0822aii

1 The Hot and Tasty Coffee chain conducts a survey of its customers at its location at the Staten Island ferry terminal. After the survey is completed, the statistical consultant states that 70% of customers who took the survey said the most important factor in choosing where to get their coffee is how fast they are served. Based on this result, Hot and Tasty Coffee can infer that

1) most of its customers in New York State care most about being served quickly

2) coffee drinkers care less about taste and more about being served quickly

3) most of its customers at the Staten Island ferry terminal care most about being served quickly

4) most of its customers at transportation terminals and stations care most about being served quickly

2 Given that i is the imaginary unit, the expression $(x - 2i)^2$ is equivalent to

1) $x^2 + 4$

2) $x^2 - 4$

3) $x^2 - 2xi - 4$

4) $x^2 - 4xi - 4$

3 The equation below can be used to model the height of a tide in feet, $H(t)$, on a beach at t hours.

$$H(t) = 4.8\sin\left[\frac{\pi}{6}(t+3)\right] + 5.1$$

Using this function, the amplitude of the tide is

1) $\frac{\pi}{6}$

2) 4.8

3) 3

4) 5.1

4 In watching auditions for lead singer in a band, Liem became curious as to whether there is an association between how animated the lead singer is and the amount of applause from the audience. He decided to watch each singer and rate the singer on a scale of 1 to 5, where 1 is the least animated and 5 is the most animated. He did this for all 5 nights of auditions and found that the more animated singers did receive louder applause. The study Liem conducted would be best described as

1) experimental

2) observational

3) a sample survey

4) a random assignment

5 In the diagram of a unit circle below, point A, $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, represents the point where the terminal side of θ intersects the unit circle.

What is $m\angle\theta$?

1) 30°
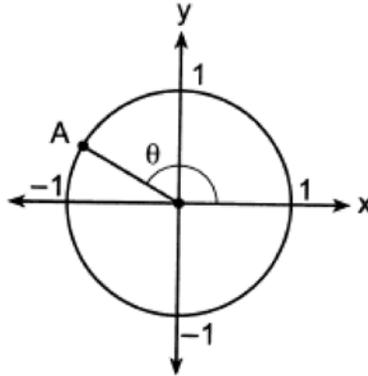
2) 120°

3) 135°

4) 150°

6 Consider the function $f(x) = 2x^3 + x^2 - 18x - 9$. Which statement is true?

1) $2x - 1$ is a factor of $f(x)$.

2) $x - 3$ is a factor of $f(x)$.

3) $f(3) \neq f\left(-\frac{1}{2}\right)$

4) $f\left(\frac{1}{2}\right) = 0$

7 Which sketch could represent the function $m(x) = -\log_{100}(x - 2)$?

1)

3)

2)

4)
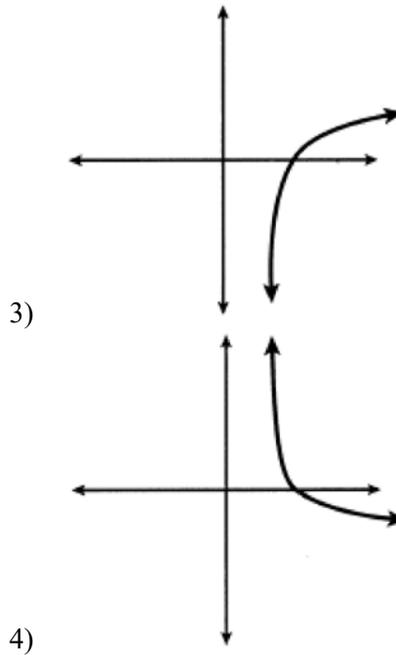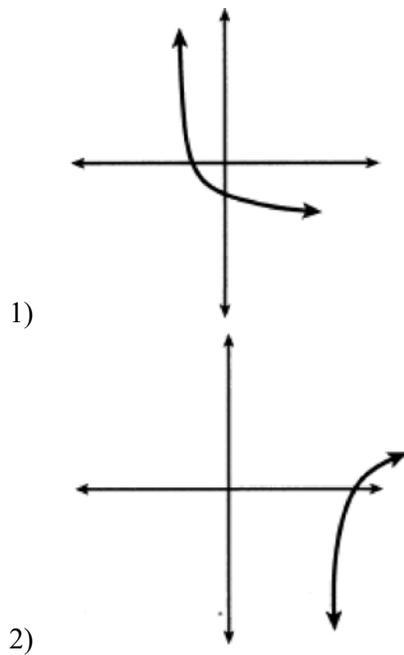
8 Which equation has roots of $3+i$ and $3-i$?

1) $x^2 - 6x + 10 = 0$

2) $x^2 + 6x - 10 = 0$

3) $x^2 - 10x + 6 = 0$

4) $x^2 + 10x - 6 = 0$

9 A local university has a current enrollment of 12,000 students. The enrollment is increasing continuously at a rate of 2.5% each year. Which logarithm is equal to the number of years it will take for the population to increase to 15,000 students?

1) $\dfrac{\ln 1.25}{0.25}$

2) $\dfrac{\ln 3000}{0.025}$

3) $\dfrac{\ln 1.25}{2.5}$

4) $\dfrac{\ln 1.25}{0.025}$

10 What is the total number of points of intersection of the graphs of the equations $y = e^x$ and $xy = 20$?

1) 1

2) 2

3) 3

4) 0

11 The amount of a substance, $A(t)$, in grams, remaining after t days is modeled by $A(t) = 50(0.5)^{\frac{t}{3}}$. Which statement is false?

