The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Friday, January 29, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name: [Fill in]

School Name: [Fill in]

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.

1. A survey is to be conducted in a small upstate village to determine whether or not local residents should fund construction of a skateboard park by raising taxes. Which segment of the population would provide the most unbiased responses?
   (1) a club of local skateboard enthusiasts
   (2) senior citizens living on fixed incomes
   (3) a group opposed to any increase in taxes
   (4) every tenth person 18 years of age or older walking down Main St.

2. Which angle does not terminate in Quadrant IV when drawn on a unit circle in standard position?
   (1) −300°
   (2) −50°
   (3) 280°
   (4) 1030°

3. The expression \( \frac{1 + \frac{3}{x}}{\frac{y}{2xy}} \) is equivalent to
   (1) \( \frac{3}{2} \)
   (2) \( \frac{3x + y}{2xy} \)
   (3) \( \frac{3xy}{2} \)
   (4) \( \frac{3x + y}{2} \)

Use this space for computations.
4 Which relation does not represent a function?

(1) Domain Range

(2) Domain Range

(3) Domain Range

(4) Domain Range

5 In the diagram below, the spinner is divided into eight equal regions.

Which expression represents the probability of the spinner landing on B exactly three times in five spins?

(1) $\binom{5}{3} \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^2$

(2) $\binom{5}{3} \left(\frac{4}{5}\right)^3$

(3) $\binom{5}{3} \left(\frac{1}{8}\right)^3 \left(\frac{7}{8}\right)^2$

(4) $\binom{5}{3} \left(\frac{1}{8}\right)^3 \left(\frac{7}{8}\right)^2$
6 The expression $\sqrt[3]{27a^{-6}b^3c^2}$ is equivalent to

(1) $\frac{3bc^3}{a^2}$

(2) $\frac{3^2c^2}{a}$

(3) $\frac{3b^6c^5}{a^3}$

(4) $\frac{3b^2c^2}{a}$

7 The amount of money in an account can be determined by the formula $A = Pe^{rt}$, where $P$ is the initial investment, $r$ is the annual interest rate, and $t$ is the number of years the money was invested. What is the value of a $5000$ investment after 18 years, if it was invested at 4% interest compounded continuously?

(1) $9367.30$

(2) $9869.39$

(3) $10,129.08$

(4) $10,272.17$

8 What is $\frac{x}{x-1} - \frac{1}{2} \div \frac{2x-2}{x-1}$ expressed as a single fraction?

(1) $\frac{x+1}{x-1}$

(2) $\frac{2x-1}{2-2x}$

(3) $\frac{2x+1}{2(x-1)}$

(4) $\frac{2x-1}{2(x-1)}$
9 What is the total number of points of intersection of the graphs of the equations $2x^2 - y^2 = 8$ and $y = x + 2$?

(1) 1
(2) 2
(3) 3
(4) 0

10 Given the sequence: $x, (x + y), (x + 2y), ...$

Which expression can be used to determine the common difference of this sequence?

(1) $x - (x + y)$
(2) $(x + 2y) - (x + y)$
(3) $\frac{x}{x + y}$
(4) $\frac{(x + 2y)}{(x + y)}$

11 In a circle with a diameter of 24 cm, a central angle of $\frac{4\pi}{3}$ radians intercepts an arc. The length of the arc, in centimeters, is

(1) $8\pi$
(2) $9\pi$
(3) $16\pi$
(4) $32\pi$

12 Which graph is the solution to the inequality $4|2x + 6| - 5 < 27$?

(1)

(2)

(3)

(4)
13 What is the sum of the roots of the equation $-3x^2 + 6x - 2 = 0$?

(1) $\frac{2}{3}$  
(2) 2  
(3) $-\frac{2}{3}$  
(4) $-2$  

14 The scores of 1000 students on a standardized test were normally distributed with a mean of 50 and a standard deviation of 5. What is the expected number of students who had scores greater than 60?

