0126aii Regents Exam

1 The value of t in the equation $27^{6-t} = 9^{t-1}$ is

1) $\frac{7}{2}$

2) $\frac{19}{5}$

3) 8

4) 4

2 Which expression is equivalent to $(x-4)^2 - 5(x-4) + 6$?

1) $(x-2)(x-1)$

2) $(x+2)(x-3)$

3) $(x-10)(x-5)$

4) $(x-6)(x-7)$

3 Researchers wanted to determine if listening to classical music can reduce math anxiety. They divided 100 students randomly into two groups and put them in identical rooms. Both groups completed a math test and a survey rating their level of math anxiety. One group then listened to classical music for 10 minutes while the other group sat quietly. Both groups then took another math test, rated their level of math anxiety, and the researchers compared their results. Is this an observational study or an experiment?

1) It is an observational study because the participants completed a survey about math anxiety.

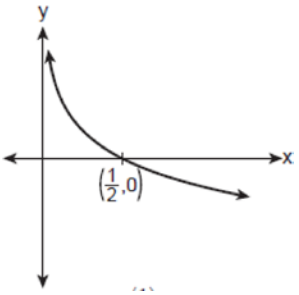
2) It is an observational study because the researchers watched the participants take math tests.

3) It is an experiment because one group was randomly assigned to listen to classical music while the other group was not.

4) It is an experiment because the participants took a math test.

4 Which graph represents the function $y = \log_{\frac{1}{2}}(x)$?

1)



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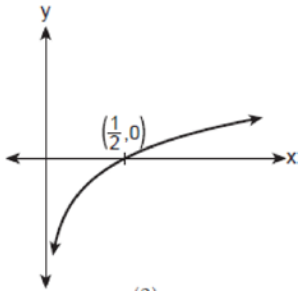

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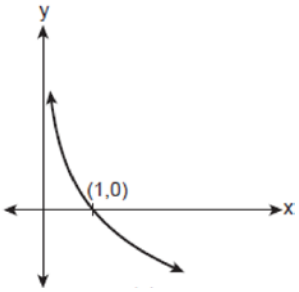
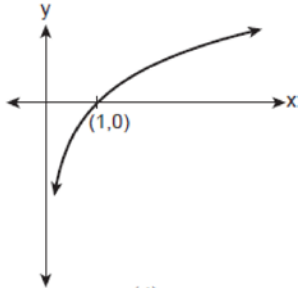
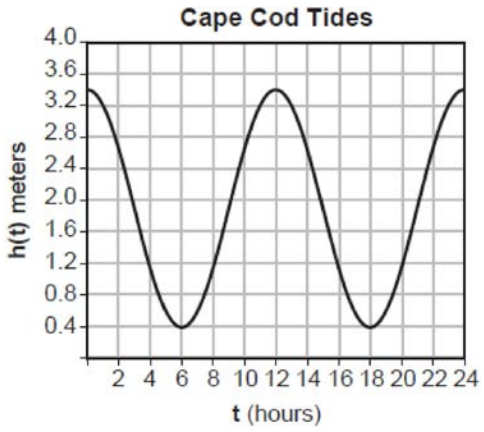


4)



5 The height of the water at Cape Cod, Massachusetts, is shown on the graph below.



If the height of the water, $h\{t\}$, is measured in meters and time, t, is measured in hours since high tide, what is an equation for the height of the water?

