

## Lesson 12-8: Combinations

### Part 1: Combinations

1. 080527a, P.I. A2.S.11

The expression  ${}_9C_2$  is equivalent to

[A]  ${}_9C_7$  [B]  ${}_9P_7$  [C]  ${}_9P_2$  [D]  $\frac{9!}{2!}$

2. 080720a, P.I. A2.S.11

The expression  ${}_8C_3$  is equivalent to

[A]  ${}_8P_3$  [B]  ${}_8C_5$  [C]  ${}_8P_5$  [D]  $\frac{8!}{3!}$

3. 010515a, P.I. A2.S.11

How many different three-member teams can be selected from a group of seven students?

[A] 5,040 [B] 1 [C] 35 [D] 210

4. 010729a, P.I. A2.S.11

If the Math Olympiad Club consists of eighteen students, how many different teams of four students can be formed for competitions?

[A] 3,060 [B] 66 [C] 73,440 [D] 72

5. 069907a, P.I. A2.S.11

How many different three-member teams can be formed from six students?

[A] 720 [B] 120 [C] 216 [D] 20

6. 010307a, P.I. A2.S.9

There are 12 people on a basketball team, and the coach needs to choose 5 to put into a game. How many different possible ways can the coach choose a team of 5 if each person has an equal chance of being selected?

[A]  ${}_{12}C_5$  [B]  ${}_{12}P_5$  [C]  ${}_5P_{12}$  [D]  ${}_5C_{12}$

7. 060320a, P.I. A2.S.11

How many different five-member teams can be made from a group of eight students, if each student has an equal chance of being chosen?

[A] 6,720 [B] 336 [C] 56 [D] 40

8. 080626a, P.I. A2.S.11

In the next Olympics, the United States can enter four athletes in the diving competition. How many different teams of four divers can be selected from a group of nine divers?

[A] 36 [B] 126 [C] 6,561 [D] 3,024

9. 080025a, P.I. A2.S.11  
Alan, Becky, Jesus, and Mariah are four students in the chess club. If two of these students will be selected to represent the school at a national convention, how many combinations of two students are possible?
10. 010424a, P.I. A2.S.11  
Five people have volunteered to work on an awards dinner at Madison High School. How many different committees of four can be formed from the five people?  
[A] 5      [B] 20      [C] 10      [D] 1
11. 010628a, P.I. A2.S.9  
A committee of five members is to be randomly selected from a group of nine freshmen and seven sophomores. Which expression represents the number of different committees of three freshmen and two sophomores that can be chosen?  
[A]  ${}_{16}C_3 \cdot {}_{16}C_2$       [B]  ${}_9C_3 + {}_7C_2$   
[C]  ${}_9C_3 \cdot {}_7C_2$       [D]  ${}_9P_3 \cdot {}_7P_2$
12. 060534a, P.I. A2.S.11  
An algebra class of 21 students must send 5 students to meet with the principal. How many different groups of 5 students could be formed from this class?
13. 060426a, P.I. A2.S.11  
In a game, each player receives 5 cards from a deck of 52 different cards. How many different groupings of cards are possible in this game?  
[A]  $5!$       [B]  ${}_{52}C_5$       [C]  ${}_{52}P_5$       [D]  $\frac{52!}{5!}$
14. 080126a, P.I. A2.S.11  
Megan decides to go out to eat. The menu at the restaurant has four appetizers, three soups, seven entrees, and five desserts. If Megan decides to order an appetizer or a soup, and one entree, and two different desserts, how many different choices can she make?
15. 080229a, P.I. A2.S.11  
On a bookshelf, there are five different mystery books and six different biographies. How many different sets of four books can Emilio choose if two of the books must be mystery books and two of the books must be biographies?
16. 060114a, P.I. A2.S.11  
If there are four teams in a league, how many games will have to be played so that each team plays every other team once?  
[A] 3      [B] 6      [C] 8      [D] 16

17. 060632a, P.I. A2.S.11  
Five friends met for lunch, and they all shook hands. Each person shook the other person's right hand only once. What was the total number of handshakes?
20. 010126a, P.I. A2.S.12  
Sal has a small bag of candy containing three green candies and two red candies. While waiting for the bus, he ate two candies out of the bag, one after another, without looking. What is the probability that both candies were the same color?

