G.GPE.A.1: Equations of Circles 2b

1. Kevin’s work for deriving the equation of a circle is shown below.
   
   \[ x^2 + 4x = -(y^2 - 20) \]
   
   STEP 1 \[ x^2 + 4x = -y^2 + 20 \]
   
   STEP 2 \[ x^2 + 4x + 4 = -y^2 + 20 - 4 \]
   
   STEP 3 \[ (x + 2)^2 = -y^2 + 20 - 4 \]
   
   STEP 4 \[ (x + 2)^2 + y^2 = 16 \]

   In which step did he make an error in his work?

2. The equation \[ x^2 + y^2 - 2x + 6y + 3 = 0 \] is equivalent to
   
   1) \[ (x - 1)^2 + (y + 3)^2 = -3 \]
   
   2) \[ (x - 1)^2 + (y + 3)^2 = 7 \]
   
   3) \[ (x + 1)^2 + (y + 3)^2 = 7 \]
   
   4) \[ (x + 1)^2 + (y + 3)^2 = 10 \]

3. The equation \[ 4x^2 - 24x + 4y^2 + 72y = 76 \] is equivalent to
   
   1) \[ 4(x - 3)^2 + 4(y + 9)^2 = 76 \]
   
   2) \[ 4(x - 3)^2 + 4(y + 9)^2 = 121 \]
   
   3) \[ 4(x - 3)^2 + 4(y + 9)^2 = 166 \]
   
   4) \[ 4(x - 3)^2 + 4(y + 9)^2 = 436 \]

4. What are the coordinates of the center of a circle whose equation is \[ x^2 + y^2 - 16x + 6y + 53 = 0 \]?

5. The equation of a circle is \[ x^2 + y^2 + 6y = 7 \]. What are the coordinates of the center and the length of the radius of the circle?

6. What are the center and radius of the circle whose equation is \[ x^2 + y^2 + 4x = 5 \]?

7. The equation of a circle is \[ x^2 + y^2 - 12y + 20 = 0 \]. What are the coordinates of the center and the length of the radius of the circle?

8. The equation of a circle is \[ x^2 + y^2 - 6y + 1 = 0 \]. What are the coordinates of the center and the length of the radius of this circle?

9. The equation of a circle is \[ x^2 + 8x + y^2 - 12y = 144 \].
   
   What are the coordinates of the center and the length of the radius of the circle?
   
   1) center \((4, -6)\) and radius 12
   
   2) center \((-4, 6)\) and radius 12
   
   3) center \((4, -6)\) and radius 14
   
   4) center \((-4, 6)\) and radius 14

10. What are the coordinates of the center and length of the radius of the circle whose equation is \[ x^2 + 6x + y^2 - 4y = 23 \]?

11. What are the coordinates of the center and the length of the radius of the circle represented by the equation \[ x^2 + y^2 - 4x + 8y + 11 = 0 \]?

12. The equation of a circle is \[ x^2 + y^2 - 6x + 2y = 6 \].
   
   What are the coordinates of the center and the length of the radius of the circle?

13. What are the coordinates of the center and the length of the radius of the circle whose equation is \[ x^2 + y^2 = 8x - 6y + 39 \]?

14. If \[ x^2 + 4x + y^2 - 6y - 12 = 0 \] is the equation of a circle, the length of the radius is

15. An equation of circle \(O\) is \[ x^2 + y^2 + 4x - 8y = -16 \].
   
   The statement that best describes circle \(O\) is the
   
   1) center is \((2, -4)\) and is tangent to the \(x\)-axis
   
   2) center is \((2, -4)\) and is tangent to the \(y\)-axis
   
   3) center is \((-2, 4)\) and is tangent to the \(x\)-axis
   
   4) center is \((-2, 4)\) and is tangent to the \(y\)-axis

16. Determine and state the coordinates of the center and the length of the radius of a circle whose equation is \[ x^2 + y^2 - 6x = 56 - 8y \].
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Answer Section

1. ANS:
   Step 2
   REF: 061603geo

2. ANS: 2
   \[ x^2 - 2x + y^2 + 6y = -3 \]
   \[ x^2 - 2x + 1 + y^2 + 6y + 9 = -3 + 1 + 9 \]
   \[ (x - 1)^2 + (y + 3)^2 = 7 \]
   REF: 061016a2

3. ANS: 4
   \[ 4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324 \]
   \[ 4(x - 3)^2 + 4(y + 9)^2 = 436 \]
   REF: 061619aii

4. ANS: (8, -3)
   \[ x^2 + y^2 - 16x + 6y + 53 = 0 \]
   \[ x^2 - 16x + 64 + y^2 + 6y + 9 = -53 + 64 + 9 \]
   \[ (x - 8)^2 + (y + 3)^2 = 20 \]
   REF: 011415a2

5. ANS:
   center (0, -3) and radius 4
   \[ x^2 + y^2 + 6y + 9 = 7 + 9 \]
   \[ x^2 + (y + 3)^2 = 16 \]
   REF: 061514geo

6. ANS: (-2, 0) and 3
   \[ x^2 + y^2 + 4x = 5 \]
   \[ x^2 + 4x + 4 + y^2 = 5 + 4 \]
   \[ (x + 2)^2 + y^2 = 9 \]
   REF: 081626a2
7 ANS:  
center (0,6) and radius 4  
\[ x^2 + y^2 - 12y + 36 = -20 + 36 \]
\[ x^2 + (y - 6)^2 = 16 \]

REF: 061712geo

8 ANS:  
center (0,3) and radius \(2\sqrt{2}\)  
\[ x^2 + y^2 - 6y + 9 = -1 + 9 \]
\[ x^2 + (y - 3)^2 = 8 \]

REF: 011718geo

9 ANS: 4  
\[ x^2 + 8x + 16 + y^2 - 12y + 36 = 144 + 16 + 36 \]
\[ (x + 4)^2 + (y - 6)^2 = 196 \]

REF: 061920geo

10 ANS:  
\((-3,2)\) and 6  
\[ x^2 + 6x + 9 + y^2 - 4y + 4 = 23 + 9 + 4 \]
\[ (x + 3)^2 + (y - 2)^2 = 36 \]

REF: 011617geo

11 ANS:  
center (2,−4) and radius 3  
\[ x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16 \]
\[ (x - 2)^2 + (y + 4)^2 = 9 \]

REF: 081616geo

12 ANS:  
center (3,−1) and radius 4  
\[ x^2 + y^2 - 6x + 2y = 6 \]
\[ x^2 - 6x + 9 + y^2 + 2y + 1 = 6 + 9 + 1 \]
\[ (x - 3)^2 + (y + 1)^2 = 16 \]

REF: 011812geo
13 ANS:
   center (4, -3) and radius 8
   \[ x^2 - 8x + y^2 + 6y = 39 \]
   \[ x^2 - 8x + 16 + y^2 + 6y + 9 = 39 + 16 + 9 \]
   \[ (x - 4)^2 + (y + 3)^2 = 64 \]

REF: 081906geo

14 ANS:
   5
   \[ x^2 + 4x + 4 + y^2 - 6y + 9 = 12 + 4 + 9 \]
   \[ (x + 2)^2 + (y - 3)^2 = 25 \]

REF: 081509geo

15 ANS: 4
   \[ x^2 + 4x + 4 + y^2 - 8y + 16 = -16 + 4 + 16 \]
   \[ (x + 2)^2 + (y - 4)^2 = 4 \]

REF: 081821geo

16 ANS:
   \[ x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16 \] (3, -4); \( r = 9 \)
   \[ (x - 3)^2 + (y + 4)^2 = 81 \]

REF: 081731geo