1) $h(t) = 1.9\cos\left(\frac{\pi}{6}t\right) + 1.5$

2) $h(t) = 1.5\cos\left(\frac{\pi}{6}t\right) + 1.9$

3) $h(t) = 1.9\cos\left(\frac{\pi}{12}t\right) + 1.5$

4) $h(t) = 1.5\cos\left(\frac{\pi}{12}t\right) + 1.9$

6 The solutions to $3x^2 - 4x + 2 = 0$ are

1) $x = \frac{2}{3} \pm \frac{\sqrt{2}}{3}i$

2) $x = \frac{2}{3} \pm \frac{\sqrt{10}}{3}$

3) $\frac{2}{3} \pm \frac{\sqrt{2}}{3}$

4) $x = \frac{2}{3} \pm \frac{\sqrt{10}}{3}i$

7 The value of $\sin\left(\frac{2\pi}{3}\right)$ is equivalent to

1) $\sin\left(\frac{4\pi}{3}\right)$

2) $\cos\left(\frac{\pi}{6}\right)$

3) $\tan\left(\frac{\pi}{3}\right)$

4) $\csc\left(\frac{2\pi}{3}\right)$

8 Given $f^{-1}(x) = \sqrt[3]{4x+1}$, which function represents $f(x)$?

1) $f(x) = \sqrt[3]{\frac{1}{4}x - 1}$

2) $f(x) = (4x+1)^3$

3) $f(x) = \frac{1}{4}(x^3 - 1)$

4) $f(x) = \frac{x^3}{4} - 1$

9 In order for a school to purchase graduation gowns, the heights of all of the high school seniors are recorded. The heights are approximately normally distributed with a mean of 64.7 inches and a standard deviation of 4.267 inches. If a senior is selected at random, what is the probability that this student has a height that falls between 65 and 68 inches?

1) 0.085
2) 0.177
3) 0.252
4) 0.271

10 Consider the system of equations below.

$$(x+1)^2+(y-4)^2=9$$

$$x=2$$

The x-value and y-value of the solution of the system are

1) one rational and one imaginary
2) both rational
3) one rational and one irrational
4) both irrational

11 Which expressions are equivalent to $36x^4-9x^2$?

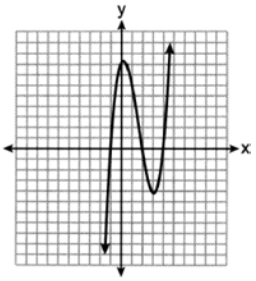
I. $x^2(6x+3)(6x-3)$
II. $36\left(x^2-\frac{1}{2}\right)\left(x^2+\frac{1}{2}\right)$
III. $9(2x^2+x)(2x^2-x)$
IV. $36x^2\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$

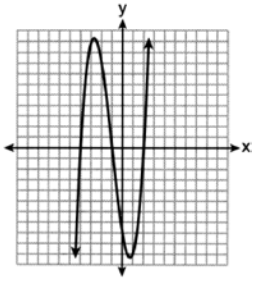
1) I and II, only
2) I and III, only
3) I, II, and III, only
4) I, III, and IV, only

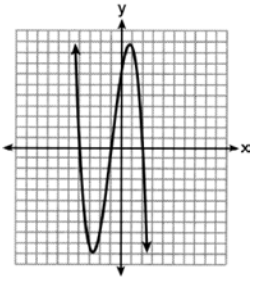
12 A deposit of \$1250 is made into a savings account that earns 1.86% annual interest. The amount in this account at any time, t, in years, is modeled by $A(t) = 1250(1.0186)^t$. Which expression best models the equivalent monthly growth of the account?

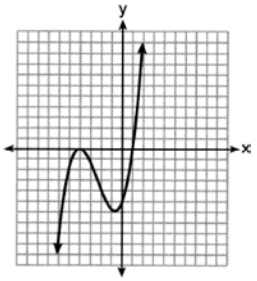
1) $1250(0.001550)^t$
2) $1250(0.001550)^{\frac{t}{12}}$
3) $1250(1.001537)^{12t}$
4) $1250(1.001537)^{\frac{t}{12}}$

13 Which graph represents a function that has a remainder of zero when divided by $(x-4)$?

1) 

3) 

2) 

4) 

