**0125aii Regents Exam**

1 The exact value of $\sin\left(\dfrac{8\pi}{3}\right)$ is

1) $\dfrac{1}{2}$

2) $-\dfrac{1}{2}$

3) $-\dfrac{\sqrt{3}}{2}$

4) $\dfrac{\sqrt{3}}{2}$

2 A teacher randomly divides all of her students into two groups. She grades the homework for one group but does not grade the homework for the other group. All homework is returned to the students. She then compares test scores for each of the groups to see if grading homework has an effect on the test scores. This method of data collection is best described as
1) an experiment
2) an unbiased survey
3) a simulation
4) an observational study

3 Which expression is equivalent to $(x-2)^2 + 27(x-2) - 90$?
1) $(x+30)(x-3)$
2) $(x+28)(x-5)$
3) $(x-30)(x+3)$
4) $(x-2)(x+25)(x-90)$

4 Given the functions $f(x) = 2x + \dfrac{5}{2}$ and $g(x) = \dfrac{3}{x}$, what are the solutions to $f(x) = g(x)$?

1) $(0.75, 4)$ or $(-2, -1.5)$
2) $x = 0.75$ or $x = -2$
3) $y = -1.5$ or $y = 4$
4) $(-2, 0.75)$

5 Given $f(x) = 2x^3 - 3x^2 - 5x - 12$ and $g(x) = x - 3$, the quotient of $\dfrac{f(x)}{g(x)}$ is

1) $2x^2 + 3x + 4$

2) $2x^3 + 3x^2 + 4x$

3) $2x^2 - 9x + 22 - \dfrac{78}{x-3}$

4) $2x^3 - 9x^2 + 22x - 78$

6 Abby is told that each day there is a 50% chance it will rain. Which simulation can Abby perform to determine the likelihood of it raining for the next seven days?
1) Flip a coin seven times, count how many heads, and repeat 50 times.
2) Roll a die seven times, count how many twos, and repeat 50 times.
3) Roll a pair of dice, count totals of seven, and repeat 50 times.
4) Flip a coin 50 times and count how many heads.

7  What are the solutions to $4x^2 - 7x - 2 = -10$

1) $-\dfrac{1}{4}, 2$

2) $\dfrac{7}{8} \pm \dfrac{\sqrt{79}}{8}i$

3) $\dfrac{7}{8} \pm \dfrac{\sqrt{241}}{8}$

4) $\dfrac{7}{8} \pm \dfrac{\sqrt{143}}{8}i$

8  If $x - 5$ is a factor of $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, then which statement must be true?
1) $p(-5) = 0$
2) $p(-5) \neq 0$
3) $p(5) = 0$
4) $p(5) \neq 0$

9  In a small city, there are 22 gas stations. The mean price for a gallon of regular gas was \$2.12 with a standard deviation of \$0.05. The distribution of the data was approximately normal. Given this information, the middle 95% of the gas stations in this small city likely charge
1) \$1.90 to \$2.34 for a gallon of gas
2) \$1.97 to \$2.27 for a gallon of gas
3) \$2.02 to \$2.22 for a gallon of gas
4) \$2.07 to \$2.17 for a gallon of gas

10  The expression $\dfrac{4x^2 - 5}{x^2 - 1}$ is equivalent to

1) $4 - \dfrac{1}{x^2 - 1}$

2) $4 + \dfrac{1}{x^2 - 1}$

3) $4 - \dfrac{9}{x^2 - 1}$

4) $4 - \dfrac{4}{x^2 - 1}$

11  For all positive values of $x$, which expression is equivalent to $\sqrt{x} \cdot \sqrt[4]{x^{11}}$ ?

1) $x^{\frac{19}{22}}$

2) $x^{\frac{11}{8}}$

3) $x^{\frac{13}{4}}$

4) $x^{\frac{2}{11}}$

12  The expression $i^2(5x - 2i)^2$ is equivalent to
1) $-25x^2 + 20xi - 4$
2) $-25x^2 + 20xi + 4$
3) $25x^2 + 20xi + 4$
4) $25x^2 + 4$

