F.LE.A.4: Exponential Decay

1 The equation for radioactive decay is $p = (0.5)^{\frac{t}{H}}$, where p is the part of a substance with half-life H remaining radioactive after a period of time, t. A given substance has a half-life of 6,000 years. After t years, one-fifth of the original sample remains radioactive. Find t, to the *nearest thousand years*.

2 One of the medical uses of Iodine–131 (I–131), a radioactive isotope of iodine, is to enhance x-ray images. The half-life of I–131 is approximately 8.02 days. A patient is injected with 20 milligrams of I–131. Determine, to the *nearest day*, the amount of time needed before the amount of I–131 in the patient's body is approximately 7 milligrams.

3 An archaeologist can determine the approximate age of certain ancient specimens by measuring the amount of carbon-14, a radioactive substance, contained in the specimen. The formula used to determine the age of a specimen is $A = A_0 2^{\frac{-t}{5760}}$, where A is the amount of carbon-14 that a specimen contains, A_0 is the original amount of carbon-14, t is time, in years, and 5760 is the half-life of carbon-14. A specimen that originally contained 120 milligrams of carbon-14 now contains 100 milligrams of this substance. What is the age of the specimen, to the *nearest hundred years*?

4 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m. Using this equation, solve for h, to the *nearest ten thousandth*. Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

F.LE.A.4: Exponential Decay
Answer Section

1 ANS:

14,000.

$$p = (.5)^{\frac{t}{H}}$$
$$.2 = (.5)^{\frac{t}{6000}}$$
$$\log .2 = \log .5^{\frac{t}{6000}}$$
$$\log .2 = \frac{t}{6000}\log .5$$
$$\frac{\log .2}{\log .5} = \frac{t}{6000}$$
$$t \approx 14000$$

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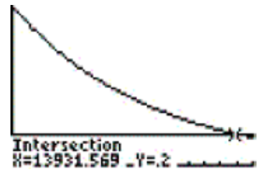

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2 ANS:

$$7 = 20(0.5)^{\frac{t}{8.02}}$$
$$\log 0.35 = \log 0.5^{\frac{t}{8.02}}$$
$$\log 0.35 = \frac{t \log 0.5}{8.02}$$
$$\frac{8.02 \log 0.35}{\log 0.5} = t$$
$$t \approx 12$$

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3 ANS:

1,500.

$$100 = 120(2)^{\frac{-t}{5760}}$$

$$\frac{5}{6} = (2)^{\frac{-t}{5760}}$$

$$\log\frac{5}{6} = \log 2^{\frac{-t}{5760}}$$

$$\log\frac{5}{6} = \frac{-t}{5760}\log 2$$

$$\frac{\log\frac{5}{6}}{\log 2} = \frac{-t}{5760}$$

$$t \approx 1500$$

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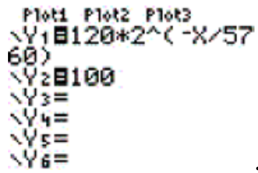

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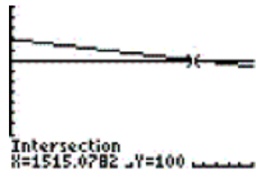

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4 ANS:

$$100 = 140\left(\frac{1}{2}\right)^{\frac{5}{h}} \quad \log\frac{100}{140} = \log\left(\frac{1}{2}\right)^{\frac{5}{h}}$$

$$\log\frac{5}{7} = \frac{5}{h}\log\frac{1}{2}$$

$$h = \frac{5\log\frac{1}{2}}{\log\frac{5}{7}} \approx 10.3002$$

$$40 = 140\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$\log\frac{2}{7} = \log\left(\frac{1}{2}\right)^{\frac{t}{10.3002}}$$

$$\log\frac{2}{7} = \frac{t\log\left(\frac{1}{2}\right)}{10.3002}$$

$$t = \frac{10.3002\log\frac{2}{7}}{\log\frac{1}{2}} \approx 18.6$$

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