

ALGEBRA 2/TRIGONOMETRY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Wednesday, June 18, 2014 — 1:15 to 4:15 p.m., only

Student Name: Mr. Sybil

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

ALGEBRA 2/TRIGONOMETRY

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [54]

Use this space for computations.

1 Which survey is *least* likely to contain bias?

- (1) surveying a sample of people leaving a movie theater to determine which flavor of ice cream is the most popular
- (2) surveying the members of a football team to determine the most watched TV sport
- (3) surveying a sample of people leaving a library to determine the average number of books a person reads in a year
- (4) surveying a sample of people leaving a gym to determine the average number of hours a person exercises per week

2 The expression $(2a)^{-4}$ is equivalent to

- (1) $-8a^4$
- (2) $\frac{16}{a^4}$
- (3) $-\frac{2}{a^4}$
- (4) $\frac{1}{16a^4}$

3 Two sides of a triangular-shaped sandbox measure 22 feet and 13 feet. If the angle between these two sides measures 55° , what is the area of the sandbox, to the *nearest square foot*?

- (1) 82
- (2) 117
- (3) 143
- (4) 234

$$\frac{1}{2}(22)(13)\sin 55 \approx 117$$

4 Expressed in simplest form, $\sqrt{-18} - \sqrt{-32}$ is

- (1) $-\sqrt{2}$
- (2) $-7\sqrt{2}$
- (3) $-i\sqrt{2}$
- (4) $7i\sqrt{2}$

$$3i\sqrt{2} - 4i\sqrt{2}$$

Use this space for computations.

5 Theresa is comparing the graphs of $y = 2^x$ and $y = 5^x$. Which statement is true?

(1) The y -intercept of $y = 2^x$ is $(0,2)$, and the y -intercept of $y = 5^x$ is $(0,5)$.

(2) Both graphs have a y -intercept of $(0,1)$, and $y = 2^x$ is steeper. $x < 0$

(3) Both graphs have a y -intercept of $(0,1)$, and $y = 5^x$ is steeper. $x > 0$

(4) Neither graph has a y -intercept.

6 The solution set of the equation $\sqrt{2x-4} = x-2$ is

(1) $\{-2, -4\}$

(3) $\{4\}$

(2) $\{2, 4\}$

(4) $\{\}$

$$2x-4 = x^2 - 4x + 4$$

$$0 = x^2 - 6x + 8$$

$$0 = (x-4)(x-2)$$

$$x = 4, 2$$

7 The expression $(2 - 3\sqrt{x})^2$ is equivalent to

(1) $4 - 9x$

(3) $4 - 12\sqrt{x} + 9x$

(2) $4 - 3x$

(4) $4 - 12\sqrt{x} + 6x$

$$(2 - 3\sqrt{x})(2 - 3\sqrt{x})$$

$$4 - 6\sqrt{x} - 6\sqrt{x} + 9x$$

$$4 - 12\sqrt{x} + 9x$$

8 Which step can be used when solving $x^2 - 6x - 25 = 0$ by completing the square?

(1) $x^2 - 6x + 9 = 25 + 9$

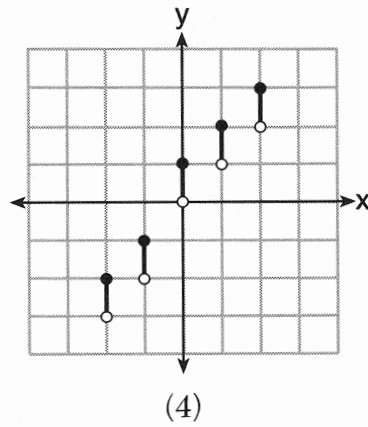
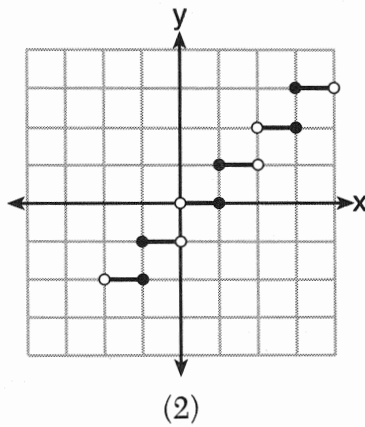
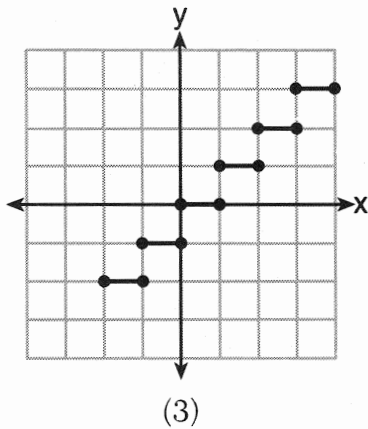
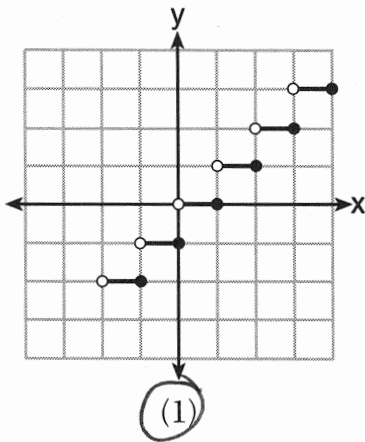
(2) $x^2 - 6x - 9 = 25 - 9$