13 Functions $f$ and $g$ are given below.
$$f(x) = \frac{7}{2}x^2 - 5x + 11$$
$$g(x) = 3x^2 - 7x + 25$$
When $2f(x)$ is subtracted from $g(x)$, the result is

1) $4x^2 - 3x - 3$ 　　　　　　　　　3) $4x^2 - 17x - 47$
2) $-4x^2 + 3x + 3$ 　　　　　　　　4) $-4x^2 - 17x + 47$

14 A manufacturer claims that the number of ounces of a beverage dispensed by one of its automatic dispensers is normally distributed with a mean of 8.0 ounces and a standard deviation of 0.04 ounces. To the *nearest tenth of a percent*, what percent of the cups filled by this company's dispenser will contain between 7.9 and 8.11 ounces?

1) 99.5 　　　　　　　　　　　　　3) 99.1
2) 99.4 　　　　　　　　　　　　　4) 97.6

15 What is the value of $x$ in the solution of the system of equations below?
$$5x + 2y - z = -14$$
$$7y - z = 31$$
$$5y + 4z - 5x = -23$$

1) -17 　　　　　　　　　　　　　3) $-\frac{1}{5}$
2) 2 　　　　　　　　　　　　　　4) $-7$

16 The graph below shows the amount of a radioactive substance left over time.



The daily rate of decay over an 8-day interval is approximately
1) 23% 　　　　　　　　　　　　　3) 5%
2) 95% 　　　　　　　　　　　　　4) 77%

3

17  If $4(10^{5x-2}) = 12$ then $x$ equals

1) $\dfrac{2.3}{5}$

2) $\dfrac{1}{3}\left(\dfrac{\log 12}{\log 40} + 5\right)$

3) $\dfrac{\log(3) + 2}{5}$

4) $\dfrac{1}{5}\left(\dfrac{\log 12}{\log 4} + 2\right)$

18  A random sample of 152 students was surveyed on a particular day about how they got to school.  The survey results are summarized in the table below.

|  | | Attendance Status | |
| --- | --- | --- | --- |
|  | | Late | On-Time |
| Method of Transportation | Car | 6 | 24 |
| | Bus | 20 | 80 |
| | Walk | 4 | 18 |

Which statement is best supported by the data?

1) The probability of being late given that a student walked is greater than the probability that a student walked given that the student was late.

2) The probability of being late given that a student walked is less than the probability that a student walked given that the student was late.

3) The probability of being late given that a student walked is equal to the probability that a student walked given that the student was late.

4) The probability of being late given that a student walked cannot be determined.

