G.SRT.B.5: Triangle Proofs 2

1. Given: \( \overline{BE} \) and \( \overline{AD} \) intersect at point \( C \)
   \( \overline{BC} \cong \overline{EC} \)
   \( \overline{AC} \cong \overline{DC} \)
   \( \overline{AB} \) and \( \overline{DE} \) are drawn
   Prove: \( \triangle ABC \cong \triangle DEC \)

2. In the diagram of \( \triangle MAH \) below, \( \overline{MH} \cong \overline{AH} \) and medians \( \overline{AB} \) and \( \overline{MT} \) are drawn.
   Prove: \( \angle MBA \cong \angle ATM \)

3. Complete the partial proof below for the accompanying diagram by providing reasons for steps 3, 6, 8, and 9.

   Given: \( \overline{AFCD}, \overline{AB} \perp \overline{BC}, \overline{DE} \perp \overline{EF}, \overline{BC} \parallel \overline{FE}, \overline{AB} \cong \overline{DE} \)
   Prove: \( \overline{AC} \cong \overline{FD} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \overline{AFCD} )</td>
<td>1 Given</td>
</tr>
<tr>
<td>2 ( \overline{AB} \perp \overline{BC}, \overline{DE} \perp \overline{EF} )</td>
<td>2 Given</td>
</tr>
<tr>
<td>3 ( \angle B ) and ( \angle E ) are right angles.</td>
<td>3</td>
</tr>
<tr>
<td>4 ( \overline{BC} \parallel \overline{FE} )</td>
<td>5 Given</td>
</tr>
<tr>
<td>5 ( \overline{BC} \parallel \overline{FE} )</td>
<td>5 Given</td>
</tr>
<tr>
<td>6 ( \overline{ABC} \cong \overline{EFD} )</td>
<td>6</td>
</tr>
<tr>
<td>7 ( \overline{AB} \cong \overline{DE} )</td>
<td>7 Given</td>
</tr>
<tr>
<td>8 ( \overline{ABC} \cong \overline{DEF} )</td>
<td>8</td>
</tr>
<tr>
<td>9 ( \overline{AC} \cong \overline{FD} )</td>
<td>9</td>
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</tbody>
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4. Given: \( \triangle ABC, \overline{AEC}, \overline{BDE} \) with \( \angle ABE \cong \angle CBE \), and \( \angle ADE \cong \angle CDE \)
   Prove: \( \overline{BDE} \) is the perpendicular bisector of \( \overline{AC} \)
Fill in the missing statement and reasons below.

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>1 $\triangle ABC, \triangle AEC, \triangle BDE$ with $\angle ABE \cong \angle CBE$, and $\angle ADE \cong \angle CDE$</td>
<td>1 Given</td>
</tr>
<tr>
<td>2 $BD \cong BD$</td>
<td>2</td>
</tr>
<tr>
<td>3 $\angle BDA$ and $\angle ADE$ are supplementary. $\angle BDC$ and $\angle CDE$ are supplementary.</td>
<td>3 Linear pairs of angles are supplementary.</td>
</tr>
<tr>
<td>4</td>
<td>4 Supplements of congruent angles are congruent.</td>
</tr>
<tr>
<td>5 $\triangle ABD \cong \triangle CBD$</td>
<td>5 ASA</td>
</tr>
<tr>
<td>6 $AD \cong CD, AB \cong CB$</td>
<td>6</td>
</tr>
<tr>
<td>7 $BD$ is the perpendicular bisector of $AC$.</td>
<td>7</td>
</tr>
</tbody>
</table>

5 Given: $\triangle ABC, \overline{BD}$ bisects $\angle ABC$, $\overline{BD} \perp \overline{AC}$
Prove: $AB \cong CB$

6 Given: $\overline{AD}$ bisects $\overline{BC}$ at $E$.
\[
\begin{align*}
\frac{AB}{BC} &= \frac{DC}{BC} \\
\text{Prove: } AB &\cong DC
\end{align*}
\]
7. Given: $\triangle ABC$ and $\triangle EDC$, $C$ is the midpoint of $BD$ and $AE$
Prove: $AB \parallel DE$

8. Given: $RS$ and $TV$ bisect each other at point $X$
$TR$ and $SV$ are drawn
Prove: $TR \parallel SV$

9. Given: $MT$ and $HA$ intersect at $B$, $MA \parallel HT$, and $MT$ bisects $HA$.
Prove: $MA \cong HT$
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Answer Section

1 ANS:
\( \overline{BE} \) and \( \overline{AD} \) intersect at point \( C \), \( \overline{BC} \cong \overline{EC}, \overline{AC} \cong \overline{DC}, \overline{AB} \) and \( \overline{DE} \) are drawn (Given). \( \angle BCA \cong \angle ECD \) (Vertical Angles). \( \triangle ABC \cong \triangle DEC \) (SAS).