(3) $x^2 - 6x + 36 = 25 + 36$

(4) $x^2 - 6x - 36 = 25 - 36$

Use this space for computations.

9 Which graph represents a function?



10 The expression $\frac{\cot x}{\csc x}$ is equivalent to

- (1) $\sin x$ (3) $\tan x$
 (2) $\cos x$ (4) $\sec x$

$$\frac{\cos x}{\frac{1}{\sin x}} = \cos x \cdot \sin x = \cos x$$

11 What is the common difference of the arithmetic sequence below?

$$-7x, -4x, -x, 2x, 5x, \dots$$

- (1) -3 (3) 3
 (2) $-3x$ (4) $3x$

$$5x - 2x = 3x$$

Use this space for computations.

16 The formula to determine continuously compounded interest is $A = Pe^{rt}$, where A is the amount of money in the account, P is the initial investment, r is the interest rate, and t is the time, in years. Which equation could be used to determine the value of an account with an \$18,000 initial investment, at an interest rate of 1.25% for 24 months?

- (1) $A = 18,000e^{1.25 \cdot 2}$ (3) $A = 18,000e^{0.0125 \cdot 2}$
 (2) $A = 18,000e^{1.25 \cdot 24}$ (4) $A = 18,000e^{0.0125 \cdot 24}$

17 What is the solution set of the equation $\frac{30}{x^2-9} + 1 = \frac{5}{x-3}$?

- (1) {2, 3}
 (2) {2}

- (3) {3}
 (4) {}

$$\frac{30}{x^2-9} + \frac{x^2-9}{x^2-9} = \frac{5(x+3)}{(x-3)(x+3)}$$

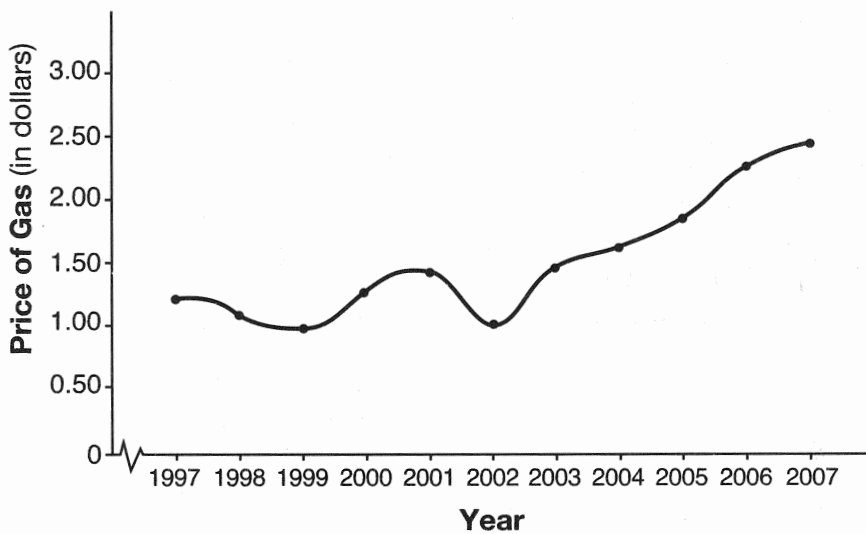
$$30 + x^2 - 9 = 5x + 15$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$x = 3, 2$

18 The graph below shows the average price of gasoline, in dollars, for the years 1997 to 2007.



What is the approximate range of this graph?

