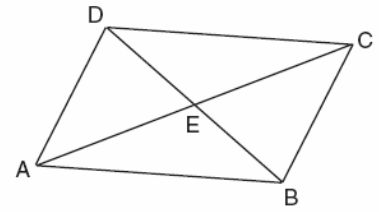
CHAPTER 9-1

SPECIAL QUADRILATERALS

1. 080735a, P.I. G.G.39
In rhombus *ABCD*, the measure, in inches, of $\overline{AB}$ is $3x+2$ and $\overline{BC}$ is $x+12$. Find the number of inches in the length of $\overline{DC}$.

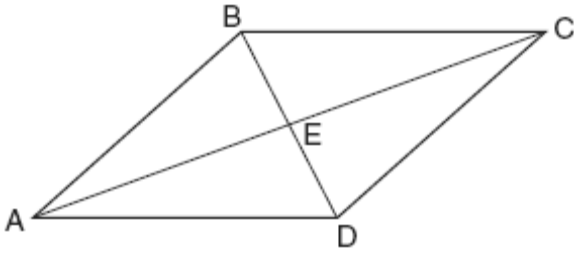
2. 080202a, P.I. G.G.38
In the accompanying diagram of parallelogram *ABCD*, diagonals $\overline{AC}$ and $\overline{DB}$ intersect at *E*, $AE = 3x-4$, and $EC = x+12$.



What is the value of *x*?

[A] 40 [B] 16 [C] 20 [D] 8

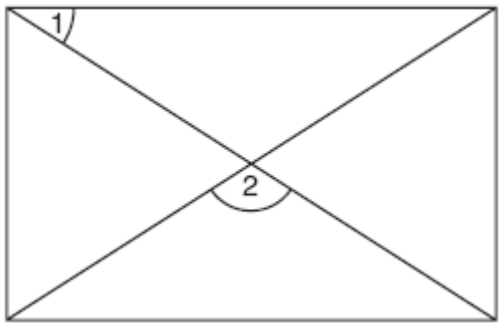
3. 060626a, P.I. G.G.38
In the accompanying diagram of parallelogram *ABCD*, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*, $BE = \frac{2}{3}x$, and $ED = x-10$.



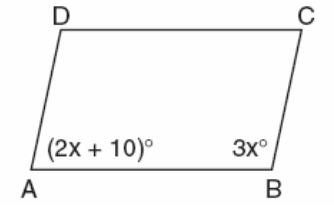
What is the value of *x*?

[A] 6 [B] 30 [C] -6 [D] -30

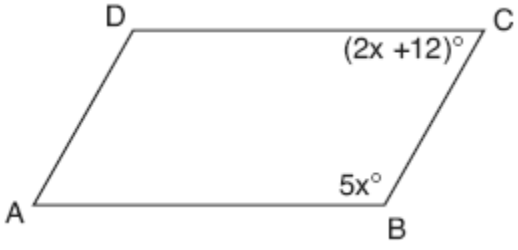
4. 010835a, P.I. G.G.38
As shown in the accompanying diagram, a rectangular gate has two diagonal supports. If $m\angle 1 = 42$, what is $m\angle 2$?



5. 060126a, P.I. G.G.38
In the accompanying diagram of parallelogram *ABCD*, $m\angle A = (2x+10)$ and $m\angle B = 3x$. Find the number of degrees in $m\angle B$.



6. 060736a, P.I. G.G.38
In the accompanying diagram of parallelogram *ABCD*, $m\angle B = 5x$ and $m\angle C = 2x+12$. Find the number of degrees in $\angle D$.



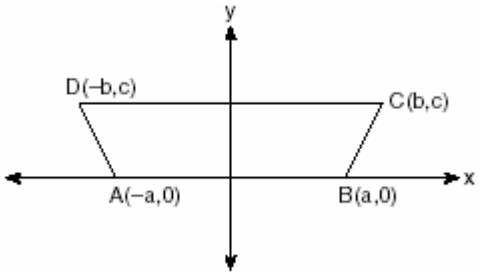
7. 080618a, P.I. G.G.38
The measures of two consecutive angles of a parallelogram are in the ratio 5:4. What is the measure of an obtuse angle of the parallelogram?

