F.IF.C.9: Comparing Functions 1

1 The quadratic functions \( r(x) \) and \( q(x) \) are given below.

\[
\begin{array}{c|c}
 x & r(x) \\
-4 & -12 \\
-3 & -15 \\
-2 & -16 \\
-1 & -15 \\
0 & -12 \\
1 & 7 \\
\end{array}
\]

\[ q(x) = x^2 + 2x - 8 \]

The function with the smaller minimum value is

1) \( q(x) \), and the value is \(-9\)
2) \( q(x) \), and the value is \(-1\)
3) \( r(x) \), and the value is \(-16\)
4) \( r(x) \), and the value is \(-2\)

2 Which statement is true about the quadratic functions \( g(x) \), shown in the table below, and \( f(x) = (x - 3)^2 + 2 \)?

\[
\begin{array}{c|c}
 x & g(x) \\
0 & 4 \\
1 & -1 \\
2 & -4 \\
3 & -5 \\
4 & -4 \\
5 & -1 \\
6 & 4 \\
\end{array}
\]

1) They have the same vertex.
2) They have the same zeros.
3) They have the same axis of symmetry.
4) They intersect at two points.
3 Given the following quadratic functions:

\[ g(x) = -x^2 - x + 6 \]

and

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
 x & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
 n(x) & -7 & 0 & 5 & 8 & 9 & 8 & 5 & 0 & -7 \\
\end{array}
\]

Which statement about these functions is true?

1) Over the interval \(-1 \leq x \leq 1\), the average rate of change for \(n(x)\) is less than that for \(g(x)\).  
2) The \(y\)-intercept of \(g(x)\) is greater than the \(y\)-intercept for \(n(x)\).  
3) The function \(g(x)\) has a greater maximum value than \(n(x)\).  
4) The sum of the roots of \(n(x) = 0\) is greater than the sum of the roots of \(g(x) = 0\).

4 The function \(h(x)\), which is graphed below, and the function \(g(x) = 2|x + 4| - 3\) are given.

Which statements about these functions are true?

I. \(g(x)\) has a lower minimum value than \(h(x)\).  
II. For all values of \(x\), \(h(x) < g(x)\).  
III. For any value of \(x\), \(g(x) \neq h(x)\).

1) I and II, only  
2) I and III, only  
3) II and III, only  
4) I, II, and III
5. Which function has the greatest y-intercept?
   1) \( f(x) = 3x \)
   2) \( 2x + 3y = 12 \)
   3) the line that has a slope of 2 and passes through \( (1, -4) \)
   4) the graph of \( f(x) \)

6. Which statement is true about the functions \( f(x) \) and \( g(x) \), given below?
   \( f(x) = -x^2 - 4x - 4 \)
   1) The minimum value of \( g(x) \) is greater than the maximum value of \( f(x) \).
   2) \( f(x) \) and \( g(x) \) have the same y-intercept.
   3) \( f(x) \) and \( g(x) \) have the same roots.
   4) \( f(x) = g(x) \) when \( x = -4 \).
7. Nancy works for a company that offers two types of savings plans. Plan A is represented on the graph below.

Plan B is represented by the function $f(x) = 0.01 + 0.05x^2$, where $x$ is the number of weeks. Nancy wants to have the highest savings possible after a year. Nancy picks Plan B. Her decision is

1) correct, because Plan B is an exponential function and will increase at a faster rate
2) correct, because Plan B is a quadratic function and will increase at a faster rate
3) incorrect, because Plan A will have a higher value after 1 year
4) incorrect, because Plan B is a quadratic function and will increase at a slower rate

8. Three functions are shown below.

A: $g(x) = -\frac{3}{2}x + 4$

B: $f(x) = (x + 2)(x + 6)$

C:

Which statement is true?

1) $B$ and $C$ have the same zeros.
2) $A$ and $B$ have the same $y$-intercept.
3) $B$ has a minimum and $C$ has a maximum.
4) $C$ has a maximum and $A$ has a minimum.
9 The functions \(f(x)\), \(q(x)\), and \(p(x)\) are shown below.

\[ q(x) = (x - 1)^2 - 6 \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(p(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

When the input is 4, which functions have the same output value?