- (1) $1997 \leq x \leq 2007$ (3) $0.97 \leq y \leq 2.38$
 (2) $1999 \leq x \leq 2007$ (4) $1.27 \leq y \leq 2.38$

Use this space for computations.

19 If $f(x) = 2x^2 - 3x + 1$ and $g(x) = x + 5$, what is $f(g(x))$?

(1) $2x^2 + 17x + 36$

(3) $2x^2 - 3x + 6$

(2) $2x^2 + 17x + 66$

(4) $2x^2 - 3x + 36$

$2(x+5)^2 - 3(x+5) + 1$
 $2(x^2 + 10x + 25) - 3x - 15 + 1$
 $2x^2 + 20x + 50 - 3x - 14$
 $2x^2 + 17x + 36$

20 A jogger ran $\frac{1}{3}$ mile on day 1, and $\frac{2}{3}$ mile on day 2, and $1\frac{1}{3}$ miles on day 3, and $2\frac{2}{3}$ miles on day 4, and this pattern continued for 3 more days. Which expression represents the total distance the jogger ran?

(1) $\sum_{d=1}^7 \frac{1}{3}(2)^{d-1}$

(3) $\sum_{d=1}^7 2\left(\frac{1}{3}\right)^{d-1}$

(2) $\sum_{d=1}^7 \frac{1}{3}(2)^d$

(4) $\sum_{d=1}^7 2\left(\frac{1}{3}\right)^d$

21 If $\sin x = \sin y = a$ and $\cos x = \cos y = b$, then $\cos(x - y)$ is equivalent to

(1) $b^2 - a^2$

(3) $2b - 2a$

(2) $b^2 + a^2$

(4) $2b + 2a$

$\cos x \cos y + \sin x \sin y$
 $b \cdot b + a \cdot a$
 $b^2 + a^2$

22 A school math team consists of three juniors and five seniors. How many different groups can be formed that consist of one junior and two seniors?

(1) 13

(3) 30

(2) 15

(4) 60

$3C_1 \cdot 5C_2$
 $3 \cdot 10$
 30

23 For which value of k will the roots of the equation $2x^2 - 5x + k = 0$ be real and rational numbers?

(1) $1 \quad (-5)^2 - 4(2)(1) = 17$

(3) $0 \quad (-5)^2 - 4(2)(0) = 25$

(2) $-5 \quad (-5)^2 - 4(2)(-5) = 65$

(4) 4

$b^2 - 4ac \geq 0$
 $(-5)^2 - 4(2)k \geq 0$
 $25 - 8k \geq 0$
 $25 \geq 8k$
 $10 \leq \frac{25}{8}$

Use this space for computations.

24 A cliff diver on a Caribbean island jumps from a height of 105 feet, with an initial upward velocity of 5 feet per second. An equation that models the height, $h(t)$, above the water, in feet, of the diver in time elapsed, t , in seconds, is $h(t) = -16t^2 + 5t + 105$. How many seconds, to the nearest hundredth, does it take the diver to fall 45 feet below his starting point?

- (1) 1.45 (3) 2.10
 (2) 1.84 (4) 2.72

$$h(t) = 60$$

$$60 = -16t^2 + 5t + 105$$

$$0 = -16t^2 + 5t + 45$$

$$t = \frac{-5 \pm \sqrt{5^2 - 4(-16)(45)}}{2(-16)}$$

$$t \approx \frac{-5 \pm 53.89}{-32}$$

$t \approx 1.84$

25 The number of possible different 12-letter arrangements of the letters in the word "TRIGONOMETRY" is represented by

- (1) $\frac{12!}{3!}$ (3) $\frac{12P_{12}}{8}$
 (2) $\frac{12!}{6!}$ (4) $\frac{12P_{12}}{6!}$

$$\frac{12P_{12}}{2!2!2!} = \frac{12P_{12}}{8}$$

26 If $2x^3 = y$, then $\log y$ equals

- (1) $\log(2x) + \log 3$ (3) $3 \log 2 + 3 \log x$
 (2) $3 \log(2x)$ (4) $\log 2 + 3 \log x$

