F.IF.C.9: Comparing Quadratic Functions

1 Which quadratic function has the largest maximum over the set of real numbers?

1) $f(x) = -x^2 + 2x + 4$

2)

x	k(x)
−1	−1
0	3
1	5
2	5
3	3
4	−1

3) $g(x) = -(x-5)^2 + 5$

4)

x	h(x)
−2	−9
−1	−3
0	1
1	3
2	3
3	1

2 Which quadratic function has the largest maximum?

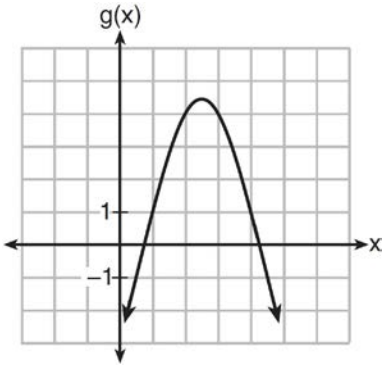
1) $h(x) = (3-x)(2+x)$

2)

x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

3) $k(x) = -5x^2 - 12x + 4$

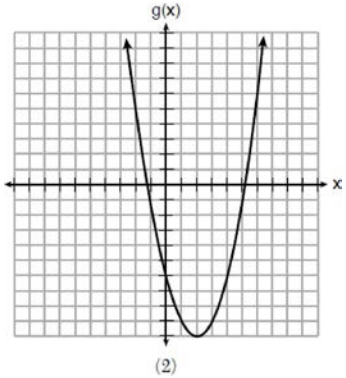
4)



3 Which of the quadratic functions below has the *smallest* minimum value?

1) $h(x) = x^2 + 2x - 6$

2)



3) $k(x) = (x+5)(x+2)$

4)

x	f(x)
−1	−2
0	−5
1	−6
2	−5
3	−2

4 Which quadratic function has the *smallest* minimum value?

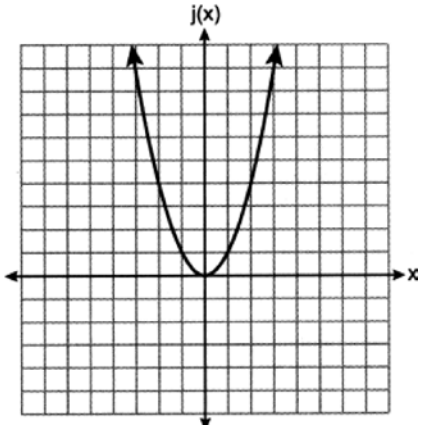
1) $f(x) = 6x^2 + 5x - 2$

2)

x	h(x)
0	6
1	2
2	0
3	0
4	2
5	6

3) $g(x) = 6(x-2)^2 - 2$

4)



5 The quadratic functions $r(x)$ and $q(x)$ are given below.

x	$r(x)$
–4	–12
–3	–15
–2	–16
–1	–15
0	–12
1	7

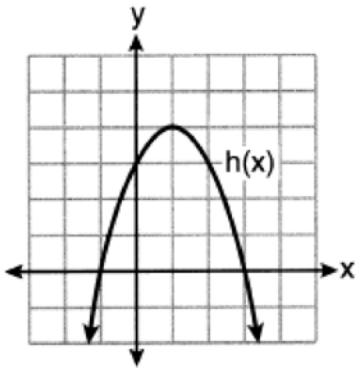
$$q(x) = x^2 + 2x - 8$$

The function with the *smaller* minimum value is

1) $q(x)$, and the value is –9
2) $q(x)$, and the value is –1
3) $r(x)$, and the value is –16
4) $r(x)$, and the value is –2

6 Four quadratic functions are shown below.

x	f(x)
−4	−4
−2	4
−1	5
0	4
2	−4



$g(x) = -(x-4)^2 + 5$

$j(x) = -\frac{1}{2}x^2 + x + 4$

Which statement is true?

1) The maximum of $f(x)$ is less than the maximum of $j(x)$.
2) The maximum of $g(x)$ is less than the maximum of $h(x)$.
3) The maximum of $f(x)$ equals the maximum of $g(x)$.
4) The maximum of $h(x)$ equals the maximum of $j(x)$.