19  If $f(x) = \sqrt[3]{x} + 4$, then $f^{-1}(x)$ equals

1) $\sqrt[3]{x-4}$

2) $(x-4)^3$

3) $x^3 + \dfrac{1}{4}$

4) $-\sqrt[3]{x} - 4$

20  Given the equation $S(x) = 1.7\sin(bx) + 12$, where the period of $S(x)$ is 12, what is the value of $b$?

1) $\dfrac{\pi}{6}$

2) $24\pi$

3) $\dfrac{\pi}{12}$

4) $6\pi$

21  Jin solved the equation $\sqrt{4-x} = x + 8$ by squaring both sides.  What extraneous solution did he find?

1) $-5$

2) $-12$

3) 3

4) 4

22 The expression $\left(x^2 + y^2\right)^2$ is *not* equivalent to

1) $\left(x^2 - y^2\right)^2 + (2xy)^2$

3) $x^2\left(x^2 + 2y^2\right) + (y^2)^2$

2) $(x+y)^4 + 2(xy)^2$

4) $\left(2x^2 + y^2\right)^2 - \left(3x^4 + 2x^2y^2\right)$

23 The height of a running trail is modeled by the quartic function $y = f(x)$ shown below, where $x$ is the distance in miles from the start of the trail and $y$ is the height in feet relative to sea level.
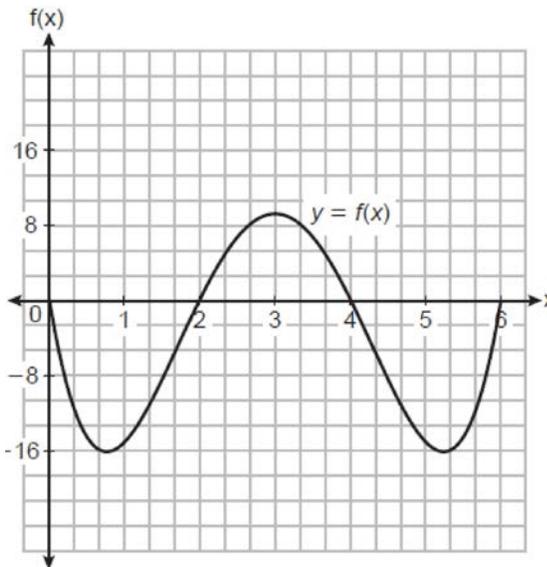


If this trail has a minimum height of 16 feet below sea level, which function(s) could represent a running trail whose minimum height is half of the minimum height of the original trail?

$$\text{I. } y = f\left(\frac{1}{2}x\right) \quad \text{II. } y = f(x) + 8 \quad \text{III. } y = \frac{1}{2}f(x)$$

1) I, only

3) I and III

2) II, only

4) II and III

24 The crew aboard a small fishing boat caught 350 pounds of fish on Monday. From that Monday through the end of the week on Friday, the weight of the fish caught increased 15% per day. The total weight, in pounds, of fish caught is approximately

1) 411

3) 1748

2) 612

4) 2360

25 Describe the translations that map $f(x) = \log x$ to $g(x) = \log(x+3) - 5$.

26  Solve algebraically for $x$: $\dfrac{1}{2x} - \dfrac{5}{6} = \dfrac{3}{x}$

27  Given $\cos\theta = -\dfrac{2}{7}$ with $\theta$ in Quadrant II, find the exact value of $\sin\theta$.

28  Given $a > 1$, use the properties of rational exponents to determine the value of $x$ for the equation below.

$$\frac{\sqrt[5]{a^{10}}}{\left(a^3\right)^{\frac{1}{2}}} = a^x$$

29  Graph *at least one* cycle of $y = 5\sin(4x) - 3$ on the set of axes below.
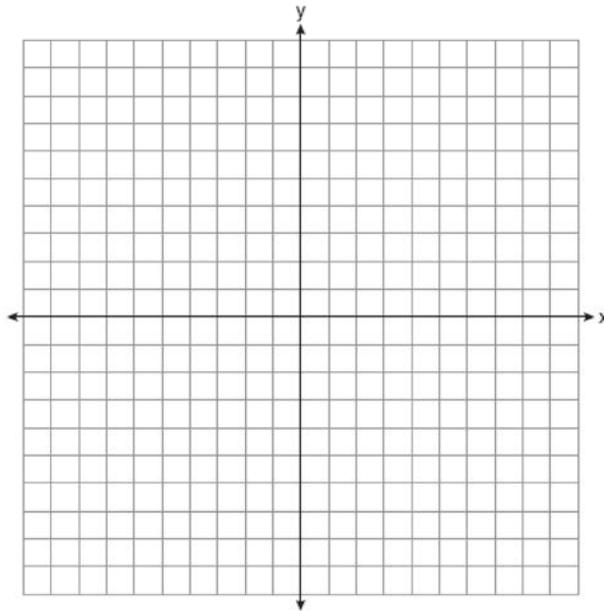


30  The cost of a brand-new electric-hybrid vehicle is listed at \$33,400, and the average annual depreciation for the vehicle is 15%. The car's value can be modeled by the function $V(x) = 33,400(0.85)^x$, where $x$ represents the years since purchase. Julia and Jacob have each written a function that is equivalent to the original.

$$\text{Jacob's function: } V(x) = 33,400(0.1422)^{\frac{1}{12}x}$$
$$\text{Julia's function: } V(x) = 33,400(0.9865)^{12x}$$

Whose function is correctly rewritten to reveal the approximate monthly depreciation rate? Justify your answer.

31  Write a recursive formula for the sequence $8, 20, 50, 125, 312.5, \ldots$

32  A grocery store orders 50 bags of oranges from a company's distribution center.  The bags have a mean weight of 3.85 pounds per bag.  The company claims that their bags of oranges have a mean weight of 4 pounds.  The grocery store ran a simulation of 50 bags, 2500 times, assuming a mean of 4 pounds.  The results are shown below.