$$\log 2x^3$$

$$\log 2 + \log x^3$$

$$\log 2 + 3 \log x$$

27 Which statement regarding the inverse function is true?

- (1) A domain of $y = \sin^{-1} x$ is $[0, 2\pi]$.
 (2) The range of $y = \sin^{-1} x$ is $[-1, 1]$.
 (3) A domain of $y = \cos^{-1} x$ is $(-\infty, \infty)$.
 (4) The range of $y = \cos^{-1} x$ is $[0, \pi]$.

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 28 In a certain school, the heights of the population of girls are normally distributed, with a mean of 63 inches and a standard deviation of 2 inches. If there are 450 girls in the school, determine how many of the girls are *shorter than* 60 inches. Round the answer to the *nearest integer*.

$$z = \frac{60 - 63}{2} = -1.5 \sigma_x$$

$$.067 \cdot 450 \approx 30$$

29 The table below shows the concentration of ozone in Earth's atmosphere at different altitudes. Write the exponential regression equation that models these data, rounding *all* values to the nearest thousandth.

Concentration of Ozone

Altitude (x)	Ozone Units (y)
0	0.7
5	0.6
10	1.1
15	3.0
20	4.9

$$y = 0.488(1.116)^x$$

30 Solve $|2x - 3| > 5$ algebraically.

$$2x - 3 > 5$$

$$2x > 8$$

$$x > 4$$

$$2x - 3 < -5$$

$$2x < -2$$

$$x < -1$$

OR

31 Convert 2.5 radians to degrees, and express the answer to the *nearest minute*.

$$2.5 \cdot \frac{180}{\pi} \approx 143^{\circ} 14'$$

32 Multiply $x + yi$ by its conjugate, and express the product in simplest form.

$$\begin{aligned} &(x + yi)(x - yi) \\ &x^2 - y^2 i^2 \\ &x^2 - y^2(-1) \\ &x^2 + y^2 \end{aligned}$$