[A] 80° [B] 160° [C] 100° [D] 20°

CHAPTER 9-2

PROOFS

8. 080534b, P.I. G.G.27
In the accompanying diagram of *ABCD*, where $a \neq b$, prove *ABCD* is an isosceles trapezoid.



CHAPTER 9-3

SPECIAL QUADRILATERALS

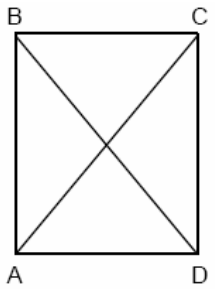
9. 060526a, P.I. G.G.39
Which quadrilateral must have diagonals that are congruent and perpendicular?

[A] rhombus [B] square

[C] trapezoid [D] parallelogram

10. 010533a, P.I. G.G.39
In rectangle *ABCD*, $AC = 3x + 15$ and $BD = 4x - 5$. Find the length of $\overline{AC}$.

11. 089909a, P.I. G.G.39
In the accompanying diagram of rectangle ABCD, $m\angle BAC = 3x + 4$ and $m\angle ACD = x + 28$.



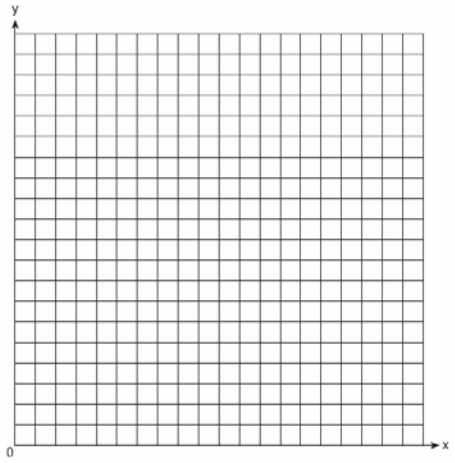
What is $m\angle CAD$?

[A] 37 [B] 50 [C] 40 [D] 12

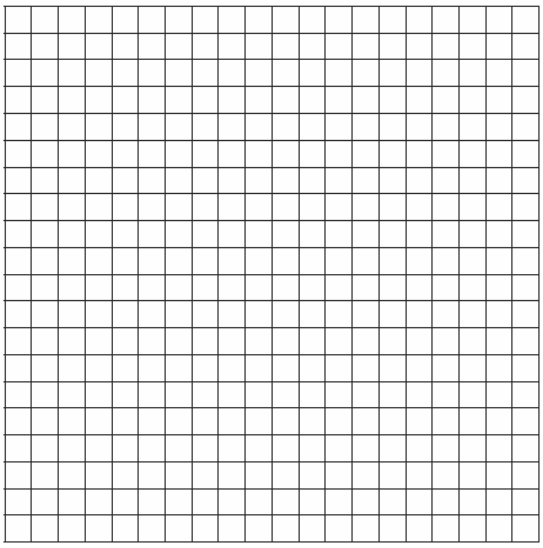
CHAPTER 9-6

PROOFS

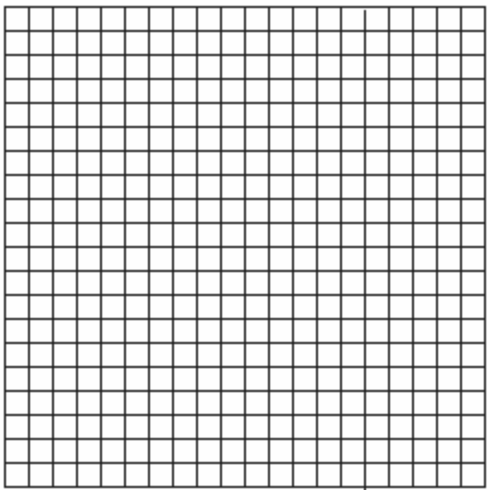
12. 080032a, P.I. G.G.69
Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram *ABCD* are *A*(0,0), *B*(5,2), and *C*(6,5). Find the coordinates of point *D* and sketch parallelogram *ABCD* on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.



