## G.C.A.2: Chords, Secants and Tangents 20

1 In the accompanying diagram,  $\triangle ABC$  is inscribed in circle  $O, \overline{AP}$  bisects  $\angle BAC, \overline{PBD}$  is tangent to circle O at B, and  $m \angle ACB : m \angle CAB : m \angle ABC = 4:3:2$ 



Find:  $m \angle ABC$ ,  $\widehat{mBF}$ ,  $m \angle BEP$ ,  $m \angle P$ ,  $m \angle PBC$ 

2 In the accompanying diagram of circle O, diameters  $\overline{BD}$  and  $\overline{AE}$ , secants  $\overline{PAB}$  and  $\overline{PDC}$ , and chords  $\overline{BC}$  and  $\overline{AD}$  are drawn; mAD = 40; and m $\overline{DC} = 80$ .



Find:  $\widehat{\text{mAB}}$ ,  $\mathbb{m}\angle BCD$ ,  $\mathbb{m}\angle BOE$ ,  $\mathbb{m}\angle P$ ,  $\mathbb{m}\angle PAD$ 

3 In the accompanying diagram of circle O, diameter  $\overline{CA}$  intersects chord  $\overline{BD}$  at F;  $\overline{AE}$  is a tangent;  $\overline{EDC}$  is a secant,  $\overline{CB}$ ,  $\overline{BA}$ , and  $\overline{AD}$  are chords;  $\overline{mBC} = 100$ ; and  $\overline{mAD} = 70$ .



Find:  $\widehat{mAB}$ ,  $\mathbb{m}\angle AEC$ ,  $\mathbb{m}\angle BCA$ ,  $\mathbb{m}\angle DFA$ ,  $\mathbb{m}\angle DAE$ .

4 In the accompanying diagram of circle *O* with inscribed isosceles triangle *ABC*,  $\overline{AB} \cong \overline{AC}$ ,  $\widehat{mCB} = 60$ ,  $\overline{FC}$  is a tangent, and secant  $\overline{FBA}$ intersects diameter  $\overline{CD}$  at *E*.



Find:  $m \angle ADC$ ,  $m \widehat{AD}$ ,  $m \angle DEB$ ,  $m \angle AFC$ ,  $m \angle BCF$ 

5 In the accompanying diagram,  $\overrightarrow{PA}$  is tangent to circle *O* at point *A*, secant  $\overrightarrow{PBD}$  intersects diameter  $\overrightarrow{AC}$  at point *E*, chord  $\overrightarrow{AB}$  is drawn, m $\angle P = 40$ , and  $\overrightarrow{mCD}:\overrightarrow{mDA} = 1:8$ .



Find:  $\widehat{mDA}$ ,  $\widehat{mAB}$ ,  $\underline{m}\angle BEA$ ,  $\underline{m}\angle BAC$ ,  $\underline{m}\angle PBA$ .

6 In the accompanying diagram, isosceles triangle ABC is inscribed in circle O, and vertex angle BAC measures 40°. Tangent  $\overline{PC}$ , secant  $\overline{PBA}$ , and diameters  $\overline{BD}$  and  $\overline{AE}$  are drawn.



Find:  $\widehat{mBC}$ ,  $\mathbb{m}\angle ABD$ ,  $\mathbb{m}\angle DOE$ ,  $\mathbb{m}\angle P$ ,  $\mathbb{m}\angle ACP$ .

7 In the accompanying diagram, regular pentagon *ABCDE* is inscribed in circle *O*, chords  $\overline{EC}$  and  $\overline{DB}$ intersect at *F*, chord  $\overline{DB}$  is extended to *G*, and tangent  $\overline{GA}$  is drawn.



Find:  $m \angle BDE$ ,  $m \angle BFC$ ,  $m \angle AGD$ 

8 In the accompanying diagram of circle *O*, chord  $\overline{AB}$ is parallel to diameter  $\overline{EC}$ , secant  $\overline{PBD}$  intersects  $\overline{EC}$  at *F*, tangent  $\overline{PA}$  is drawn,  $\overline{mAB} = \overline{mBC}$ , and  $\overline{mCD} = 80$ .



Find:  $\widehat{\mathsf{mAE}}$ ,  $\mathsf{m}\angle ABD$ ,  $\mathsf{m}\angle DFC$ ,  $\mathsf{m}\angle P$ ,  $\mathsf{m}\angle PAB$ .

9 In the accompanying diagram of circle O, diameter  $\overline{EOC}$  is extended through C to point P; diameter  $\overline{AFOD}$ , tangent  $\overline{PD}$ , and chords  $\overline{AC}$ ,  $\overline{CD}$ ,  $\overline{BFE}$  are drawn; m $\angle COD = 60$ ; and m $\angle AFB = 100$ .



Find:  $\widehat{\text{mDE}}$ ;  $\mathbb{m} \angle P$ ;  $\mathbb{m} \angle ACE$ ,  $\widehat{\mathbb{mAB}}$ ,  $\mathbb{m} \angle ACD$ .