(1) 1.7  
(2) 23  
(3) 46  
(4) 304

15 If $T = \frac{10x^2}{y}$, then $\log T$ is equivalent to

(1) $(1 + 2\log x) - \log y$  
(2) $\log(1 + 2x) - \log y$  
(3) $(1 - 2\log x) + \log y$  
(4) $2(1 - \log x) + \log y$

16 Which statement regarding correlation is not true?

(1) The closer the absolute value of the correlation coefficient is to one, the closer the data conform to a line.

(2) A correlation coefficient measures the strength of the linear relationship between two variables.

(3) A negative correlation coefficient indicates that there is a weak relationship between two variables.

(4) A relation for which most of the data fall close to a line is considered strong.
17 What is the value of \( \sum_{n=1}^{3} \cos \frac{n\pi}{2} \)?

(1) 1  
(2) -1  
(3) 0  
(4) \(-\frac{1}{2}\)

18 The roots of the equation \(4(x^2 - 1) = -3x\) are

(1) imaginary  
(2) real, rational, equal  
(3) real, rational, unequal  
(4) real, irrational, unequal

19 If \( f(x) = 2x^2 - 3x + 4 \), then \( f(x + 3) \) is equal to

(1) \(2x^2 - 3x + 7\)  
(2) \(2x^2 - 3x + 13\)  
(3) \(2x^2 + 9x + 13\)  
(4) \(2x^2 + 9x + 25\)

20 The expression \( x(3i^2)^3 + 2x^{12} \) is equivalent to

(1) \(2x + 27xi\)  
(2) \(-7x\)  
(3) \(-25x\)  
(4) \(-29x\)

21 If the terminal side of angle \( \theta \) passes through the point \((-3, -4)\), what is the value of \( \sec \theta \)?

(1) \(\frac{5}{3}\)  
(2) \(-\frac{5}{3}\)  
(3) \(\frac{5}{4}\)  
(4) \(-\frac{5}{4}\)
22 When the inverse of \( \tan \theta \) is sketched, its domain is
(1) \(-1 \leq \theta \leq 1\)    (3) \(0 \leq \theta \leq \pi\)
(2) \(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\)  (4) \(-\infty < \theta < \infty\)

24 What is the equation of a circle with its center at \((0, -2)\) and passing through the point \((3, -5)\)?
(1) \(x^2 + (y + 2)^2 = 9\)    (3) \(x^2 + (y + 2)^2 = 18\)
(2) \((x + 2)^2 + y^2 = 9\)    (4) \((x + 2)^2 + y^2 = 18\)

25 If angles \(A\) and \(B\) are complementary, then sec \(B\) equals
(1) \(\csc(90^\circ - B)\)    (3) \(\cos(B - 90^\circ)\)
(2) \(\csc(B - 90^\circ)\)    (4) \(\cos(90^\circ - B)\)
26 The legs of a right triangle are represented by \( x + \sqrt{2} \) and \( x - \sqrt{2} \). The length of the hypotenuse of the right triangle is represented by

\[
\begin{align*}
(1) & \quad \sqrt{2x^2 + 4} \\
(2) & \quad 2x^2 + 4 \\
(3) & \quad x\sqrt{2} + 2 \\
(4) & \quad \sqrt{x^2 - 2}
\end{align*}
\]

27 What are the amplitude and the period of the graph represented by the equation \( y = -3\cos \frac{\theta}{3} \)?

(1) amplitude: -3; period: \( \frac{\pi}{3} \)

(2) amplitude: -3; period: \( 6\pi \)

(3) amplitude: 3; period: \( \frac{\pi}{3} \)