Mean = 4.001
SD = 0.026
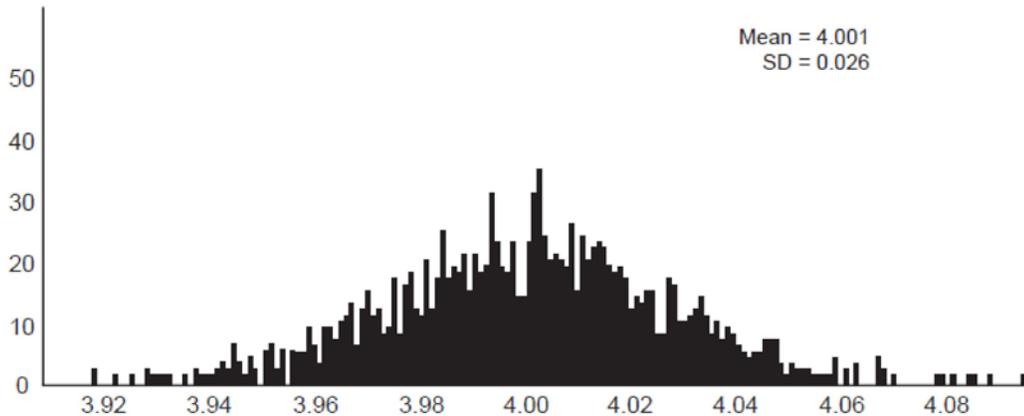


Is the mean weight of the grocery store's sample unusual?  Explain using the results of the simulation.

33  At the Lakeside Resort, the probability that a guest room has a view of the lake is 0.24.  The probability that a guest room has a queen-size bed is 0.74.  Let $A$ be the event that the guest room has a view of the lake, and let $B$ be the event that the guest room has a queen-size bed.  Events $A$ and $B$ are found to be independent of each other.  Determine the exact probability that a randomly selected guest room has a view of the lake and a queen-size bed.  Determine the exact probability that a randomly selected guest room has a view of the lake or a queen-size bed.

34  Which function has a greater average rate of change on the interval $[-1, 4]$?  Justify your answer.

| x | m(x) |
|---|---|
| −2 | −3 |
| −1 | 1 |
| 0 | 1 |
| 1 | 3 |
| 2 | 13 |
| 3 | 37 |
| 4 | 81 |
| 5 | 151 |

$$p(x) = 3^x + 1$$

35  Determine an equation for the parabola with focus $(-2,4)$ and directrix $y = 10$.  (The use of the grid below is optional.)
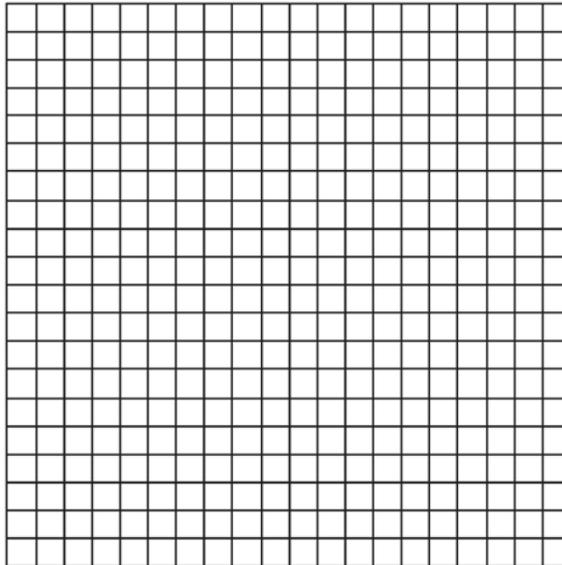
36  Algebraically find the zeros of $c(x) = x^3 + 2x^2 - 16x - 32$.  On the axes below, sketch $y = c(x)$.
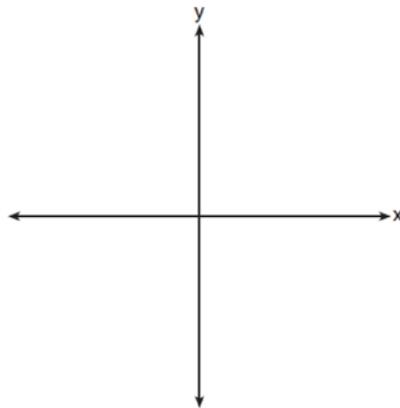
37  The populations of honeybees in two different colonies are studied for four months.  During this time, the colony
population can be approximated by $P(t) = P_0 e^{rt}$, where $P(t)$ is the colony population of bees at $t$ months, $P_0$ is the
initial population, and $r$ is the growth rate.  Colony $A$ has an initial population of 10,000 bees and a continuous
growth rate of 0.25.  Colony $B$ has an initial population of 6000 bees and a continuous growth rate of 0.45.  Write
functions for both $A(t)$ and $B(t)$ that model the honeybee populations of the colonies after $t$ months.  Graph $A(t)$
and $B(t)$ for $0 \le t \le 4$.