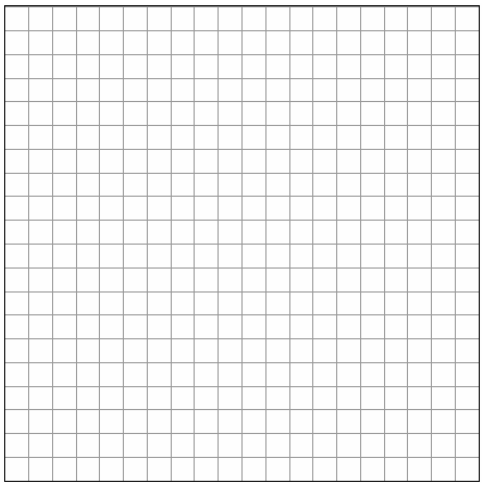
13. 060733b, P.I. G.G.69
Given: quadrilateral *ABCD* with vertices $A(-2,2)$, $B(8,-4)$, $C(6,-10)$, and $D(-4,-4)$. State the coordinates of *A'B'C'D'*, the image of quadrilateral *ABCD* under a dilation of factor $\frac{1}{2}$. Prove that *A'B'C'D'* is a parallelogram. [The use of the grid is optional.]



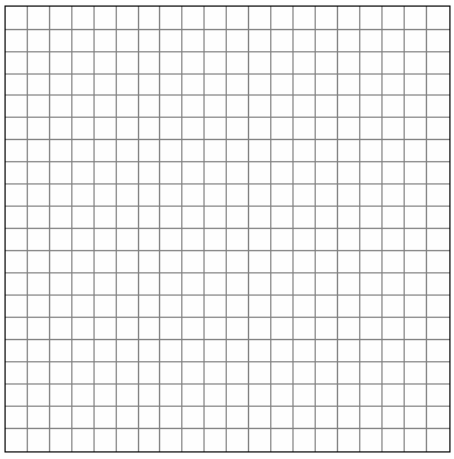
14. 060633b, P.I. G.G.69
Given: *A*(-2,2), *B*(6,5), *C*(4,0), *D*(-4,-3)
Prove: *ABCD* is a parallelogram but not a rectangle. [The use of the grid is optional.]



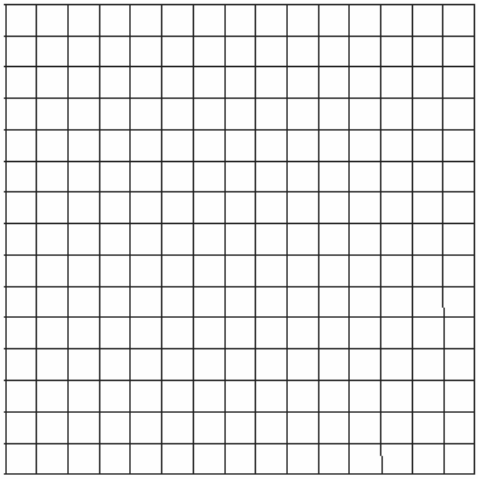
15. 060327b, P.I. G.G.69
The coordinates of quadrilateral *ABCD* are *A*(-1,-5), *B*(8,2), *C*(11,13), and *D*(2,6). Using coordinate geometry, prove that quadrilateral *ABCD* is a rhombus. [The use of the grid is optional.]



