $C(x)$ represents the total cost and $R(x)$ represents the total revenue for the month. The functions are modeled by $C(x) = 300x + 250$ and $R(x) = -0.5x^2 + 800x - 100$. The profit is the difference between revenue and cost where $P(x) = R(x) - C(x)$. What is the total profit, $P(x)$, for the month?
1) $P(x) = -0.5x^2 + 500x - 150$
2) $P(x) = -0.5x^2 + 500x - 350$
3) $P(x) = -0.5x^2 - 500x + 350$
4) $P(x) = -0.5x^2 + 500x + 350$

3 If $p(x) = ab^x$ and $r(x) = cd^x$, then $p(x) \bullet r(x)$ equals
1) $ac(b+d)^x$
2) $ac(b+d)^{2x}$
3) $ac(bd)^x$
4) $ac(bd)^{x^2}$

4 If $g(c) = 1 - c^2$ and $m(c) = c + 1$, then which statement is *not* true?
1) $g(c) \cdot m(c) = 1 + c - c^2 - c^3$
2) $g(c) + m(c) = 2 + c - c^2$
3) $m(c) - g(c) = c + c^2$
4) $\dfrac{m(c)}{g(c)} = \dfrac{-1}{1-c}$

5 A company calculates its profit by finding the difference between revenue and cost. The cost function of producing x hammers is $C(x) = 4x + 170$. If each hammer is sold for $10, the revenue function for selling x hammers is $R(x) = 10x$. How many hammers must be sold to make a profit? How many hammers must be sold to make a profit of $100?

6 A small, open-top packing box, similar to a shoebox without a lid, is three times as long as it is wide, and half as high as it is long. Each square inch of the bottom of the box costs $0.008 to produce, while each square inch of any side costs $0.003 to produce. Write a function for the cost of the box described above. Using this function, determine the dimensions of a box that would cost $0.69 to produce.

7 A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$. The company's profit, $P(x) = R(x) - C(x)$, could be modeled by
1) $0.15x^3 + 0.02x^2 - 28x + 120$
2) $-0.15x^3 - 0.02x^2 + 28x - 120$
3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$
4) $-0.15x^3 + 32x + 120$

F.BF.A.1: Operations with Functions
Answer Section

1 ANS: 3

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 35x - (20x + 500) \\ &= 15x - 500 \end{aligned}$$

REF: 010220b

2 ANS: 2

$P(x) = -0.5x^2 + 800x - 100 - (300x + 250) = -0.5x^2 + 500x - 350$

REF: 081406ai

3 ANS: 3 REF: 011710aii

4 ANS: 4

$$\frac{m(c)}{g(c)} = \frac{c+1}{1-c^2} = \frac{c+1}{(1+c)(1-c)} = \frac{1}{1-c}$$

REF: 061608aii

5 ANS:

29, 45. $\begin{aligned} R(x) &= C(x) \\ 10x &= 4x + 170 \\ x &= 28.3 \end{aligned}$. If you round down to 28 hammers, the company does not make a profit. Round up to 29. To make a profit of \$100, $\begin{aligned} R(x) - C(x) &= 100 \\ 10x - (4x + 170) &= 100 \\ 6x - 170 &= 100 \\ x &= 45 \end{aligned}$.

REF: 080332b

6 ANS:
$f(w) = .06w^2$, $\sqrt{11.5} \times 3\sqrt{11.5} \times \frac{3}{2}\sqrt{11.5}$. The area of the bottom is $w \times 3w = 3w^2$, where w is the width in inches. If each square inch of the bottom costs \$0.008 to produce, the cost of the bottom may be expressed as $.024w^2$. The area of the two smaller sides is $2 \times w \times \frac{3}{2}w = 3w^2$ and the area of the two larger sides is $2 \times 3w \times \frac{3}{2}w = 9w^2$ for a total area of $12w^2$. If each square inch of a side of the box costs \$0.003 to produce, the cost of the bottom may be expressed as $.036w^2$. Adding the cost of the bottom and sides equals $.06w^2$. A function for the cost of the box is $f(w) = .06w^2$. A box that would cost \$0.69 to produce would have the following dimensions

$$\begin{aligned} .69 &= .06w^2 \\ w &= \sqrt{11.5} \\ l &= 3\sqrt{11.5} \\ h &= \frac{3}{2}\sqrt{11.5} \end{aligned}$$

REF: 080130b

7 ANS: 2

$$\begin{aligned} x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120) &= 30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120 \\ &= -0.15x^3 - 0.02x^2 + 28x - 120 \end{aligned}$$

REF: 061709aii