Bee Population (in thousands)
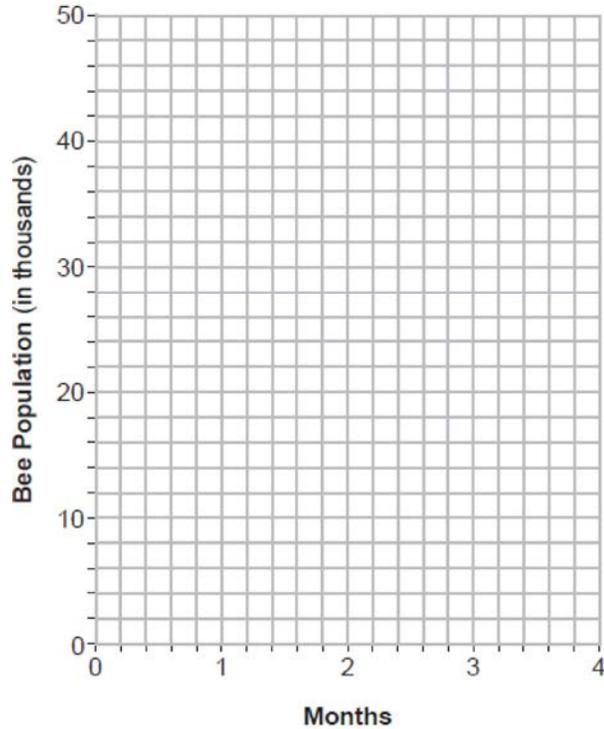
Months

State, to the *nearest tenth of a month*, when the colonies will have the same population.  Determine algebraically
how long it will take, to the *nearest tenth of a month*, for the population in Colony $A$ to triple.

9

## 0125aii Regents Exam
## Answer Section

1  ANS: 4 PTS: 2 REF: 012501aii NAT: F.TF.A.2
   TOP: Determining Trigonometric Functions KEY: radians

2  ANS: 1 PTS: 2 REF: 012502aii NAT: S.IC.B.3
   TOP: Analysis of Data

3  ANS: 2

$u = x - 2$    $u^2 + 27u - 90$

$(u + 30)(u - 3)$

$(x - 2 + 30)(x - 2 - 3)$

$(x + 28)(x - 5)$

   PTS: 2 REF: 012503aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

4  ANS: 2

$2x\left(2x + \dfrac{5}{2} = \dfrac{3}{x}\right)$

$4x^2 + 5x = 6$

$4x^2 + 5x - 6 = 0$

$(4x - 3)(x + 2) = 0$

$x = \dfrac{3}{4}, -2$

   PTS: 2 REF: 012504aii NAT: A.REI.D.11 TOP: Other Systems
   KEY: rational

5  ANS: 1

$$\begin{array}{r} 2x^2 + 3x + 4 \\ x - 3 \overline{\smash{\big)}\ 2x^3 - 3x^2 - 5x - 12} \end{array}$$

