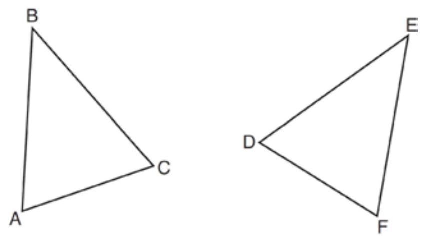
G.CO.B.7: Triangle Congruency

1 Which statement is sufficient evidence that $\triangle DEF$ is congruent to $\triangle ABC$?

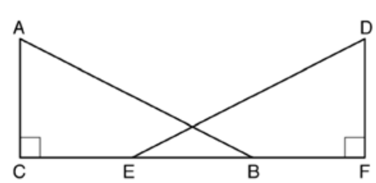


1) $AB = DE$ and $BC = EF$
2) $\angle D \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle F$
3) There is a sequence of rigid motions that maps $\overline{AB}$ onto $\overline{DE}$, $\overline{BC}$ onto $\overline{EF}$, and $\overline{AC}$ onto $\overline{DF}$.
4) There is a sequence of rigid motions that maps point A onto point D, $\overline{AB}$ onto $\overline{DE}$, and $\angle B$ onto $\angle E$.

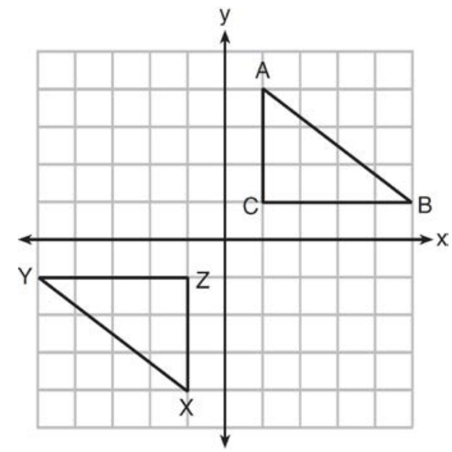
2 In the two distinct acute triangles ABC and DEF, $\angle B \cong \angle E$. Triangles ABC and DEF are congruent when there is a sequence of rigid motions that maps

1) $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$
2) $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$
3) $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$
4) point A onto point D, and $\overline{AB}$ onto $\overline{DE}$

3 Given right triangles ABC and DEF where $\angle C$ and $\angle F$ are right angles, $\overline{AC} \cong \overline{DF}$ and $\overline{CB} \cong \overline{FE}$. Describe a precise sequence of rigid motions which would show $\triangle ABC \cong \triangle DEF$.

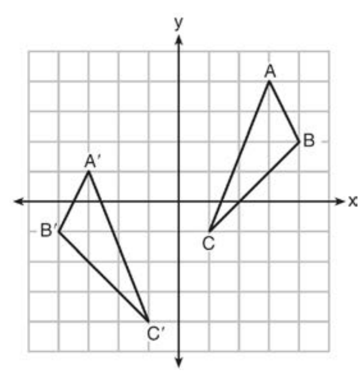


4 In the diagram below, $\triangle ABC$ and $\triangle XYZ$ are graphed.



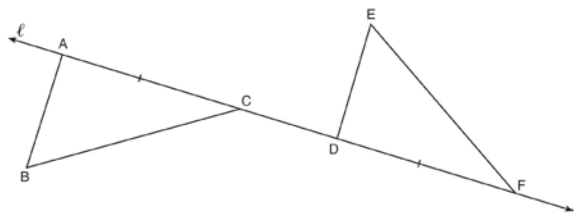
Use the properties of rigid motions to explain why $\triangle ABC \cong \triangle XYZ$.

5 As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.



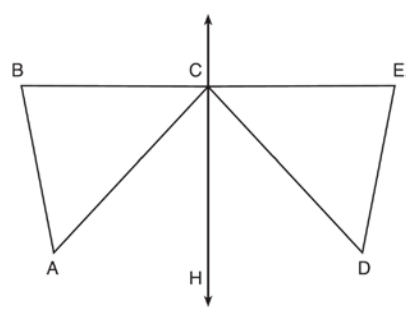
Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.