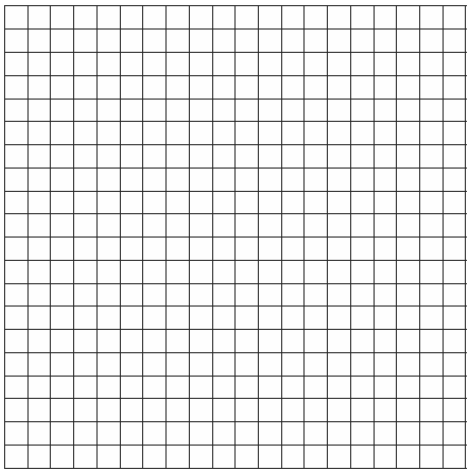
16. 010533b, P.I. G.G.69
Jim is experimenting with a new drawing program on his computer. He created quadrilateral *TEAM* with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$. Jim believes that he has created a rhombus but not a square. Prove that Jim is correct. [The use of the grid is optional.]



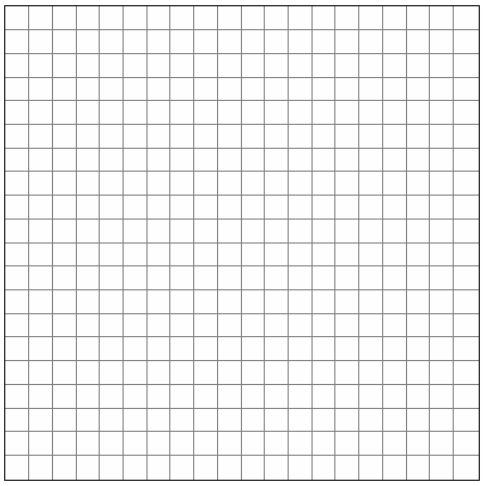
17. 080134b, P.I. G.G.69
Given: *A*(1,6), *B*(7,9), *C*(13,6), and *D*(3,1)
Prove: *ABCD* is a trapezoid. [*The use of the accompanying grid is optional.*]



18. 010333b, P.I. G.G.69
Quadrilateral *KATE* has vertices *K*(1,5), *A*(4,7), *T*(7,3), and *E*(1,-1).
a Prove that *KATE* is a trapezoid. [The use of the grid is optional.]
b Prove that *KATE* is *not* an isosceles trapezoid.



19. 080434b, P.I. G.G.69
The coordinates of quadrilateral *JKLM* are *J*(1,-2), *K*(13,4), *L*(6,8), and *M*(-2,4). Prove that quadrilateral *JKLM* is a trapezoid but not an isosceles trapezoid. [The use of the grid is optional.]



[2] 17, and appropriate work is shown, such as solving the equation $x+12=3x+2$.
[1] Appropriate work is shown, but one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] A correct equation is written and solved for x, but no further correct work is shown.
or [1] 17, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[1]

[2] D

[3] B

[2] 96, and appropriate work is shown, such as an algebraic solution or a correctly labeled diagram.
[1] Appropriate work is shown, but one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] 96, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[4]

[3] 102, and appropriate work is shown, such as using the equation $2x + 10 + 3x = 180$ or an equivalent equation.
[2] The equation $2x + 10 + 3x = 180$ is solved correctly for x, but $m\angle B$ is not determined or is determined incorrectly.
[1] Appropriate work is shown, but one computational error is made or x is not determined.
or [1] The equation $2x + 10 + 3x = 360$ is solved correctly, and an answer of 210 is found.
or [1] 102, but no work is shown.
[0] The equation $2x + 10 = 3x$ where $x = 10$ is given.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[5]

[3] 120, and appropriate work is shown, such as solving the equation $5x+2x+12=180$.
[2] Appropriate work is shown, but one computational error is made.
or [2] The correct equation is solved for x, but no further correct work is shown.
[1] Appropriate work is shown, but two or more computational errors are made.
or [1] Appropriate work is shown, but one conceptual error is made.
or [1] A correct equation is written, but no further correct work is shown.
or [1] An incorrect equation of equal difficulty is solved appropriately, and an appropriate measure is found for $\angle D$.
or [1] 120, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[6]