$\underline{2x^3 - 6x^2}$

$3x^2 - 5x$

$\underline{3x^2 - 9x}$

$4x - 12$

$\underline{4x - 12}$

$0$

   PTS: 2 REF: 012505aii NAT: A.APR.D.6 TOP: Rational Expressions
   KEY: division

6  ANS: 1 PTS: 2 REF: 012506aii NAT: S.IC.B.3
   TOP: Analysis of Data

7  ANS: 2

$$4x^2 - 7x + 8 = 0 \quad x = \frac{7 \pm \sqrt{(-7)^2 - 4(4)(8)}}{2(4)} = \frac{7 \pm \sqrt{-79}}{8}$$

PTS: 2          REF: 012507aii     NAT: A.REI.B.4     TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

8  ANS: 3          PTS: 2          REF: 012508aii     NAT: A.APR.B.2
TOP: Remainder and Factor Theorems

9  ANS: 3
$2.12 \pm 2(.05)$

PTS: 2          REF: 012509aii     NAT: S.ID.A.4     TOP: Normal Distributions

10  ANS: 1

$$\frac{4x^2 - 5}{x^2 - 1} = \frac{4(x^2 - 1)}{x^2 - 1} - \frac{1}{x^2 - 1}$$

PTS: 2          REF: 012510aii     NAT: A.APR.D.7     TOP: Addition and Subtraction of Rationals

11  ANS: 3

$$\sqrt{x} \bullet \sqrt[4]{x^{11}} = x^{\frac{1}{2}} \bullet x^{\frac{11}{4}} = x^{\frac{2}{4}} \bullet x^{\frac{11}{4}} = x^{\frac{13}{4}}$$

PTS: 2          REF: 012511aii     NAT: N.RN.A.2     TOP: Radicals and Rational Exponents

12  ANS: 2
$i^2(5x - 2i)^2 = -(25x^2 - 20xi - 4)$

PTS: 2          REF: 012512aii     NAT: N.CN.A.2     TOP: Operations with Complex Numbers

13  ANS: 2
$3x^2 - 7x + 25 - (7x^2 - 10x + 22) = -4x^2 + 3x + 3$

PTS: 2          REF: 012513aii     NAT: F.BF.A.1     TOP: Operations with Functions

14  ANS: 3



$\overline{x} + 2\sigma$ represents approximately 99.1% of the data.

PTS: 2          REF: 012514aii     NAT: S.ID.A.4     TOP: Normal Distributions
KEY: percent

15  ANS:  4

$z = 7y - 31$   $5x + 2y - (7y - 31) = -14$   $\rightarrow$   $5x - 5y = -45$   $\rightarrow$   $x - y = -9$   $\rightarrow$   $y = x + 9$

$$5y + 4(7y - 31) - 5x = -23 \ \rightarrow \ -5x + 33y = 101$$

$$-5x + 33(x + 9) = 101$$
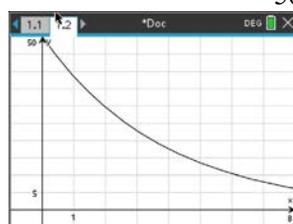
$$28x = -196$$

$$x = -7$$

PTS:  2          REF:  012515aii       NAT:  A.REI.C.6       TOP:  Solving Linear Systems
KEY:  three variables

16  ANS:  1

Estimate $(0, 50)$ and $(1, 38)$ as points on the graph.  $\dfrac{38}{50} = 76\%$ implies an estimated 24% rate of decay.  Confirmed



with graph of $y = 50(.77)^x$ :

PTS:  2          REF:  012516aii       NAT:  F.LE.B.5       TOP:  Modeling Exponential Functions

17  ANS:  3

$$10^{5x-2} = 3$$

$$\log 10^{5x-2} = \log 3$$

$$(5x - 2)\log 10 = \log 3$$

$$5x - 2 = \log 3$$

$$5x = \log 3 + 2$$

$$x = \frac{\log 3 + 2}{5}$$

PTS:  2          REF:  012517aii       NAT:  F.LE.A.4       TOP:  Exponential Equations
KEY:  without common base

18  ANS:  1

The probability of being late given that a student walked is $\dfrac{4}{22}$.  The probability that student walked given that the

student was late is $\dfrac{4}{30}$.