10 In the accompanying diagram of circle O,  $\widehat{\mathbf{mAB}}:\widehat{\mathbf{mBC}} = 1:2$ ; diameter  $\overline{CA}$  and chord  $\overline{AE}$  are drawn; chord  $\overline{EC}$  is parallel to chord  $\overline{AB}$ ; chord  $\overline{BC}$ is extended through C to D; and tangent  $\overline{DE}$  is drawn.



Find:  $\widehat{mBC}$ ,  $\widehat{mCE}$ ,  $\underline{m}\angle AEC$ ,  $\underline{m}\angle CED$ ,  $\underline{m}\angle BDE$ .

11 In the accompanying diagram of circle O,  $\widehat{mAC} = 140, \ \widehat{mAE} = 130, \ \widehat{mAB}: \widehat{mBC} = 6:4, \ \overline{PD}$  is a tangent, secant  $\overline{PCE}$  intersects diameter  $\overline{AD}$  at F, and secant  $\overline{PBA}$  is drawn.



Find m $\widehat{ED}$ , m $\widehat{AB}$ , m $\angle BAD$ , m $\angle APE$ , m $\angle EFD$ 

12 In the accompanying diagram of circle *O*, diameter  $\overline{AE}$  is extended through *E* to *C*; tangent  $\overline{CB}$ , chord  $\overline{AB}$ , and radius  $\overline{OB}$  are drawn; and  $\widehat{mAB}:\widehat{mBE} = 2:1$ .



*a* Find:  $\widehat{mAB}$ ,  $\underline{m}\angle BAC$ ,  $\underline{m}\angle C$ ,  $\underline{m}\angle ABC$ . *b* Is  $\triangle OBC$  acute, right, obtuse or equiangular? Explain your answer.

13 In the accompanying diagram of circle *O*, diameter  $\overline{AD}$ , chord  $\overline{AE}$ , and secants  $\overline{CBA}$  and  $\overline{CDE}$  are drawn; m $\angle BAD = 40$ ; and m $\overline{AE} = 5(\overline{mED})$ .



Find:  $\widehat{mBD}$ ,  $\widehat{mAE}$ ,  $\underline{m}\angle ACE$ ,  $\underline{m}\angle AED$ ,  $\underline{m}\angle ADC$ .

14 In the accompanying diagram of circle O,  $\overline{AOED}$  is a diameter,  $\overline{PD}$  is a tangent,  $\overline{PBA}$  is a secant, chords  $\overline{BD}$  and  $\overline{BEC}$  are drawn, m $\angle DAB = 43$ , and m $\angle DEC = 72$ .



Find:  $m \angle BDP$ ,  $m \widehat{AB}$ ,  $m \widehat{AC}$ ,  $m \angle P$ ,  $m \angle CBD$ 

15 In the accompanying diagram of circle *O*, *AOEC* is a diameter,  $\overrightarrow{PC}$  is a tangent,  $\overrightarrow{PBA}$  is a secant,  $\overrightarrow{BED}$ is a chord,  $\overrightarrow{AO} = 8$ , and  $\overrightarrow{mAB}:\overrightarrow{mBC}:\overrightarrow{mCD}:\overrightarrow{mDA} = 3:2:1:4.$ 



Find:  $\widehat{mBC}$ ,  $m \angle P$ ,  $m \angle BEC$ , AP to the *nearest* tenth

16 In the accompanying diagram of circle *O*, tangent  $\overline{PA}$ , secant  $\overline{PGFB}$ , diameter  $\overline{AOEB}$ , and chord  $\overline{CEFD}$  are drawn;  $\widehat{mCA} = 70$ ;  $\widehat{mDG} = 90$ ; and  $\underline{m\angle CEA} = 40$ .



Find:  $\widehat{mCB}$ ,  $\widehat{mBD}$ ,  $m\angle APB$ ,  $m\angle PAB$ ,  $m\angle ABG$ 

17 In the accompanying diagram of circle O, tangent  $\overrightarrow{PA}$ , secant  $\overrightarrow{PBEC}$ , and chords  $\overrightarrow{AB}$ ,  $\overrightarrow{AD}$ , and  $\overrightarrow{CD}$  are drawn; m $\angle C = 30$ , m $\overrightarrow{AB} = 100$ ; m $\overrightarrow{AC} : m\overrightarrow{CD} = 4:1$ .



Find:  $\widehat{mCD}$ ,  $\underline{m}\angle BAP$ ,  $\underline{m}\angle CDA$ ,  $\underline{m}\angle AEB$ ,  $\underline{m}\angle P$ 

18 In the accompanying diagram of circle *O*, tangent  $\overline{AB}$  and chord  $\overline{BC}$  are drawn, secant  $\overline{ACD}$  intersects diameter  $\overline{EB}$  at *F*,  $\widehat{mBD} = 160$ , and  $\widehat{mBC} = 80$ .



Find:  $m \angle A$ ,  $m \angle ABE$ ,  $m \angle ABC$ ,  $m \angle EFC$ ,  $m \angle ACB$ 

19 In the accompanying diagram of circle *O*, secant  $\overline{PFCQ}$ , secant  $\overline{PAOEB}$ , tangent  $\overline{QB}$ , and chord  $\overline{CEG}$  are drawn;  $\widehat{mBC}:\widehat{mCF}:\widehat{mFA}=7:8:3$ ; and  $\underline{m\angle AEG}=95$ .