14 Given $a > 0$, which expression is equivalent to $\dfrac{\sqrt[3]{a}\cdot\sqrt[5]{a^2}}{\sqrt{a}}$

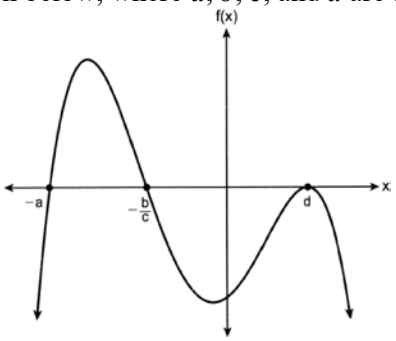
1) $a^{\frac{2}{13}}$

3) $a^{\frac{4}{15}}$

2) $a^{\frac{7}{30}}$

4) $a^{\frac{7}{3}}$

15 Which function could represent the sketch below, where a, b, c, and d are all greater than zero?



1) $f(x) = -(x+a)(cx+b)(x-d)^2$

3) $f(x) = -(x-a)(cx-b)(x+d)^2$

2) $f(x) = -(x+a)(bx+c)(2x-d)$

4) $f(x) = -(x-a)(bx-c)(2x+d)$

16 Before leaving for summer vacation, 447 students at a high school were asked whether or not they had a job for the summer. The results of the survey, broken down by the students' averages, are shown in the table below.

	Below 70%	**70%-79%**	**80%-89%**	**90%-100%**
Has a Job	35	73	78	52
Does Not Have a Job	42	62	70	35

The probability that a student has a job, given that the student's average is between 90% and 100%, rounded to the *nearest hundredth*, is

1) 0.11
2) 0.12
3) 0.59
4) 0.60

17 The expression $\frac{3x^3 - 4x^2 - 17x + 6}{3x - 1}$, where $x \neq \frac{1}{3}$, is equivalent to

1) $(x + 3)(x - 2)$
2) $(x - 3)(x + 2)$
3) $(x - 3)(x - 2)$
4) $x^2 - x - 6 + \frac{12}{3x - 1}$

18 Three students put their first steps to solve the equation $x^2 - 4x = 9$ on the board at school.

Max	Gus	Rosie
$\sqrt{x^2 - 4x} = \sqrt{9}$	$x^2 - 4x$ __ $= 9$ __	$x(x - 4) = 9$
$x - 2x = 3$	$x^2 - 4x + 4 = 9 + 4$	$x = 9 \quad x - 4 = 9$

Correct steps are shown for

1) Max, only
2) Gus, only
3) Max and Rosie, only
4) Gus and Rosie, only

19 The average depreciation rate of a laptop is approximately 33.3% each year. If Garrett purchased a new laptop for $700, which recursive formula represents the value of his laptop n years after it was purchased?

1) $a_0 = 700$
$a_n = a_{n-1}(0.667)$
2) $a_0 = 700$
$a_n = a_{n-1}(0.333)$
3) $a_n = 700(0.667)^{n-1}$
4) $a_n = 700(0.333)^{n-1}$

20 Given that the directrix of a parabola is $y = 7$ and its focus is 6 units vertically away from it, the vertex of this parabola could be the point

1) $(5, 13)$
2) $(2, 1)$
3) $(3, 10)$
4) $(4, 6)$

21 How many values of x satisfy the equation $\frac{x}{x-1} = -|2x-3| + 5$?

1) 1
2) 2
3) 3
4) 0

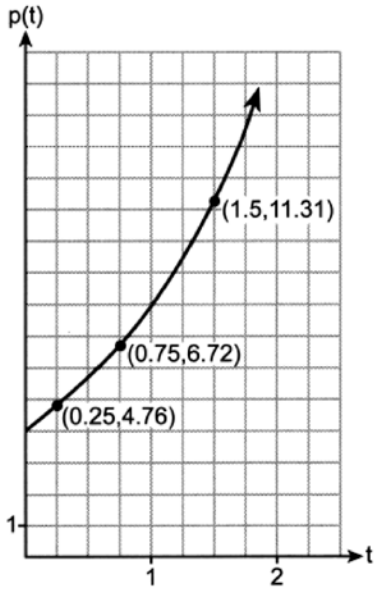
22 The tangent of an angle in standard position that terminates in Quadrant II is $-\sqrt{6}$. What is the cosine of this angle?