[7] C

[6] A complete and correct proof that includes a concluding statement is written, such as showing that $\overline{AB}$ is parallel to $\overline{CD}$ and that $\overline{BC}$ is not parallel to $\overline{AD}$ by finding their slopes and using the distance formula to show that the two nonparallel sides are equal.
[5] Appropriate work is shown, but one computational error is made.
[4] Appropriate work is shown, but two or more computational errors are made.
or [4] Appropriate work is shown, but one conceptual error is made, such as using an incorrect formula.
or [4] The slopes of all four sides are found correctly and the lengths of $\overline{AD}$ and $\overline{BC}$ are found correctly, and appropriate work is shown, but no conclusion is stated.
or [4] A proof is written that correctly shows *ABCD* is a trapezoid, but it is not proved to be isosceles.
[3] The slopes of only one pair of sides are found correctly, but the lengths of $\overline{AD}$ and $\overline{BC}$ are found correctly, and appropriate work is shown, and an appropriate conclusion is stated.
or [3] A correct numerical illustration is given in lieu of a proof of the general case.
[2] The slopes of only one pair of sides are found correctly, but the lengths of $\overline{AD}$ and $\overline{BC}$ are found correctly, and appropriate work is shown, but no conclusion is stated.
[1] Either the slopes or the lengths of $\overline{AD}$ and $\overline{BC}$ are found correctly, but no conclusion is stated.
or [1] The correct definition of an isosceles trapezoid is written, but no further correct work is shown.
[0] The slopes of $\overline{AB}$ and $\overline{DC}$ are found correctly, but no further correct work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[8]

[9] B

[2] 75, and appropriate work is shown, such as $3x+15=4x-5$.
[1] Appropriate work is shown, but one computational error is made.
or [1] Appropriate work is shown, but one conceptual error is made, such as showing $\overline{AC}$ and $\overline{BD}$ as congruent opposite sides.
or [1] A correct equation is written, but no further correct work is shown.
or [1] A correct equation is written and solved for *x*, but the length of $\overline{AC}$ is not found.
or [1] An incorrect equation of equal difficulty, such as $3x+15+4x-5=180$, is solved appropriately, and an appropriate length of $\overline{AC}$ is found.
or [1] 75, but no work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[10]

[11] B

[4] The student draws parallelogram *ABCD* correctly, identifies *D*(1,3), and justifies the coordinates of *D* by using any appropriate method to show *ABCD* is a parallelogram.
[3] The student draws parallelogram *ABCD* incorrectly but justifies *D* appropriately.
or [3] The student draws parallelogram *ABCD* correctly and identifies *D*(1,3), but the justification is incomplete.
[2] The student draws parallelogram *ABCD* correctly and identifies *D*(1,3), but no justification is shown.
[1] The student either draws parallelogram *ABCD* correctly or identifies *D*(1,3) correctly.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[12]

[6] The vertices $A'(-1,1)$, $B'(4,-2)$, $C'(3,-5)$, and $D'(-2,-2)$ are stated and a complete and correct proof that includes a conclusion is written.
[5] The vertices are stated, and a proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one reason is missing or is incorrect.
or [5] A complete proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but the vertices of $A'B'C'D'$ are not stated.
[4] The vertices are stated, and a proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.
[3] The vertices are stated, and a proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two reasons are missing or are incorrect.
[2] The vertices are stated, and some correct relevant statements about the proof are made, but three or four statements or reasons are missing or are incorrect.
[1] The vertices $A'(-1,1)$, $B'(4,-2)$, $C'(3,-5)$, and $D'(-2,-2)$ are stated, but no proof is written.
[0] The "given" and/or the "prove" statements are rewritten in the style of a formal proof, but no further correct relevant statements are written.
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[13]