PTS:  2          REF:  012518aii       NAT:  S.CP.A.4       TOP:  Conditional Probability

19  ANS:  2

$$y = \sqrt[3]{x} + 4$$

$$x = \sqrt[3]{y} + 4$$

$$x - 4 = \sqrt[3]{y}$$

$$(x - 4)^3 = y$$

PTS:  2          REF:  012519aii     NAT:  F.BF.B.4     TOP:  Inverse of Functions
KEY:  cubic

20  ANS:  1

$$\frac{2\pi}{b} = 12$$

$$12b = 2\pi$$

$$b = \frac{\pi}{6}$$

PTS:  2          REF:  012520aii     NAT:  F.IF.C.7     TOP:  Graphing Trigonometric Functions
KEY:  period

21  ANS:  2

$$\sqrt{4 - x} = x + 8 \qquad\qquad -12 + 8 = -4$$

$$4 - x = x^2 + 16x + 64$$

$$0 = x^2 + 17x + 60$$

$$x = (x + 12)(x - 5)$$

$$x = -12, 5$$

PTS:  2          REF:  012521aii     NAT:  A.REI.A.2     TOP:  Solving Radicals

22  ANS:  2

1) $x^4 - 2x^2y^2 + y^4 + 4x^2y^2$;  3) $x^4 + 2x^2y^2 + y^4$;  4) $4x^4 + 4x^2y^2 + y^4 - 3x^4 - 2x^2y^2$

PTS:  2          REF:  012522aii     NAT:  A.APR.C.4     TOP:  Polynomial Identities

23  ANS:  4

I. Minimum does not change, only period;  II. $-16 + 8 = -8$;  III. $\frac{1}{2}(-16) = -8$

PTS:  2          REF:  012523aii     NAT:  F.BF.B.3     TOP:  Transformations with Functions

24  ANS:  4

$$S_5 = \frac{350 - 350(1.15)^5}{1 - 1.15} \approx 2360$$

PTS:  2          REF:  012524aii     NAT:  F.BF.B.7     TOP:  Series
KEY:  geometric

25  ANS:
left 3, down 5

PTS: 2          REF: 012525aii          NAT: F.IF.C.7          TOP: Graphing Logarithmic Functions

26  ANS:

$$6x\left(\frac{1}{2x} - \frac{5}{6} = \frac{3}{x}\right)$$

$$3 - 5x = 18$$

$$-15 = 5x$$

$$-3 = x$$

PTS: 2          REF: 012526aii          NAT: A.REI.A.2          TOP: Solving Rationals

27  ANS:

$$\left(-\frac{2}{7}\right)^2 + \sin^2\theta = 1 \qquad \frac{3\sqrt{5}}{7} \text{ as sin is positive in Quadrant II.}$$

$$\frac{4}{49} + \sin^2\theta = \frac{49}{49}$$

$$\sin^2\theta = \frac{45}{49}$$

$$\sin\theta = \pm\frac{3\sqrt{5}}{7}$$

PTS: 2          REF: 012527aii          NAT: F.TF.C.8          TOP: Determining Trigonometric Functions

28  ANS:

$$\frac{\sqrt[5]{a^{10}}}{\left(a^3\right)^{\frac{1}{2}}} = \frac{a^{\frac{10}{5}}}{a^{\frac{3}{2}}} = \frac{a^{\frac{20}{10}}}{a^{\frac{15}{10}}} = a^{\frac{5}{10}} \qquad x = \frac{1}{2}$$

PTS: 2          REF: 012528aii          NAT: N.RN.A.2          TOP: Radicals and Rational Exponents

29  ANS:
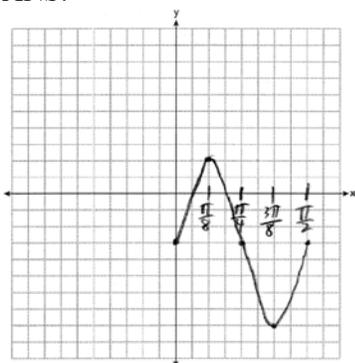


PTS:  2          REF:  012529aii          NAT:  F.IF.C.7          TOP:  Graphing Trigonometric Functions
KEY:  graph