Part 2: Probability with Counting Techniques

18. 010034a, P.I. A2.S.12  
Three roses will be selected for a flower vase. The florist has 1 red rose, 1 white rose, 1 yellow rose, 1 orange rose and 1 pink rose from which to choose.  
*a* How many different three rose selections can be formed from the 5 roses?  
*b* What is the probability that 3 roses selected at random will contain 1 red rose, 1 white rose, and 1 pink rose?  
*c* What is the probability that 3 roses selected at random will *not* contain an orange rose?
19. 060034a, P.I. A2.S.12  
Paul orders a pizza. Chef Carl randomly chooses two different toppings to put on the pizza from the following: pepperoni, onion, sausage, mushrooms, and anchovies. If Paul will not eat pizza with mushrooms, determine the probability that Paul will *not* eat the pizza Chef Carl has made.
21. 060234a, P.I. A2.S.12  
Alexi's wallet contains four \$1 bills, three \$5 bills, and one \$10 bill. If Alexi randomly removes two bills without replacement, determine whether the probability that the bills will total \$15 is greater than the probability that the bills will total \$2.
22. 069932a, P.I. A2.S.12  
A bookshelf contains six mysteries and three biographies. Two books are selected at random without replacement.  
*a* What is the probability that both books are mysteries?  
*b* What is the probability that one book is a mystery and the other is a biography?

[1] A

[2] B

[3] C

[4] A

[5] D

[6] A

[7] C

[8] B

[2] 6 and appropriate work is shown, such as using the combination  ${}_4C_2$ , listing all six possible outcomes, or drawing a correct tree diagram.

[1] A correct setup of combinations is shown, but an incorrect solution, such as leaving  ${}_4C_2$ , or no integral solution is found.

or [1] An appropriate list or tree diagram is shown, but an incorrect solution is found, such as 5, by omitting one of the possible combinations.

or [1] 12 but a complete list or tree diagram is shown.

or [1] 6 but no work is shown.

[0] The answer is completely incorrect, such as  ${}_4P_2$  or  $4 \times 3$ .

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[9] obviously incorrect procedure.

[10] A

[11] C

[2] 20,349, and appropriate work is shown, such as  ${}_{21}C_5 = 20,349$ .

[1] Appropriate work is shown, but one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made, such as determining the value of  ${}_{21}P_5$ .

or [1] 20,349, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[12] incorrect procedure.

[13] B

[3] 490, and appropriate work is shown, such as  $7 \cdot 7 \cdot 10$ .

[2] Appropriate work is shown, but one computational error is made.

or [2] Appropriate work is shown, but an incorrect answer is found, based on an incorrect

number of possible dessert combinations or an incorrect number of soup or appetizer choices.

or [2] Appropriate work is shown, but an incorrect answer is found, based on one error in the tree diagram.

or [2]  $\frac{1}{490}$ , but appropriate work is shown.

[1] 7, 7, and 10 are added instead of multiplied.

or [1] The counting principle is used correctly, but incorrect substitutions are made, but

an appropriate answer is shown.

or [1] 490, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[14] incorrect procedure.

[3] 150, and appropriate work is shown, such as  ${}_5C_2 \bullet {}_6C_2$ .

[2] Appropriate work is shown, but one computational error is made.

or [2] All the possible combinations of two mystery books and all the possible combinations of two biographies are calculated, but the answers are not multiplied.

[1] Appropriate work is shown, but more than one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made, such as the computation  ${}_{11}C_4 = 330$ .

or [1] 150, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[15] incorrect procedure.

[16] B

[2] 10, and appropriate work is shown, such as  ${}_5C_2$  or a diagram or a list.

[1] Appropriate work is shown, but one computational error is made.

or [1] Appropriate work is shown, but one conceptual error is made.

or [1] 10, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[17] incorrect procedure.

a [2] 10 and an appropriate tree diagram, list, sample space, or  ${}_5C_3 = 10$  is shown.

