

F.TF.C.9: Angle Sum and Difference Identities 2b

- 1 If A and B are positive acute angles, $\sin A = \frac{5}{13}$, and $\cos B = \frac{4}{5}$, what is the value of $\sin(A + B)$?
- 2 If $\sin x = \frac{12}{13}$, $\cos y = \frac{3}{5}$, and x and y are acute angles, the value of $\cos(x - y)$ is
- 3 If $\sin A = \frac{3}{5}$, $\sin B = \frac{5}{13}$, and angles A and B are acute angles, what is the value of $\cos(A - B)$?
- 4 If A and B are both acute angles, $\sin A = \frac{5}{13}$ and $\sin B = \frac{4}{5}$, then $\sin(A - B)$ is
- 5 If $\cos x = \frac{12}{13}$ and $\sin y = \frac{4}{5}$, then $\sin(x - y)$ equals
- 6 Given angle A in Quadrant I with $\sin A = \frac{12}{13}$ and angle B in Quadrant II with $\cos B = -\frac{3}{5}$, what is the value of $\cos(A - B)$?
- 7 If $\sin A = \frac{4}{5}$, $\tan B = \frac{5}{12}$, and angles A and B are in Quadrant I, what is the value of $\sin(A + B)$?
- 8 If $\tan A = 8$ and $\tan B = \frac{1}{2}$, what is the value of $\tan(A + B)$?
- 9 If $\tan A = \frac{2}{3}$ and $\tan B = \frac{1}{2}$, what is the value of $\tan(A + B)$?
- 10 If $\tan A = \frac{2}{3}$ and $\sin B = \frac{5}{\sqrt{41}}$ and angles A and B are in Quadrant I, find the value of $\tan(A + B)$.
- 11 If $\tan A = \frac{2}{3}$ and $\tan B = 3$, express $\tan(A - B)$ as a fraction in simplest form.
- 12 When $\sin x = -\frac{8}{17}$ and x lies in Quadrant III and $\cos y = -\frac{4}{5}$ and y lies in Quadrant II, what is $\cos(x - y)$?
- 13 Using the formula for $\cos(x - y)$, find the exact value of $\cos 15^\circ$ in radical form if $m\angle x = 45$ and $m\angle y = 30$.
- 14 Express as a single fraction the exact value of $\sin 75^\circ$.
- 15 If $\sin x = \frac{4}{5}$, where $0^\circ < x < 90^\circ$, find the value of $\cos(x + 180^\circ)$.

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Answer Section

1 ANS:

$$\frac{56}{65}$$

If $\sin A = \frac{5}{13}$ and A is a positive acute angle, $\cos A = \frac{12}{13}$. If $\cos B = \frac{4}{5}$ and B is a positive acute angle, $\sin B = \frac{3}{5}$.

$$\sin(A + B) = \frac{5}{13} \cdot \frac{4}{5} + \frac{12}{13} \cdot \frac{3}{5} = \frac{56}{65}$$

REF: 060312b

2 ANS:

$$\frac{63}{65}$$

If $\sin x = \frac{12}{13}$ and x is an acute angle, $\cos x = \frac{5}{13}$. If $\cos y = \frac{3}{5}$ and y is an acute angle, $\sin y = \frac{4}{5}$.

$$\cos(x - y) = \frac{5}{13} \cdot \frac{3}{5} + \frac{12}{13} \cdot \frac{4}{5} = \frac{63}{65}$$

REF: 080316b

3 ANS:

$$\frac{63}{65}$$

REF: 018620siii

4 ANS:

$$-\frac{33}{65}$$

REF: 069020siii

5 ANS:

$$-\frac{33}{65}$$

REF: 089432siii

6 ANS:

$$\frac{33}{65}$$

$$\cos(A - B) = \left(\frac{5}{13}\right)\left(-\frac{3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right) = -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$$

REF: 011214a2

7 ANS:

$$\frac{63}{65}$$

If $\sin A = \frac{4}{5}$ and angle A is in Quadrant I, $\cos A = \frac{3}{5}$. If $\tan B = \frac{5}{12}$, and angle B is in Quadrant I, then

$$\tan^2 B + 1 = \sec^2 B$$

$$\left(\frac{5}{12}\right)^2 + 1 = \sec^2 B$$

$$\frac{25}{144} + 1 = \sec^2 B$$

$$\frac{169}{144} = \sec^2 B$$

$$\frac{13}{12} = \sec B$$

$$\frac{12}{13} = \cos B$$

$$\cos^2 B + \sin^2 B = 1$$

$$\left(\frac{12}{13}\right)^2 + \sin^2 B = 1$$

$$\frac{144}{169} + \sin^2 B = 1$$

$$\sin^2 B = \frac{25}{169}$$

$$\sin B = \frac{5}{13}$$

$$\cdot \text{ Then } \sin(A+B) = \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13} = \frac{63}{65}.$$

REF: 080409b

8 ANS:

$$-\frac{17}{6}$$

REF: 069525siii

9 ANS:

$$\frac{7}{4}$$

REF: 010018siii

10 ANS:

$$\frac{23}{2} \quad \cos^2 B + \sin^2 B = 1 \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{5}{\sqrt{41}}}{\frac{4}{\sqrt{41}}} = \frac{5}{4} \quad \tan(A+B) = \frac{\frac{2}{3} + \frac{5}{4}}{1 - \left(\frac{2}{3}\right)\left(\frac{5}{4}\right)} = \frac{\frac{8+15}{12}}{\frac{12}{12} - \frac{10}{12}} = \frac{\frac{23}{12}}{\frac{2}{12}} = \frac{23}{2}$$

$$\cos^2 B + \left(\frac{5}{\sqrt{41}}\right)^2 = 1$$

$$\cos^2 B + \frac{25}{41} = \frac{41}{41}$$

$$\cos^2 B = \frac{16}{41}$$

$$\cos B = \frac{4}{\sqrt{41}}$$

REF: 081037a2

11 ANS:

$$-\frac{7}{9}$$

REF: 088405siii

12 ANS:

$$\frac{36}{85}$$

REF: 060141siii

13 ANS:

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

REF: 089041siii

14 ANS:

$$\sin(45 + 30) = \sin 45 \cos 30 + \cos 45 \sin 30$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

REF: 061136a2

15 ANS:

$$\begin{aligned} \cos(A + B) &= \cos x \cos 180^\circ - \sin x \sin 180^\circ \\ -\frac{3}{5}. \text{ If } \sin x &= \frac{4}{5} \text{ and } 0^\circ < x < 90^\circ, \text{ then } \cos x = \frac{3}{5}. &= \frac{3}{5}(-1) - \frac{4}{5}(0) \\ &= -\frac{3}{5} \end{aligned}$$

REF: 080126b