1) $\frac{1}{\sqrt{6}}$
2) $-\sqrt{7}$
3) $-\frac{1}{37}$
4) $-\frac{1}{\sqrt{7}}$

23 Miranda joins a gym and pays an annual membership of $400. Due to rising costs, the gym operator has told her the membership will increase 8% each year after the first year. How much will Miranda have paid in total, to the *nearest dollar*, if she is a member for 6 years?

1) 635
2) 2592
3) 2934
4) 4320

24 Ms. Miles observed slides of bacteria under a microscope. She found that over a specific amount of time, t, in hours, the population was growing. She graphed the results of the population, $p(t)$, in thousands, of the bacteria over time on the graph below.



Assuming exponential growth, which statement is true?

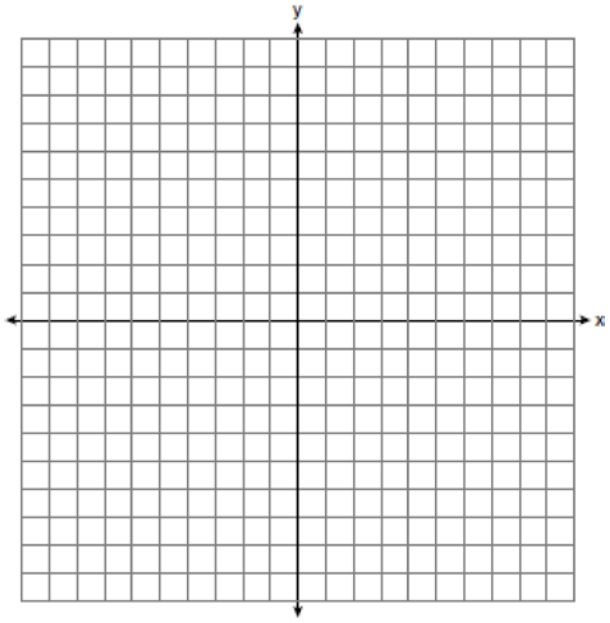
1) The population can be modeled by $p(t) = 2(4)^t$.
2) The population can be modeled by $p(t) = 3.92t + 3.78$.
3) The population approximately doubled every hour.
4) The population approximately doubled every half hour.

25 The number of people at a concert t hours after the doors open can be modeled by the equation $B(t) = 250(1.8)^t$. Determine how much time, to the *nearest hundredth of an hour*, must pass before the number of people reaches 2000.

26 On a day in June 2017 on Staten Island, the function $T(h) = 8\sin\left(\frac{\pi}{12}h\right) + 82$ was used to model the temperature, T, in degrees Fahrenheit, h hours after 9 a.m. State the value of $T(6)$ and explain its meaning in this context.

27 Use properties of exponents to show why $(-64)^{\frac{2}{3}} = 16$. Justify your answer.

28 Graph $y = 2^x - 2$ on the axes below.



State the equation of the asymptote.

29 In a survey of students at a large high school, 58% speak English fluently, 24% speak Mandarin fluently, and 16% speak English and Mandarin fluently. Determine the percentage of students at the school who speak English or Mandarin fluently.

30 The following table represents the number of years after 1980, x, and the median value of a home in the United States in thousands of dollars, y.