7 Three quadratic functions are given below.

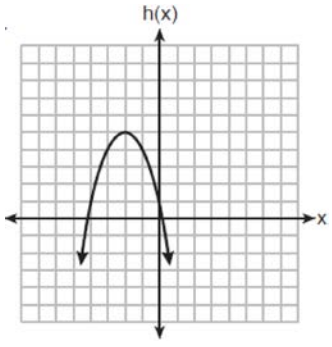
I.

$f(x) = (x+2)^2 + 5$

II.

x	−4	−3	−2	−1	0	1
g(x)	−3	2	5	5	2	−3

III.



Which of these functions have the same vertex?

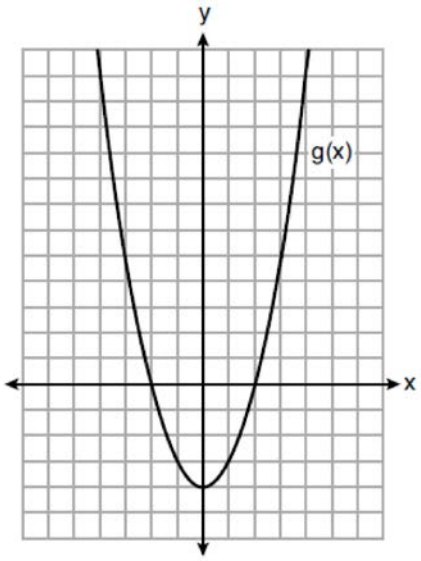
1) I and II, only
2) II and III, only
3) I and III, only
4) I, II, and III

8 Which statement is true about the quadratic functions $g(x)$, shown in the table below, and $f(x) = (x-3)^2 + 2$?

x	g(x)
0	4
1	–1
2	–4
3	–5
4	–4
5	–1
6	4

1) They have the same vertex.
2) They have the same zeros.
3) They have the same axis of symmetry.
4) They intersect at two points.

9 Which statement is true about the functions $f(x)$ and $g(x)$, given below?



$f(x) = -x^2 - 4x - 4$

1) The minimum value of $g(x)$ is greater than the maximum value of $f(x)$.
2) $f(x)$ and $g(x)$ have the same y-intercept.
3) $f(x)$ and $g(x)$ have the same roots.
4) $f(x) = g(x)$ when $x = -4$.

10 Given the following quadratic functions:

$$g(x) = -x^2 - x + 6$$

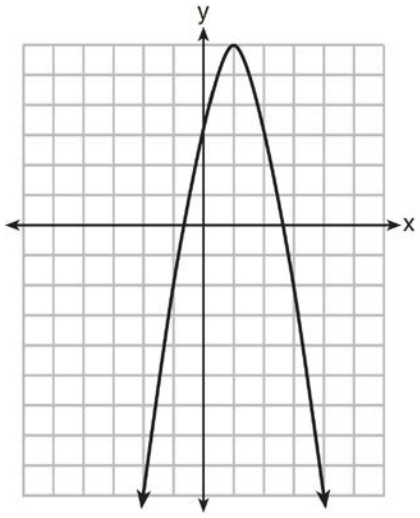
and

x	–3	–2	–1	0	1	2	3	4	5
n(x)	–7	0	5	8	9	8	5	0	–7

Which statement about these functions is true?

1) Over the interval $-1 \le x \le 1$, the average rate of change for $n(x)$ is less than that for $g(x)$.
2) The y-intercept of $g(x)$ is greater than the y-intercept for $n(x)$.
3) The function $g(x)$ has a greater maximum value than $n(x)$.
4) The sum of the roots of $n(x) = 0$ is greater than the sum of the roots of $g(x) = 0$.

11 Let f be the function represented by the graph below.



Let g be a function such that $g(x) = -\frac{1}{2}x^2 + 4x + 3$. Determine which function has the larger maximum value. Justify your answer.

F.IF.C.9: Comparing Quadratic Functions
Answer Section

1 ANS: 2

1) $x = \frac{-2}{2(-1)} = 1$; 2) $h = \frac{3}{2}$ Using (0,3), $3 = a\left(0 - \frac{3}{2}\right)^2 + k$; Using (1,5), $5 = a\left(1 - \frac{3}{2}\right)^2 + k$

$y = -1^2 + 2(1) + 4 = 5$ $3 = \frac{9}{4}a + k$ $5 = \frac{1}{4}a + k$

vertex (1,5)