Find:  $\widehat{\mathrm{mCF}}$ ,  $\widehat{\mathrm{mAG}}$ ,  $\mathbb{m} \angle P$ ,  $\mathbb{m} \angle FCG$ ,  $\mathbb{m} \angle FQB$ 

20 In the accompanying diagram of circle *O*, secant  $\overrightarrow{ABP}$ , secant  $\overrightarrow{CDP}$ , and chord  $\overrightarrow{AC}$  are drawn; chords  $\overrightarrow{AD}$  and  $\overrightarrow{BD}$  intersect at *E*, tangent  $\overrightarrow{GCF}$  intersects circle *O* at *C*, and  $\overrightarrow{mAB}:\overrightarrow{mBD}:\overrightarrow{mDC}:\overrightarrow{mCA}=8:2:5:3.$ 



Find:  $\widehat{\mathbf{mCA}}$ ,  $\mathbf{m} \angle ACB$ ,  $\mathbf{m} \angle P$ ,  $\mathbf{m} \angle AEB$ ,  $\mathbf{m} \angle DCF$ 

21 In the accompanying diagram of circle *O*, tangent  $\overrightarrow{PB}$ , secant  $\overrightarrow{AECP}$ , chord  $\overrightarrow{DEB}$ , and chord  $\overrightarrow{CB}$  are drawn;  $\overrightarrow{mDC} = 90$ ;  $\overrightarrow{m\angle DEC} = 85$ ; BP = 15; and CB = 8.



Find:  $\widehat{mAB}$ ;  $\underline{m}\angle ACB$ ;  $\underline{m}\angle P$  to the *nearest degree*.

22 In the accompanying diagram of circle *O*, chords  $\overline{BD}$ ,  $\overline{BC}$ , and  $\overline{AC}$ , tangent  $\overline{PC}$ , and secant  $\overline{ABP}$  are drawn; m $\angle DBC = 40$ , m $\angle AEB = 110$ ; and m $\widehat{AD}$ :m $\widehat{CB} = 9$ :5.



Find:  $\widehat{\mathsf{mAB}}$ ,  $\widehat{\mathsf{mAD}}$ ,  $\mathbb{m}\angle P$ ,  $\mathbb{m}\angle BCP$ ,  $\mathbb{m}\angle ACP$ 

23 In the accompanying diagram of circle O,  $\overrightarrow{PA}$  is tangent to the circle at A;  $\overrightarrow{PDC}$  is a secant; diameter  $\overrightarrow{AEOC}$  intersects chord  $\overrightarrow{BD}$  at E; chords  $\overrightarrow{AB}$ ,  $\overrightarrow{BC}$ , and  $\overrightarrow{DA}$  are drawn;  $\overrightarrow{mDA} = 46$ ; and  $\overrightarrow{mBC}$ is 32 more than  $\overrightarrow{mAB}$ .



Find:  $\widehat{\mathsf{mAB}}$ ;  $\mathbb{m}\angle BAC$ ;  $\mathbb{m}\angle P$ ;  $\mathbb{m}\angle DEC$ ;  $\mathbb{m}\angle PDA$ 

## ID: A

## G.C.A.2: Chords, Secants and Tangents 20 Answer Section

1 ANS: 40, 60, 70, 50, 60 REF: 010436siii 2 ANS: 140, 90, 40, 30, 90 REF: 080036siii 3 ANS: 80, 55, 40, 85, 35 REF: 019639siii 4 ANS: 75, 30, 135, 45, 30 REF: 069636siii 5 ANS: 160, 80, 50, 50, 100 REF: 089636siii 6 ANS: 80, 20, 140, 30, 110 REF: 069737siii 7 ANS: 72, 72, 36 REF: 089738siii 8 ANS: 60, 80, 100, 50, 30 REF: 019839siii 9 ANS: 120, 30, 30, 80, 90 REF: 069837siii 10 ANS: 120, 60, 90, 30, 60 REF: 089842siii 11 ANS: 50, 84, 48, 37, 95 REF: 019937siii

12 ANS: 120, 30, 30, 120, right because  $m \angle OBC = 90$ REF: 069939siii 13 ANS: 80, 150, 35, 90, 105 REF: 089937siii 14 ANS: 43, 94, 130, 47, 25 REF: 010036siii 15 ANS: 72, 54, 108, 19.8 REF: 010136siii 16 ANS: 110, 10, 50, 90, 40 REF: 060136siii 17 ANS: 40, 50, 80, 70, 30 REF: 080140siii 18 ANS: 40, 90, 40, 130, 100 REF: 010239siii 19 ANS: 80, 120, 20, 75, 70 REF: 060240siii 20 ANS: 60, 80, 10, 130, 50 REF: 080242siii 21 ANS: 80, 40, 20 REF: 010336siii 22 ANS: 140, 90, 60, 25, 95 REF: 060336siii 23 ANS: 74, 53, 67, 104, 90 REF: 080338siii