x	1	10	20	30	40
y	59.07	97.02	131.6	180.7	269.2

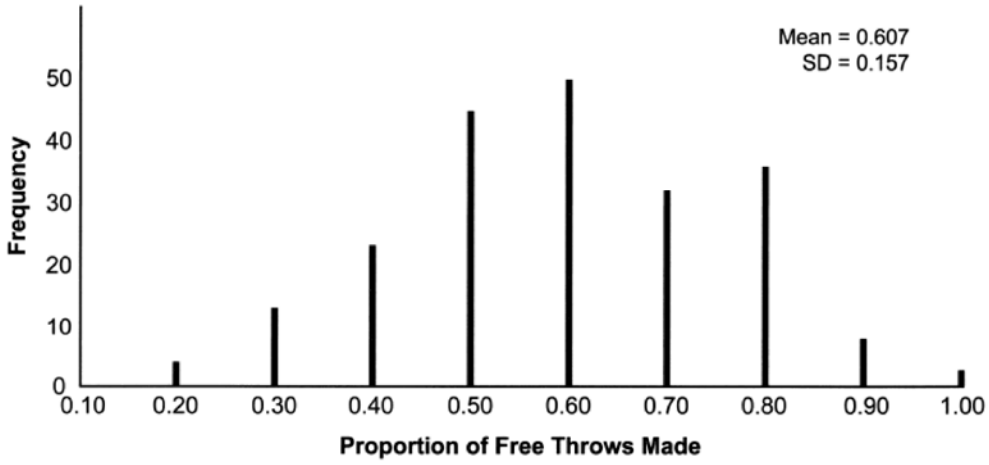
Based on these data, write an exponential regression equation to model the median home value, in thousands of dollars, x years after 1980. Round all coefficients to the *nearest hundredth*.

31 Solve algebraically for all values of x that satisfy the equation below.

$$\frac{24}{x^2+x}+\frac{x}{x+4}=\frac{5}{x}$$

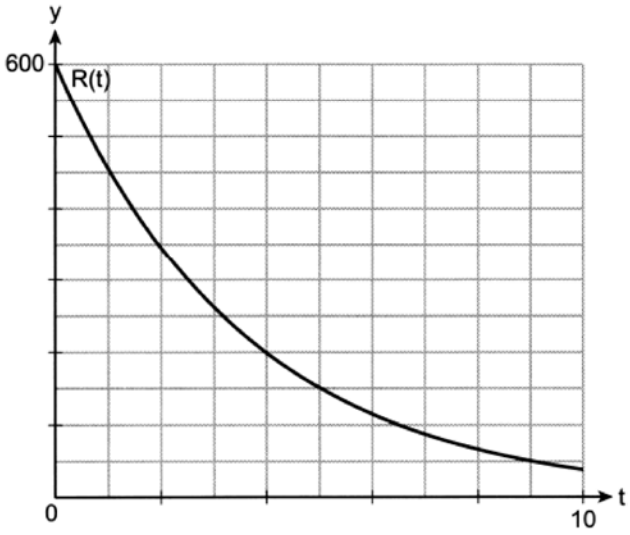
32 Express $i(x+i)-(x-i)^2$ in simplest $a+bi$ form.

33 On a high school basketball team, Alex typically makes 60% of his free throws. In the last four minutes of the game, Alex took 10 free throws and made 30% of them. Fans were saying he missed on purpose. A simulation was run of sample size 10, simulated 200 times, based on the premise that 60% of the free throws were made. The approximately normal results are shown below.



Based on the simulation, determine an interval containing the middle 95% of simulated values. State your answer to three decimal places. Do the fans have a valid argument? Explain using statistical evidence.

34 The breakdown of a drug is modeled by $A(t) = A_0 e^{-rt}$ where $A(t)$ is the amount of the drug in the body, A_0 is the initial dosage, r is the rate of decay, and t is the time in hours. A patient is given 450 mg of a drug that has a decay rate of 0.205. Write a function, $A(t)$, to model the amount of the drug remaining in the body after t hours. The sketch below shows the function $R(t)$, which models the breakdown of a different drug administered to the same patient. Graph $A(t)$ on the grid below.



Using the graph, approximate to the *nearest hour* when the patient has the same amount of both drugs remaining.

35 Consider the sinusoidal function below.

$$d(t) = 6800\cos\left(\frac{4}{3}\pi t\right) + 6400$$

State the maximum value of $d(t)$. State the period of $d(t)$. Determine the average rate of change from $\frac{3}{4} \le t \le 2$.