6 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A, C, D, and F are collinear on line ℓ.



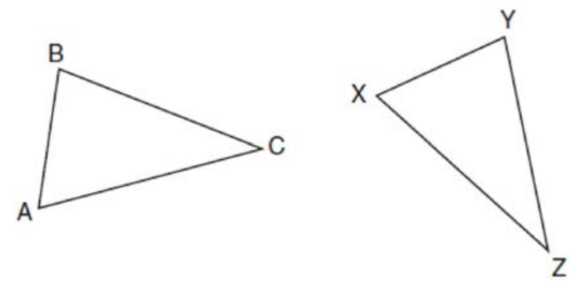
Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along ℓ, such that point D is mapped onto point A. Determine and state the location of F'. Explain your answer. Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line ℓ. Suppose that E'' is located at B. Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

7 Given: D is the image of A after a reflection over $\overleftrightarrow{CH}$.
$\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BCE}$
$\triangle ABC$ and $\triangle DEC$ are drawn
Prove: $\triangle ABC \cong \triangle DEC$



8 After a reflection over a line, $\triangle A'B'C'$ is the image of $\triangle ABC$. Explain why triangle ABC is congruent to triangle $\triangle A'B'C'$.

9 In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.



Determine and state whether $\overline{BC} \cong \overline{YZ}$. Explain why.

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Answer Section

1 ANS: 3 REF: 061524geo

2 ANS: 3

NYSED has stated that all students should be awarded credit regardless of their answer to this question.

REF: 061722geo

3 ANS:

Translate $\triangle ABC$ along $\overline{CF}$ such that point C maps onto point F, resulting in image $\triangle A'B'C'$. Then reflect $\triangle A'B'C'$ over $\overline{DF}$ such that $\triangle A'B'C'$ maps onto $\triangle DEF$.

or

Reflect $\triangle ABC$ over the perpendicular bisector of $\overline{EB}$ such that $\triangle ABC$ maps onto $\triangle DEF$.

REF: fall1408geo

4 ANS:

The transformation is a rotation, which is a rigid motion.

REF: 081530geo

5 ANS:

Yes. The sequence of transformations consists of a reflection and a translation, which are isometries which preserve distance and congruency.

REF: 011628geo

6 ANS:

Translations preserve distance. If point D is mapped onto point A, point F would map onto point C. $\triangle DEF \cong \triangle ABC$ as $\overline{AC} \cong \overline{DF}$ and points are collinear on line ℓ and a reflection preserves distance.

REF: 081534geo

7 ANS:

It is given that point D is the image of point A after a reflection in line CH. It is given that $\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BCE}$ at point C. Since a bisector divides a segment into two congruent segments at its midpoint, $\overline{BC} \cong \overline{EC}$. Point E is the image of point B after a reflection over the line CH, since points B and E are equidistant from point C and it is given that $\overleftrightarrow{CH}$ is perpendicular to $\overline{BE}$. Point C is on $\overleftrightarrow{CH}$, and therefore, point C maps to itself after the reflection over $\overleftrightarrow{CH}$. Since all three vertices of triangle ABC map to all three vertices of triangle DEC under the same line reflection, then $\triangle ABC \cong \triangle DEC$ because a line reflection is a rigid motion and triangles are congruent when one can be mapped onto the other using a sequence of rigid motions.

REF: spr1414geo

8 ANS:

Reflections are rigid motions that preserve distance.

REF: 061530geo

9 ANS:
Yes. $\angle A \cong \angle X$, $\angle C \cong \angle Z$, $\overline{AC} \cong \overline{XZ}$ after a sequence of rigid motions which preserve distance and angle measure, so $\triangle ABC \cong \triangle XYZ$ by ASA. $\overline{BC} \cong \overline{YZ}$ by CPCTC.

REF: 081730geo