(4) amplitude: 3; period: \( 6\pi \)

\[
\sqrt{(x + \sqrt{2})^2 + (x - \sqrt{2})^2} = \sqrt{x^2 + 2\sqrt{2}x + 2 + x^2 - 2\sqrt{2}x + 2} = \sqrt{2x^2 + 4}
\]
Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Solve algebraically for $x$:

$$\sqrt{2x + 1} + 4 = 8$$

$$\sqrt{2x + 1} = 4$$

$$2x + 1 = 16$$

$$x = \frac{15}{2}$$
Factor completely:

\[ x^3 + 3x^2 + 2x + 6 = x^2(x+3) + 2(x+3) = (x^2 + 2)(x+3) \]
30 Solve algebraically for the exact value of $x$:

$$\log_8 16 = x + 1$$

$$8^{x+1} = 16$$

$$2^{3(x+1)} = 2^4$$

$$3x + 3 = 4$$

$$3x = 1$$

$$x = \frac{1}{3}$$
31 Determine how many eleven-letter arrangements can be formed from the word "CATTARAUGUS."

\[
\frac{11!}{3! \cdot 2! \cdot 2!} = 1,663,200
\]
32 Express $-130^\circ$ in radian measure, to the nearest hundredth.

$$-130 \cdot \frac{\pi}{180} \approx -2.27$$
33 Determine the area, to the nearest integer, of $\triangle SRO$ shown below.

$$\frac{1}{2} (15)(31.6) \sin 125 \approx 194$$
34 Prove that the equation shown below is an identity for all values for which the functions are defined:

\[
\csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta
\]
35 Find the difference when \( \frac{4}{3}x^3 - \frac{5}{8}x^2 + \frac{7}{9}x \) is subtracted from \( 2x^3 + \frac{3}{4}x^2 - \frac{2}{9} \).

\[
\frac{2}{3} - \frac{4}{3} = \frac{6}{3} - \frac{4}{3} = \frac{2}{3}
\]

\[
\frac{3}{4} - \left(-\frac{5}{8}\right) = \frac{6}{8} + \frac{5}{8} = \frac{11}{8}
\]

\[
\frac{2}{3}x^3 + \frac{11}{8}x^2 - \frac{7}{9}x - \frac{2}{9}
\]
Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Find the exact roots of \( x^2 + 10x - 8 = 0 \) by completing the square.

\[
\begin{align*}
  x^2 + 10x + 25 &= 8 + 25 \\
  (x + 5)^2 &= 33 \\
  x + 5 &= \pm \sqrt{33} \\
  x &= -5 \pm \sqrt{33}
\end{align*}
\]
37 The table below gives the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4.2</td>
<td>33.5</td>
<td>113.1</td>
<td>268.1</td>
<td>523.6</td>
</tr>
</tbody>
</table>

Use exponential regression to find an equation for $y$ as a function of $x$, rounding all values to the nearest hundredth.

$$y = 2.19 \cdot (3.23)^x$$

Using this equation, predict the value of $x$ if $y$ is 426.21, rounding to the nearest tenth. [Only an algebraic solution can receive full credit.]

$$426.21 = 2.19 \cdot (3.23)^x$$

$$\log \frac{426.21}{2.19} = x \log 3.23$$

$$\log 3.23$$

$$4.5 \approx x$$
38 Solve the equation \( \cos 2x = \cos x \) algebraically for all values of \( x \) in the interval \( 0° \leq x < 360° \).

\[
2\cos^2 x - 1 = \cos x \\
2\cos^2 x - \cos x - 1 = 0 \\
(2\cos x + 1)(\cos x - 1) = 0 \\
\cos x = -\frac{1}{2} \quad \cos x = 1
\]

\( x = 120°, 240° \quad x = 0 \)
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 Given: \( DC = 10, AG = 15, BE = 6, FE = 10, \)
\( m\angle ABG = 40, m\angle GBD = 90, m\angle C < 90, \)
\( \frac{BE}{ED}, \) and \( GF = FB \)

Find \( m\angle A \) to the nearest tenth.

Find \( BC \) to the nearest tenth. 11.7

\[
\frac{16}{\sin A} = \frac{15}{\sin 40} \\
\sin A = \frac{16\sin 40}{15} \\
A \approx 43.3
\]