36 Solve algebraically for all values of m: $m - 2\sqrt{4m-3} = 3$

37 Xander and Yvette are each practicing for their road tests. Xander decides to drive for 15 minutes the first day and plans to increase the amount of time he spends driving by five minutes each day leading up to the day of his test. Yvette decides to drive for 10 minutes the first day, and she plans to increase the amount of time she spends driving by 15% each day leading up to the day of her test. State whose plan for the amount of time driving per day can be modeled by an arithmetic sequence, and whose plan can be modeled by a geometric sequence. Explain your reasoning. Write an equation for x_n that represents the amount of time spent driving on the nth day for Xander's plan and an equation for y_n that represents the amount of time spent driving on the nth day for Yvette's plan. Who will spend more time driving on the 19th day of practicing? Justify your answer.

0126aii Regents Exam
Answer Section

1 ANS: 4

$$27^{6-t} = 9^{t-1}$$
$$(3^3)^{6-t} = (3^2)^{t-1}$$
$$3^{18-3t} = 3^{2t-2}$$
$$18 - 3t = 2t - 2$$
$$20 = 5t$$
$$4 = t$$

PTS: 2 REF: 012601aii NAT: F.LE.A.4 TOP: Exponential Equations

2 ANS: 2

$u = x - 4 \qquad u^2 - 5u + 6$

$$(u-3)(u-2)$$
$$(x-4-3)(x-4-2)$$
$$(x-7)(x-6)$$

PTS: 2 REF: 012602aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

3 ANS: 3 PTS: 2 REF: 012603ai NAT: S.IC.B.6
TOP: Analysis of Data KEY: bias

4 ANS: 2 PTS: 2 REF: 012604aii NAT: F.IF.C.7
TOP: Graphing Logarithmic Functions

5 ANS: 2

$a = \frac{3.4 - .4}{2} = 1.5,\ 12 = \frac{2\pi}{b},\ \text{midline} = \frac{3.4 + .4}{2} = 1.9$

$$b = \frac{\pi}{6}$$

PTS: 2 REF: 012605aii NAT: F.TF.B.5 TOP: Modeling Trigonometric Functions

6 ANS: 1

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(2)}}{2(3)} = \frac{4 \pm \sqrt{16 - 24}}{6} = \frac{4 \pm \sqrt{-8}}{6} = \frac{4 \pm 2i\sqrt{2}}{6} = \frac{2 \pm i\sqrt{2}}{3}$$

PTS: 2 REF: 012606aii NAT: A.REI.B.4 TOP: Solving Quadratics

7 ANS: 2

$$\sin\left(\frac{2\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

PTS: 2 REF: 012607aii NAT: F.TF.A.2 TOP: Determining Trigonometric Functions
KEY: radians

8 ANS: 3

$$y = \sqrt[3]{4x+1}$$

$$x = \sqrt[3]{4y+1}$$

$$x^3 = 4y + 1$$

$$x^3 - 1 = 4y$$

$$\frac{x^3 - 1}{4} = y$$

PTS: 2 REF: 012608aii NAT: F.BF.B.4 TOP: Inverse of Functions
KEY: cubic

9 ANS: 3

normCdf(65,68,64.7,4.267)

PTS: 2 REF: 012609aii NAT: S.ID.A.4 TOP: Normal Distributions

10 ANS: 2

$$(2+1)^2 + (y-4)^2 = 9$$

$$9 + (y-4)^2 = 9$$

$$(y-4)^2 = 0$$

$$y = 4$$

PTS: 2 REF: 012610aii NAT: A.REI.C.7 TOP: Quadratic-Linear Systems

11 ANS: 4

I. $x^2(36x^2 - 9) = x^2(6x+3)(6x-3)$; II. $36\left(x^4 - \frac{x^2}{4}\right) = 36\left(x^2 + \frac{x}{2}\right)\left(x^2 - \frac{x}{2}\right)$;

III. $9(4x^4 - x^2) = 9(2x^2 + x)(2x^2 - x)$; IV. $36x^2\left(x^2 - \frac{1}{4}\right) = 36x^2\left(x - \frac{1}{2}\right)\left(x + \frac{1}{2}\right)$