$k = 3 - \frac{9}{4}a$ $k = 5 - \frac{1}{4}a$

$5 - \frac{1}{4}a = 3 - \frac{9}{4}a$ $k = 5 - \frac{1}{4}(-1) = \frac{21}{4}$; 3) vertex (5,5); 4) Using $c = 1$ $-9 = (-2)^2 a + (-2)b + 1$

$20 - a = 12 - 9a$ vertex $\left(\frac{3}{2}, \frac{21}{4}\right)$ $-10 = 4a - 2b$

$8a = -8$ $b = 2a + 5$

$a = -1$

$-3 = (-1)^2 a + (-1)b + 1$ $2a + 5 = a + 4$ $x = \frac{-3}{2(-1)} = \frac{3}{2}$ vertex $\left(\frac{3}{2}, \frac{13}{4}\right)$

$-3 = a - b + 1$ $a = -1$

$b = a + 4$ $b = -1 + 4 = 3$ $y = -\left(\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) + 1 = -\frac{9}{4} + \frac{18}{4} + \frac{4}{4} = \frac{13}{4}$

REF: 011823ai

2 ANS: 3

$h(x) = -x^2 + x + 6$ Maximum of $f(x) = 9$ $k(x) = -5x^2 - 12x + 4$ Maximum of $g(x) < 5$

$x = \frac{-1}{2(-1)} = \frac{1}{2}$ $x = \frac{12}{2(-5)} = -\frac{6}{5}$

$y = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $y = -5\left(-\frac{6}{5}\right)^2 - 12\left(-\frac{6}{5}\right) + 4$

$= -\frac{1}{4} + \frac{2}{4} + 6$ $= -\frac{36}{5} + \frac{72}{5} + \frac{20}{5}$

$= 6\frac{1}{4}$ $= \frac{56}{5}$

$= 11\frac{1}{5}$

REF: 061514ai

3 ANS: 2

1) $x = \frac{-2}{2(1)} = -1$, $h(-1) = (-1)^2 + 2(-1) - 6 = -7$; 2) $y = -10$; 3) $k\left(\frac{-5+-2}{2}\right) = (-3.5+5)(-3.5+2) = -2.25$; 4) $y = -6$

REF: 061813ai

4 ANS: 1

1) $f\left(\frac{-5}{2(6)}\right) \approx -3.04$; 2) $h(2.5) = (2.5-2)(2.5-3) = -0.25$; 3) $g(2) = -2$; 4) 0

REF: 012320ai

5 ANS: 3

The minimum of $r(x)$ is -16. The minimum of $q(x)$ is -9 $\left(x = \frac{-2}{2(1)} = -1, q(-1) = -9\right)$.

REF: 081917ai

6 ANS: 3

Maximum of $f(x) = 5$ Maximum of $h(x) = 4$ Maximum of $g(x) = 5$ $j(x) = -\frac{1}{2}x^2 + x + 4$

$$x = \frac{-1}{2\left(-\frac{1}{2}\right)} = 1$$

$$j(1) = -\frac{1}{2}(1)^2 + 1 + 4 = 4\frac{1}{2}$$

REF: 062219ai

7 ANS: 3

f and h's vertex is $(-2,5)$. g's axis of symmetry is $x = -1.5$.

REF: 062319ai

8 ANS: 3

$x = 3$

REF: 061717ai

9 ANS: 2

The y-intercept of both $f(x)$ and $g(x)$ is -4.

REF: 012013ai

10 ANS: 4

1) $\dfrac{g(1)-g(-1)}{1--1}=\dfrac{4-6}{2}=\dfrac{-2}{2}=-1$ 2) $g(0)=6$ 3) $x=\dfrac{-(-1)}{2(-1)}=-\dfrac{1}{2};\ g\left(-\dfrac{1}{2}\right)=-\left(-\dfrac{1}{2}\right)^2+\dfrac{1}{2}+6=6\dfrac{1}{4}$

$\dfrac{n(1)-n(-1)}{1--1}=\dfrac{9-5}{2}=\dfrac{4}{2}=2$ $n(0)=8$ $x=1;\ n(1)=9$

4) $g: S=\dfrac{-(-1)}{-1}=-1$

$n: S=-2+4=2$

REF: 081521ai

11 ANS:

g. The maximum of f is 6. For g, the maximum is 11. $x=\dfrac{-b}{2a}=\dfrac{-4}{2\left(-\frac{1}{2}\right)}=\dfrac{-4}{-1}=4$

$$y=-\frac{1}{2}(4)^2+4(4)+3=-8+16+3=11$$

REF: 081429ai