33 Solve algebraically for x :

$$\log_{5x-1} 4 = \frac{1}{3}$$

$$(5x-1)^{1/3} = 4$$

$$5x-1 = 4^3$$

$$5x-1 = 64$$

$$5x = 65$$

$$x = 13$$

34 Solve $\sec x - \sqrt{2} = 0$ algebraically for all values of x in $0^\circ \leq x < 360^\circ$.

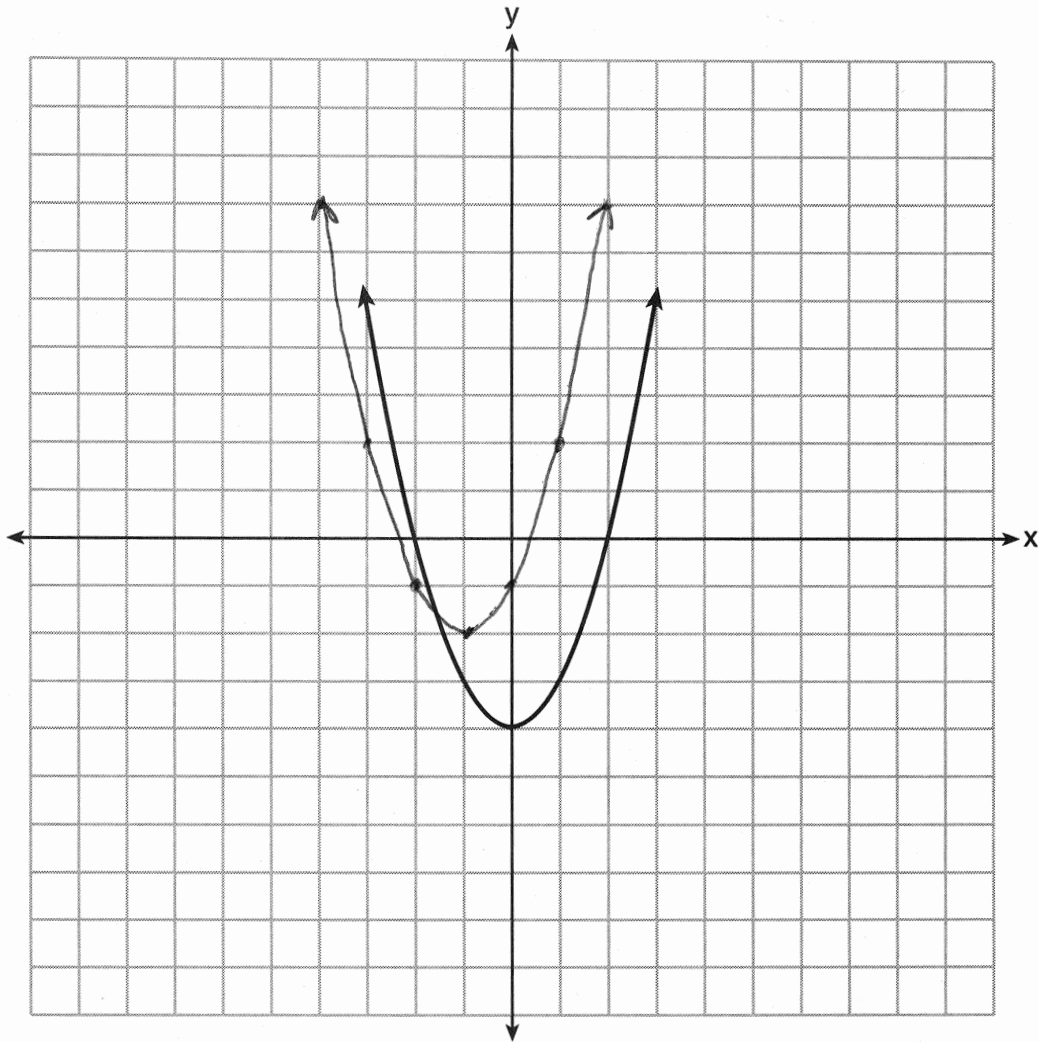
$$\sec x = \sqrt{2}$$

$$\cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$x = 45^\circ, 315^\circ$$

35 The function $f(x)$ is graphed on the set of axes below.

On the same set of axes, graph $f(x + 1) + 2$.



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Express in simplest terms:

$$\frac{1 + \frac{3}{x}}{1 - \frac{5}{x} - \frac{24}{x^2}} \cdot \frac{x^2}{x^2}$$

$$\frac{x^2 + 3x}{x^2 - 5x - 24}$$

$$\frac{x \cancel{(x+3)}}{(x-8) \cancel{(x+3)}}$$

$$\frac{x}{x-8}$$

37 Solve $x^3 + 5x^2 = 4x + 20$ algebraically.

$$\begin{aligned}x^3 + 5x^2 - 4x - 20 &= 0 \\x^2(x+5) - 4(x+5) &= 0 \\(x^2-4)(x+5) &= 0 \\(x+2)(x-2)(x+5) &= 0 \\x &= \pm 2, -5\end{aligned}$$

38 Whenever Sara rents a movie, the probability that it is a horror movie is 0.57. Of the next five movies she rents, determine the probability, to the *nearest hundredth*, that *no more than* two of these rentals are horror movies.

$${}_5C_0 (.57)^0 (.43)^5 \approx .0147$$

$${}_5C_1 (.57)^1 (.43)^4 \approx .0974$$

$${}_5C_2 (.57)^2 (.43)^3 \approx \frac{.2583}{.3704}$$

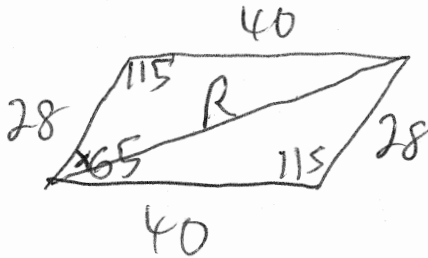
$$0.37$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 Two forces of 40 pounds and 28 pounds act on an object. The angle between the two forces is 65° . Find the magnitude of the resultant force, to the nearest pound.

Using this answer, find the measure of the angle formed between the resultant and the smaller force, to the nearest degree.



$$R = \sqrt{28^2 + 40^2 - 2(28)(40)\cos 115}$$
$$\approx 58$$

$$\frac{58}{\sin 115} = \frac{40}{\sin X}$$

$$X \approx 39$$