30  ANS:

Julia: $V(x) = 33,400(0.85^{\frac{1}{12}})^{12x} \approx 33,400(0.9865)^{12x}$

PTS:  2          REF:  012530aii          NAT:  A.SSE.B.3          TOP:  Modeling Exponential Functions

31  ANS:
$\frac{20}{8} = 2.5$  $a_1 = 8$
$a_n = 2.5 \cdot a_{n-1}$

PTS:  2          REF:  012531aii          NAT:  F.BF.A.2          TOP:  Sequences
KEY:  recursive

32  ANS:
Yes.  Using a 95% confidence interval, values outside the interval $3.95 - 4.05$ are unusual.

PTS:  2          REF:  012532aii          NAT:  S.IC.A.2          TOP:  Analysis of Data

33  ANS:
$.74 \cdot .24 = .1776$  $.74 + .24 - .1776 = .8024$

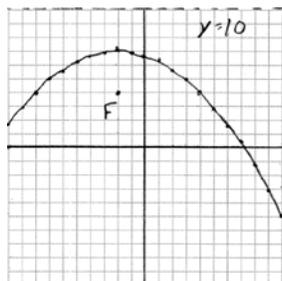PTS:  4          REF:  012533aii          NAT:  S.CP.B.7          TOP:  Addition Rule

34  ANS:
$\frac{m(4) - m(-1)}{4 - -1} = \frac{81 - 1}{5} = 16$  $p(x)$ has a greater rate of change

$\frac{p(4) - p(-1)}{4 - -1} = 16.1\overline{3}$

PTS:  4          REF:  012534aii          NAT:  F.IF.B.6          TOP:  Rate of Change
KEY:  exponential

35 ANS:

$\dfrac{10-4}{2} = 7$, so the vertex is $(-2,7)$ and $p = 3$. $y = -\dfrac{1}{4(3)}(x+2)^2 + 7 = y = -\dfrac{1}{12}(x+2)^2 + 7$
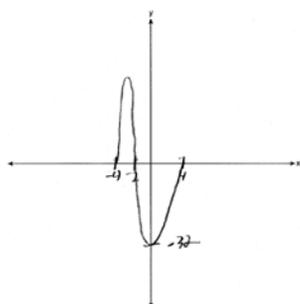


PTS: 4       REF: 012535aii     NAT: G.GPE.A.2    TOP: Graphing Quadratic Functions

36 ANS:



$x^3 + 2x^2 - 16x - 32 = 0$

$x^2(x+2) - 16(x+2) = 0$

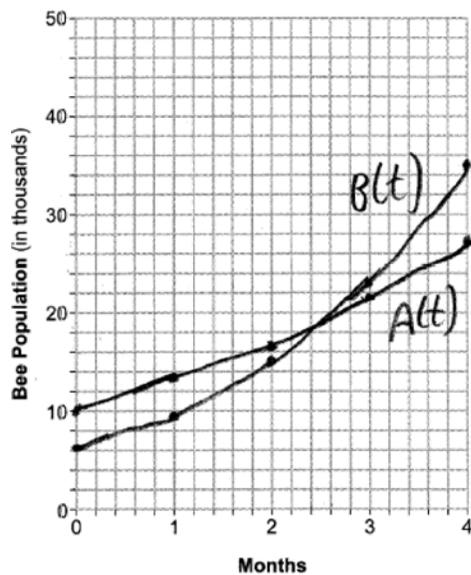$(x^2 - 16)(x+2) = 0$

$(x+4)(x-4)(x+2) = 0$

$x = -4, 4, -2$

PTS: 4       REF: 012536aii     NAT: F.IF.C.7      TOP: Graphing Polynomial Functions

37 ANS:



$A(t) = 10000e^{0.25t}$

$B(t) = 6000e^{0.45t}$

2.6 months for same. $30000 = 10000e^{0.25t}$

$$\ln 3 = \ln e^{0.25t}$$

$$\ln 3 = 0.25t \ln e$$

$$\frac{\ln 3}{0.25} = t$$

$$4.4 \approx t$$

PTS: 6        REF: 012537aii        NAT: A.REI.D.11        TOP: Other Systems
KEY: exponential