1) \(f(x)\) and \(q(x)\), only
2) \(f(x)\) and \(p(x)\), only
3) \(q(x)\) and \(p(x)\), only
4) \(f(x), q(x),\) and \(p(x)\)
10 Three functions are shown below.

\[ g(x) = 3^x + 2 \]

<table>
<thead>
<tr>
<th>x</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>30</td>
</tr>
<tr>
<td>-4</td>
<td>14</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1.5</td>
</tr>
<tr>
<td>2</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

Which statement is true?

1) The y-intercept for \( h(x) \) is greater than the y-intercept for \( f(x) \).

2) The y-intercept for \( f(x) \) is greater than the y-intercept for \( g(x) \).

3) The y-intercept for \( h(x) \) is greater than the y-intercept for both \( g(x) \) and \( f(x) \).

4) The y-intercept for \( g(x) \) is greater than the y-intercept for both \( f(x) \) and \( h(x) \).
11 Given the functions $g(x)$, $f(x)$, and $h(x)$ shown below:

$$g(x) = x^2 - 2x$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

The correct list of functions ordered from greatest to least by average rate of change over the interval $0 \leq x \leq 3$ is

1) $f(x)$, $g(x)$, $h(x)$
2) $h(x)$, $g(x)$, $f(x)$
3) $g(x)$, $f(x)$, $h(x)$
4) $h(x)$, $f(x)$, $g(x)$
12. Which graph does not represent a function that is always increasing over the entire interval \(-2 < x < 2\)?

1)  
2)  
3)  
4)  

13. Which function has a constant rate of change equal to \(-3\)?

1) \(\begin{array}{c|c} x & y \\ \hline 0 & 2 \\ 1 & 5 \\ 2 & 8 \\ 3 & 11 \end{array}\)
2) \{(1,5),(2,2),(3,5),(4,4)\}
3) \(2y = -6x + 10\)
4) \(2y = -6x + 10\)

14. Which quadratic function has the largest maximum?

1) \(h(x) = (3 - x)(2 + x)\)
2) \(g(x) = -3\)
3) \(k(x) = -5x^2 - 12x + 4\)
4) \(g(x) = -3\)
15 Which of the quadratic functions below has the smallest minimum value?

1) \( h(x) = x^2 + 2x - 6 \)

2) \( g(x) = (x - 2)(x + 3) \)

3) \( k(x) = (x + 5)(x + 2) \)

4) \( f(x) = -x^2 + 2x + 6 \)

16 Which quadratic function has the largest maximum over the set of real numbers?

1) \( f(x) = -x^2 + 2x + 4 \)

2) \( k(x) = (x - 1)(x + 3) \)

3) \( g(x) = -(x - 5)^2 + 5 \)

4) \( h(x) = x^2 - 2x + 3 \)
17 The graph representing a function is shown below.

Which function has a minimum that is less than the one shown in the graph?

1) $y = x^2 - 6x + 7$
2) $y = |x + 3| - 6$
3) $y = x^2 - 2x - 10$
4) $y = |x - 8| + 2$

18 Let $f$ be the function represented by the graph below.

Let $g$ be a function such that $g(x) = \frac{1}{2} x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.
F.IF.C.9: Comparing Functions 1
Answer Section

1 ANS: 3
The minimum of \( r(x) \) is \(-16\). The minimum of \( g(x) \) is \(-9\) \( \left\{ x = \frac{-2}{2(1)} = -1; g(-1) = -9 \right\} \).

REF: 081917ai

2 ANS: 3
\( x = 3 \)

REF: 061717ai

3 ANS: 4
1) \[ \frac{g(1) - g(-1)}{1 - (-1)} = \frac{4 - 6}{2} = -\frac{2}{2} = -1 \]
2) \( g(0) = 6 \)
3) \( x = \frac{-(-1)}{2(-1)} = \frac{-1}{2}; g\left( \frac{1}{2} \right) = \left( -\frac{1}{2} \right)^2 + \frac{1}{2} + 6 = 6 \frac{1}{4} \)
4) \( n(0) = 8 \)
   \( n(1) = 9 \)
   \( x = 1; n(1) = 9 \)

REF: 081521ai

4 ANS: 2

REF: 081718ai

5 ANS: 4
1) \( b = 0 \); 2) \( b = 4 \); 3) \( b = -6 \); 4) \( b = 5 \)

REF: 081611ai

6 ANS: 2
The \( y \)-intercept of both \( f(x) \) and \( g(x) \) is \(-4\).

REF: 012013ai

7 ANS: 2

REF: 011723ai
8 ANS: 3  
1) $B$’s zeros are $-2$ and $-6$ and $C$’s zeros are $-4$ and $-2$; 2) $A$’s $y$-intercept is 4 and $B$’s $y$-intercept is 12; 3) $B$ in standard form, $a > 0$ and $C$ in standard form, $a < 0$; d) $A$ has no minimum

REF: 061914ai

9 ANS: 4  
f(4) = q(4) = p(4) = 3

REF: 011921ai

10 ANS: 4  
The $y$-intercept for $f(x)$ is $(0,1)$. The $y$-intercept for $g(x)$ is $(0,3)$. The $y$-intercept for $h(x)$ is $(0,-1)$.