PTS: 2 REF: 012611aii NAT: A.SSE.A.2 TOP: Factoring Polynomials

12 ANS: 3

$1.0186^{\frac{1}{12}} \approx 1.001537$

PTS: 2 REF: 012612aii NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

13 ANS: 1 PTS: 2 REF: 012613aii NAT: A.APR.B.2
TOP: Remainder and Factor Theorems

14 ANS: 2

$$\frac{\sqrt[3]{a}\cdot\sqrt[5]{a^2}}{\sqrt{a}}=\frac{a^{\frac{1}{3}}\cdot a^{\frac{2}{5}}}{a^{\frac{1}{2}}}=\frac{a^{\frac{10}{30}}\cdot a^{\frac{12}{30}}}{a^{\frac{15}{30}}}=a^{\frac{7}{30}}$$

PTS: 2 REF: 012614aii NAT: N.RN.A.2 TOP: Radicals and Rational Exponents

15 ANS: 1 PTS: 2 REF: 012615aii NAT: A.APR.B.3
TOP: Graphing Polynomial Functions

16 ANS: 4

$$\frac{52}{52+35}\approx.60$$

PTS: 2 REF: 012616aii NAT: S.CP.A.4 TOP: Conditional Probability

17 ANS: 2

$$\begin{array}{r} x^2-x-6 \\ 3x-1\overline{)\,3x^3-4x^2-17x+6} \\ \underline{3x^3-x^2} \\ -3x^2-17x \\ \underline{-3x^2+x} \\ -18x+6 \\ \underline{-18x+6} \end{array}$$

PTS: 2 REF: 012617aii NAT: A.APR.D.6 TOP: Rational Expressions
KEY: division

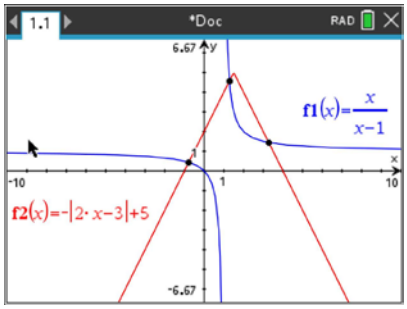
18 ANS: 2 PTS: 2 REF: 012618aii NAT: A.REI.B.4
TOP: Solving Quadratics

19 ANS: 1 PTS: 2 REF: 012619aii NAT: F.BF.A.2
TOP: Sequences KEY: recursive, geometric

20 ANS: 3
The vertical distance from the directrix to the focus, p, is 6. The vertical distance from the vertex to the focus is half this distance, or 3. $y = 7 + 3 = 10$

PTS: 2 REF: 012620aii NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

21 ANS: 3



PTS: 2 REF: 012621aii NAT: A.REI.D.11 TOP: Other Systems
KEY: rational

22 ANS: 4

$\tan^2 x + 1 = \sec^2 x \quad \cos x = \pm\frac{1}{\sqrt{7}}$ Cosine is negative in Quadrant II.

$(-\sqrt{6})^2 + 1 = \sec^2 x$

$7 = \sec^2 x$

$\pm\sqrt{7} = \sec x$

PTS: 2 REF: 012622aii NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

23 ANS: 3

$S_6 = \frac{400(1 - 1.08^6)}{1 - 1.08} \approx 2934$

PTS: 2 REF: 012623aii NAT: F.BF.B.7 TOP: Series
KEY: geometric

24 ANS: 3 PTS: 2 REF: 012624aii NAT: F.LE.B.5
TOP: Modeling Exponential Functions

25 ANS:

$2000 = 250(1.8)^t$

$8 = 1.8^t$

$\log 8 = \log 1.8^t$

$\log 8 = t \log 1.8$

$\frac{\log 8}{\log 1.8} = t$

$3.54 \approx t$

PTS: 2 REF: 012625aii NAT: F.LE.A.4 TOP: Exponential Growth

26 ANS:
The temperature at 3pm is 90º.

PTS: 2 REF: 012626aii NAT: F.IF.B.4 TOP: Graphing Trigonometric Functions

27 ANS:

$$(-64)^{\frac{2}{3}} = 16$$

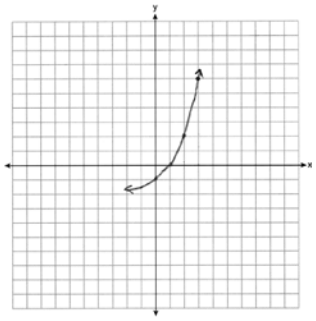
$$\left((-64)^{\frac{1}{3}}\right)^2 = 16$$