[1] 10 and no work is shown.

or [1] An appropriate method is shown, but not all 10 possible combinations are listed

b [1]  $\frac{1}{10}$

or [1] An appropriate answer is found for an incorrect response in part a.

c [1]  $\frac{4}{10}$  or  $\frac{2}{5}$  or 0.4

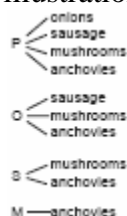
or [1] An appropriate answer is found for an incorrect response in part a.

a and b and c [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by

[18] an obviously incorrect procedure.

[4]  $\frac{4}{10}$  and appropriate work is shown, such

as the following illustration or any other correct method:



[3] Appropriate work is shown, but one computational error is made.

or [3] Appropriate work and complement  $\frac{6}{10}$

are shown.

or [3] Appropriate work is shown, but the answer is incomplete.

[2]  ${}_5C_2$  and the work is appropriate but incomplete.

or [2] 10 but appropriate work is shown.

or [2] A correct sample space or tree diagram is shown.

[1] Incorrect work leading to  $0 \leq \text{fraction} \leq 1$  or  $0 \leq \text{percent} \leq 100$  is shown.

or [1]  $\frac{4}{10}$  but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[19] incorrect procedure.

[3]  $\frac{8}{20}$  or an equivalent answer, and

appropriate work is shown, such as using a tree diagram or writing the equation

$$\frac{6}{20} + \frac{2}{20} = \frac{8}{20}.$$

[2] One computational error is made in

finding  $\frac{6}{20}$  or  $\frac{2}{20}$ , but an appropriate sum is found.

or [2]  $\frac{2}{20}$  and  $\frac{6}{20}$  are found, but no sum is shown.

[1]  $\frac{6}{20}$  or  $\frac{2}{20}$ , and appropriate work is shown.

or [1] An appropriate answer is found, using replacement with a tree diagram or an

equation such as  $\frac{3}{5} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{2}{5} = \frac{13}{25}.$

or [1]  $\frac{8}{20}$ , but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[20] incorrect procedure.

[4] No, and an appropriate explanation is given, such as  $P(15) = \frac{6}{56} < P(2) = \frac{12}{56}$ .

[3] One of the two probabilities is found correctly, but one computational error is made in finding the other, but an appropriate conclusion is drawn, based on the values found.

or [3] Replacement is used to conclude

$$P(15) = \frac{6}{64} < P(2) = \frac{12}{64}.$$

or [3] The two probabilities are found correctly, but no conclusion or the incorrect conclusion is drawn.

[2] One of the probabilities is found correctly, but one computational error is made in finding the other, and no conclusion or the incorrect conclusion is drawn.

[1] An appropriate method is used, such as a tree diagram or sample space, but the probabilities are not determined or are determined incorrectly.

or [1]  $P(15) = \frac{6}{56} < P(2) = \frac{12}{56}$ , but no work is shown.

[0] No, but no work is shown.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct

response that was obtained by an obviously

[21] incorrect procedure.

*a* [2]  $\frac{30}{72}$  or an equivalent value is found and an appropriate explanation is given.

[1] An acceptable method is used correctly, such as a tree diagram, sample space, or combinations, but the correct answer is not given.

or [1] Replacement is used, and an answer of  $\frac{36}{81}$  or an equivalent is found.

or [1]  $\frac{30}{72}$  and no work is shown.

*b* [2]  $\frac{36}{72}$  or an equivalent value is found and an appropriate explanation is given.

or [2] An appropriate probability for an incorrect denominator for part *a* is found.

[1] An appropriate method is shown, but one computational mistake is made.

or [1] Replacement is shown, and the answer  $\frac{36}{81}$  or an equivalent is found.

or [1] The student does not take into account both orders and answers  $\frac{18}{72}$  or an equivalent.

or [1]  $\frac{36}{72}$  and no work is shown.

or *a* and *b*

[1] An error in method is made but the erroneous answer is interpreted correctly in either part *a* or *b* or both.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[22] incorrect procedure.