REF: 081811ai

11 ANS: 4  
Over the interval $0 \leq x \leq 3$, the average rate of change for $h(x) = \frac{9 - 2}{3 - 0} = \frac{7}{3}$, $f(x) = \frac{7 - 1}{3 - 0} = \frac{6}{3} = 2$, and  
g(x) = \frac{3 - 0}{3 - 0} = \frac{3}{3} = 1.

REF: spr1301ai

12 ANS: 3  
REF: 061820ai

13 ANS: 4  
1) $y = 3x + 2$; 2) $-\frac{5 - 2}{3 - 2} = -7$; 3) $y = -2x + 3$; 4) $y = -3x + 5$

REF: 081615ai

14 ANS: 3  
$h(x) = -x^2 + x + 6$ Maximum of $f(x) = 9$  
$k(x) = -5x^2 - 12x + 4$ Maximum of $g(x) < 5$

\[
x = \frac{-1}{2(-1)} = \frac{1}{2} \quad x = \frac{12}{2(-5)} = -\frac{6}{5}
\]

\[
y = \left(-\frac{1}{2}\right)^2 + \frac{1}{2} + 6 \quad y = -5\left(-\frac{6}{5}\right)^2 - 12\left(-\frac{6}{5}\right) + 4
\]

\[
= \frac{1}{4} + \frac{2}{4} + 6 \quad = \frac{36}{5} + \frac{72}{5} + \frac{20}{5}
\]

\[
= 6\frac{1}{4} \quad = \frac{56}{5}
\]

\[
= 11\frac{1}{5}
\]

REF: 061514ai
15 ANS: 2

1) \( x = \frac{-2}{2(-1)} = -1, h(-1) = (-1)^2 + 2(-1) - 6 = -7; \) 2) \( y = -10 \); 3) \( k \left( \frac{-5 + 2}{2} \right) = (-3.5 + 5)(-3.5 + 2) = -2.25 \); 4) \( y = -6 \)

REF: 061813ai

16 ANS: 2

1) \( x = \frac{-2}{2(-1)} = 1 \); 2) \( h = \frac{3}{2} \) 3) \( a \left( 0 - \frac{3}{2} \right)^2 + k \); 4) \( c = 1 \)

\( y = -1^2 + 2(1) + 4 = 5 \)

vertex (1,5)

\( \frac{5 - \frac{1}{4}a}{4} = 3 - \frac{9}{4}a \)

\( k = 3 - \frac{9}{4}a \)

Using (0,3), \( y = -12 + \frac{2}{4} + 4 = 5 \)

vertex (1,5)

\( 5 = \frac{1}{4}a + k \)

Using (5,5), \( k = 5 - \frac{1}{4}a \)

Using \( c = 1 \)

\( -9 = (-2)^2a + (-2)b + 1 \)

\( 20 - a = 12 - 9a \)

\( 8a = -8 \)

\( a = -1 \)

\( -3 = (-1)^2a + (-1)b + 1 \)

\( 2a + 5 = a + 4 \)

\( x = \frac{-3}{2(-1)} = \frac{3}{2} \)

vertex \( \left( \frac{3}{2} \right) \)

\( -3 = a - b + 1 \)

\( a = -1 \)

\( b = a + 4 \)

\( b = -1 + 4 = 3 \)

\( y = \left( \frac{3}{2} \right)^2 + 3 \left( \frac{3}{2} \right) + 1 = \frac{9}{4} + 18 + 4 = \frac{13}{4} \)

REF: 011823ai

17 ANS: 3

REF: 011622ai

18 ANS:

\( g \). The maximum of \( f \) is 6. For \( g \), the maximum is 11. \( x = \frac{-b}{2a} = \frac{-4}{-1} = 4 \)

\( y = \frac{1}{2}(4)^2 + 4(4) + 3 = -8 + 16 + 3 = 11 \)

REF: 081429ai