REF: 011529ge

2 ANS:
\( \triangle MAH, \overline{MH} \cong \overline{AH} \) and medians \( \overline{AB} \) and \( \overline{MT} \) are given. \( \overline{MA} \cong \overline{AM} \) (reflexive property). \( \triangle MAH \) is an isosceles triangle (definition of isosceles triangle). \( \angle AMB \cong \angle MAT \) (isosceles triangle theorem). \( B \) is the midpoint of \( \overline{MH} \) and \( T \) is the midpoint of \( \overline{AH} \) (definition of midpoint). \( \overline{MB} \cong \overline{AT} \) (multiplication postulate). \( \triangle MBA \cong \triangle ATM \) (SAS). \( \triangle MBA \cong \triangle ATM \) (CPCTC).

REF: 061338ge

3 ANS:
3 Perpendicular line segments form right angles; 6 If two parallel lines are cut by a transversal, the alternate interior angles are congruent; 8 AAS; 9 CPCTC.

REF: 060229b

4 ANS:
2 Reflexive; 4 \( \angle BDA \cong \angle BDC \); 6 CPCTC; 7 If points \( B \) and \( D \) are equidistant from the endpoints of \( \overline{AC} \), then \( B \) and \( D \) are on the perpendicular bisector of \( \overline{AC} \).

REF: 081832geo

5 ANS:
\( \triangle ABC, \overline{BD} \) bisects \( \angle ABC, \overline{BD} \perp \overline{AC} \) (Given). \( \angle CBD \cong \angle ABD \) (Definition of angle bisector). \( \overline{BD} \cong \overline{BD} \) (Reflexive property). \( \angle CDB \) and \( \angle ADB \) are right angles (Definition of perpendicular). \( \angle CDB \cong \angle ADB \) (All right angles are congruent). \( \triangle CDB \cong \triangle ADB \) (SAS). \( \overline{AB} \cong \overline{CB} \) (CPCTC).

REF: 081335ge

6 ANS:
\( \angle B \) and \( \angle C \) are right angles because perpendicular lines form right angles. \( \angle B \cong \angle C \) because all right angles are congruent. \( \angle AEB \cong \angle DEC \) because vertical angles are congruent. \( \triangle ABE \cong \triangle DCE \) because of ASA. \( \angle B \cong \angle D \) because CPCTC.

REF: 061235ge
7 ANS: 
\[ \overline{AC} \cong \overline{EC} \] and \[ \overline{DC} \cong \overline{BC} \] because of the definition of midpoint. \[ \angle ACB \cong \angle ECD \] because of vertical angles. 
\[ \triangle ABC \cong \triangle EDC \] because of SAS. \[ \angle CDE \cong \angle CBA \] because of CPCTC. \[ BD \] is a transversal intersecting \( AB \) and \( ED \). Therefore \( AB \parallel DE \) because \( \angle CDE \) and \( \angle CBA \) are congruent alternate interior angles.

REF: 060938ge

8 ANS: 
\( RS \) and \( TV \) bisect each other at point \( X \); \( TR \) and \( SV \) are drawn (given); \( TX \cong XV \) and \( RX \cong XS \) (segment bisectors create two congruent segments); \( \angle TXR \cong \angle VXS \) (vertical angles are congruent); \( \triangle TXR \cong \triangle VXS \) (SAS); \( \angle T \cong \angle V \) (CPCTC); \( TR \parallel SV \) (a transversal that creates congruent alternate interior angles cuts parallel lines).

REF: 061733geo

9 ANS: 
\( MT \) and \( HA \) intersect at \( B \), \( MA \parallel HT \), and \( MT \) bisects \( HA \) (Given). \( \angle MBA \cong \angle TBH \) (Vertical Angles). \[ \angle A \cong \angle H \] (Alternate Interior Angles). \( BH \cong BA \) (The bisection of a line segment creates two congruent segments). \( \triangle MAB \cong \triangle THB \) (ASA). \( MA \cong HT \) (CPCTC).

REF: 081435ge