$$(-4)^2 = 16$$

$$16 = 16$$

PTS: 2 REF: 012627aii NAT: N.RN.A.1 TOP: Radicals and Rational Exponents

28 ANS:



$y = -2$

PTS: 2 REF: 012628aii NAT: F.IF.C.7 TOP: Graphing Exponential Functions

29 ANS:

$58\% + 24\% - 16\% = 66\%$

PTS: 2 REF: 012629aii NAT: S.CP.B.7 TOP: Addition Rule

30 ANS:

$y = 61.14(1.04)^x$

PTS: 2 REF: 012630aii NAT: S.ID.B.6 TOP: Regression
KEY: exponential

31 ANS:

$$\frac{24}{x^2 + x} + \frac{x}{x + 4} = \frac{5}{x}$$

$$\frac{24}{x^2 + x} + \frac{x^2}{x(x + 4)} = \frac{5(x + 4)}{x(x + 4)}$$

$$24 + x^2 = 5x + 20$$

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

$$x = 1, 4$$

PTS: 2 REF: 012631aii NAT: A.REI.A.2 TOP: Solving Rationals

32 ANS:

$i(x+i)-(x-i)^2 = ix+i^2-(x^2-2ix+i^2) = ix-1-x^2+2ix+1 = -x^2+3xi$

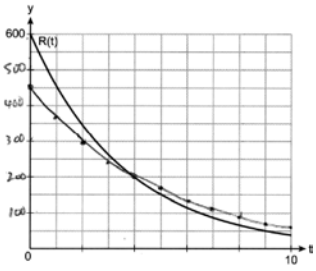
PTS: 2 REF: 012632aii NAT: N.CN.A.2 TOP: Operations with Complex Numbers

33 ANS:

$0.607 \pm 2 \cdot 0.157 \approx 0.293 - 0.921$. No, since 0.30 falls within the 95% interval.

PTS: 4 REF: 012633aii NAT: S.IC.A.2 TOP: Analysis of Data

34 ANS:

$A(t) = 450e^{-0.205t}$ 

4

PTS: 4 REF: 012634aii NAT: A.REI.D.11 TOP: Other Systems
KEY: exponential

35 ANS:

$6800(1)+6400 = 13.200.$ $\dfrac{2\pi}{\frac{4}{3}\pi} = 1.5.$ $\dfrac{d(2)-d\left(\frac{3}{4}\right)}{2-\frac{3}{4}} = 2,720.$

PTS: 4 REF: 012635aii NAT: F.IF.B.6 TOP: Rate of Change
KEY: trigonometric

36 ANS:

$m-2\sqrt{4m-3} = 3$ 1 is extraneous.

$m-3 = 2\sqrt{4m-3}$

$m^2-6m+9 = 4(4m-3)$

$m^2-6m+9 = 16m-12$

$m^2-22m+21 = 0$

$(m-1)(m-21) = 0$

$m = 1, 21$

PTS: 4 REF: 012636aii NAT: A.REI.A.2 TOP: Solving Radicals

37 ANS:
Xander's plan is arithmetic because the time increases by 5 minutes every day. Yvette's plan is geometric because the time increases by 15% every day. $x_n = 15 + (n-1)5$. Yvette, as $x_{19} = 15 + (19-1)5 = 105$

$$y_n = 10(1.15)^{n-1} \qquad y_{19} = 10(1.15)^{19-1} \approx 124$$

PTS: 6 REF: 012637aii NAT: F.BF.A.2 TOP: Sequences
KEY: explicit