[6] Appropriate work is shown, such as using slopes to prove *ABCD* is a parallelogram but not a rectangle, and an appropriate concluding statement is made.
[5] Appropriate work is shown, but one computational or graphing error is made.
[4] Appropriate work is shown, but two or more computational or graphing errors are made.
or [4] Appropriate work is shown, but one conceptual error is made, such as using an incorrect formula.
or [4] A proof is written that correctly shows either *ABCD* is a parallelogram or it is not a rectangle, but not both.
[3] Appropriate work is shown, but two or more computational or graphing errors are made, and the concluding statement is incomplete.
[2] Appropriate work is shown, but one conceptual error and one computational or graphing error are made.
or [2] All four slopes are found correctly or the lengths of all four sides are found correctly, and appropriate work is shown, but no further correct work is shown.
[1] The slopes of all four sides are identified or the lengths of all four lines are identified, but no work is shown and no proof is written.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[14]

[4] Appropriate work is shown, and an appropriate concluding statement is made to prove quadrilateral *ABCD* is a rhombus.
[3] The proof is completed appropriately, but one computational error is made, but an appropriate concluding statement is made.
or [3] Appropriate work is shown to prove quadrilateral *ABCD* is a rhombus, but the concluding statement is missing, incomplete, or incorrect.
[2] The proof is completed appropriately, but more than one computational error is made, but an appropriate concluding statement is made.
or [2] Appropriate work is shown, but one of the formulas used is incorrect.
or [2] Appropriate work is shown to prove quadrilateral *ABCD* is a parallelogram, and an appropriate concluding statement is made, but the sides are not proved to be equal.
or [2] Quadrilateral *ABCD* is proved to be a rhombus by assuming quadrilateral *ABCD* is a parallelogram.
[1] Appropriate work is shown to prove quadrilateral *ABCD* is a parallelogram, and the concluding statement is missing, incomplete, or incorrect.
or [1] The definition of a rhombus is stated, but no proof is given.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[15]

[6] A complete and correct proof is shown.
[5] Appropriate work is shown, but one computational error is made.
or [5] Appropriate work is shown, but the final conclusion is not justified or is justified incorrectly.
[4] Appropriate work is shown, but two or more computational errors are made.
or [4] Appropriate work is shown to prove TEAM is a parallelogram and not a square, but no work is shown to prove it is a rhombus.
or [4] Appropriate work is shown to prove TEAM is a rhombus, and partial work is shown to prove TEAM is not a square, but the conclusion is not adequately justified.
[3] Appropriate work is shown to prove TEAM is a rhombus, but no further correct work is shown.
or [3] Appropriate work is shown to prove TEAM is not a square, but an incorrect method is used to prove TEAM is a rhombus.
or [3] An accurate explanation of the process required to complete the proof is stated, and needed formulas are given, but no further correct work is shown.
[2] Appropriate work is shown to prove TEAM is a parallelogram, but no further correct work is shown.
[1] A complete explanation of the method of the proof is written, but no further correct work is shown.
or [1] A statement that TEAM is not a square and a correct reason are written, but no further correct work is shown.
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[16]

[6] The correct slopes of $AB = \frac{1}{2}$ and $CD = \frac{1}{2}$ are found, $\overline{AB} \parallel \overline{CD}$ is stated, and an explanation of why they are parallel is given. The correct slopes of $AD = -\frac{5}{2}$ and $BC = -\frac{1}{2}$ are found, $\overline{AD}$ is not parallel to $\overline{BC}$ is stated, and an explanation of why they are not parallel is given. An explanation that *ABCD* is a trapezoid is given.

[5] The correct slopes of $\overline{AB}$, $\overline{CD}$, $\overline{AD}$, and $\overline{BC}$ are found, and $\overline{AB} \parallel \overline{CD}$ and $\overline{AD}$ not $\parallel \overline{BC}$ are stated, but an explanation that *ABCD* is a trapezoid is not given.
or [5] One computational error is made in finding the slopes, but all further work is appropriate, based on the calculated slopes.

[4] The correct slope of $\overline{AB}$ and $\overline{CD}$ are found, and $\overline{AB} \parallel \overline{CD}$ is stated, but incorrect slopes of $\overline{AD}$ and $\overline{BC}$ are found, but an explanation of why they are not parallel is given, but an explanation that *ABCD* is a trapezoid is not given.
or [4] More than one computational error is made in finding the slopes, but $\overline{AB}$ and $\overline{CD}$ are found to have equal slopes and $\overline{AD}$ and $\overline{BC}$ to have different slopes, but an explanation that *ABCD* is a trapezoid is given.

[3] Incorrect slopes of $\overline{AB}$, $\overline{CD}$, $\overline{AD}$, and $\overline{BC}$ are found, such as by using an incorrect formula, $\overline{AB}$ and $\overline{CD}$ are found to have equal slopes and $\overline{AD}$ and $\overline{BC}$ to have different slopes, but an explanation that *ABCD* is a trapezoid is given.

[2] Only the correct slopes of $\overline{AB}$, $\overline{CD}$, $\overline{AD}$, and $\overline{BC}$ are found, and appropriate work is shown.

[1] Only two correct slopes are found, and appropriate work is shown.

[17]

or [1] $AB = \frac{1}{2}$, $CD = \frac{1}{2}$, $AD = -\frac{5}{2}$, and $BC = -\frac{1}{2}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

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[6] $\overline{KA} \parallel \overline{ET}$, $\overline{AT}$ not $\parallel \overline{KE}$, and $\overline{KE} \neq \overline{AT}$, and appropriate work is shown.

[5] Appropriate work is shown, but one computational error leads to incorrect conclusions that are appropriate, based on that error.

[4] Appropriate work is shown to find $\overline{KA} \parallel \overline{ET}$ or $\overline{AT}$ not $\parallel \overline{KE}$ and $\overline{KE} \neq \overline{AT}$, but no further correct work is shown.

[3] Appropriate work is shown to find $\overline{KE} \neq \overline{AT}$, and at least three of the four slopes are found correctly, but no statement regarding parallelism is made.
or [3] Appropriate work is shown to find the four slopes, and correct statements of parallelism are made, but no further correct work is shown.

[2] Appropriate work is shown to find unequal sides, but no further correct work is shown.
or [2] Appropriate work is shown to find the four slopes, but no conclusion is drawn.
or [2] The four slopes are correct, but no work is shown, but appropriate opposite sides are stated to be parallel and nonparallel.
or [2] The slope and distance formulas are used, but more than one computational error is made, but one accurate conclusion is drawn.

[1] Only two correct slopes or distances are found.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[18]

[6] $\overline{JK} \| \overline{ML}, \overline{MJ}$ not $\| \overline{\text{KL}}$, $\overline{MJ} \neq \overline{KL}$, and appropriate work is shown or a complete and correct proof is written, and a concluding statement is written.

[5] Appropriate work is shown and a correct concluding statement is written, but one computational error is made in determining the slopes or the lengths of the legs.

or [5] Appropriate work is shown, but the concluding statement is missing or is incomplete.

[4] Appropriate work is shown and a correct concluding statement is written, but two or more computational errors are made.

or [4] The quadrilateral is proved to be a trapezoid, but the two nonparallel sides are not proved to be unequal.

or [4] A proof is written that shows that $\overline{JK} \| \overline{ML}$ and $\overline{MJ} \neq \overline{KL}$, but the difference between a quadrilateral and a trapezoid is not addressed.

[3] Appropriate work is shown, but one conceptual error is made.

[2] The quadrilateral is proved to be a trapezoid, but one conceptual error is made, and the two nonparallel sides are not proved to be unequal.

or [2] The lengths of all four sides are found correctly, but no further correct work is shown.

or [2] The two nonparallel sides are proved to be unequal, but no further correct work is shown.

[1] The proof shows that the first set of sides is parallel, but no further correct work is shown.

or [1] JKLM is graphed correctly and the definition of an isosceles trapezoid is written, but no proof is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

[19]