1) In 20 days, there is no substance remaining.

2) After two half-lives, there is 25% of the substance remaining.

3) The amount of the substance remaining can also be modeled by $A(t) = 50(2)^{\frac{-t}{3}}$.

4) After one week, there is less than 10g of the substance remaining.

12 A parabola that has a vertex at $(2,1)$ and a focus of $(2,-3)$ has an equation of

1) $y = \dfrac{1}{16}(x-2)^2 + 1$

2) $y = -\dfrac{1}{16}(x+2)^2 - 1$

3) $y = -\dfrac{1}{16}(x-2)^2 + 1$

4) $y = -\dfrac{1}{16}(x-2)^2 - 3$

13 The expression $\left(a\sqrt[3]{2b^2}\right)\left(\sqrt[3]{4a^2 b}\right)$ is equivalent to

1) $2ab\sqrt[3]{a^2}$

2) $2ab$

3) $2ab\sqrt[3]{2a^2}$

4) $2a^2 b\sqrt[3]{2b}$

14 Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

1) $f(x) \to -1$

2) $f(x) \to 0$

3) $f(x) \to 2$

4) $f(x) \to -\infty$

15 For all values of x for which the expression is defined, $\dfrac{x^2 + 3x}{x^2 + 5x + 6}$ is equivalent to

1) $1 - \dfrac{x}{x + 2}$

2) $\dfrac{x}{x + 2}$

3) $\dfrac{3x}{5x + 6}$

4) $1 + \dfrac{1}{2x + 6}$

16 A recursive formula for the sequence $64, 48, 36, \ldots$ is

1) $a_n = 64(0.75)^{n-1}$

2) $a_1 = 64$

    $a_n = a_{n-1} - 16$

3) $a_n = 64 + (n-1)(-16)$

4) $a_1 = 64$

    $a_n = 0.75a_{n-1}$

17 Which expression is equivalent to $\dfrac{x^3 - 2}{x - 2}$?

1) x^2

2) $x^2 + 2x + 4 + \dfrac{6}{x - 2}$

3) $x^2 - 2$

4) $x^2 - 2x + 4 - \dfrac{10}{x - 2}$

18 What is the solution set of the equation $\dfrac{4}{k^2 - 8k + 12} = \dfrac{k}{k - 2} + \dfrac{1}{k - 6}$?

1) $\{-1, 6\}$

2) $\{1, -6\}$

3) $\{-1\}$

4) $\{1\}$

19 Given the polynomial identity $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$, which equation must also be true for all values of x and y?

1) $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$

2) $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$

3) $(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

4) $(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$

20 Given $p(\theta) = 3\sin\left(\dfrac{1}{2}\,\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p

1) decreases, then increases

2) increases, then decreases

3) decreases throughout the interval

4) increases throughout the interval

21 A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

Year	Amount Saved (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

Which expression determines the total amount of money saved by the company over 5 years?

1) $\dfrac{59,000 - 59,000(1.1)^5}{1 - 1.1}$

3) $\sum\limits_{n=1}^{5} 59,000(1.1)^n$

2) $\dfrac{59,000 - 59,000(0.1)^5}{1 - 0.1}$

4) $\sum\limits_{n=1}^{5} 59,000(0.1)^{n-1}$

22 A rush-hour commuter train has arrived on time 64 of its first 80 days. As arrivals continue, which equation can be used to find x, the number of consecutive days that the train must arrive on schedule to raise its on-time performance rate to 90%?

1) $\dfrac{64}{80+x} = \dfrac{90}{100}$

3) $\dfrac{64+x}{80} = \dfrac{90}{100}$

2) $\dfrac{64+x}{80+x} = \dfrac{90}{100}$

4) $\dfrac{x}{80+x} = \dfrac{90}{100}$

23 Given $f(x) = -\dfrac{2}{5}x + 4$, which statement is true of the inverse function $f^{-1}(x)$?

1) $f^{-1}(x)$ is a line with slope $\dfrac{5}{2}$.

3) $f^{-1}(x)$ passes through the point $(6,-5)$.

2) $f^{-1}(x)$ is a line with slope $\dfrac{2}{5}$.

4) $f^{-1}(x)$ has a y-intercept at $(0,-4)$.

24 The amount of a substance, $A(t)$, that remains after t days can be given by the equation $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is
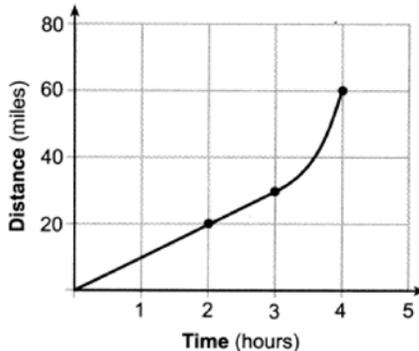
1) $A(t) = A_0(0.000178)^t$

3) $A(t) = A_0(0.04015)^t$

2) $A(t) = A_0(0.945861)^t$

4) $A(t) = A_0(1.08361)^t$

25 Determine the average rate of change, in mph, from 2 to 4 hours on the graph shown below.

26 Factor the expression $x^3 - 2x^2 - 9x + 18$ completely.

27 Solve algebraically for all values of x: $\sqrt{4x + 1} = 11 - x$

28 Given that $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}} \right)^{-4} = y^n$, where $y > 0$, determine the value of n.

29 Given $\cos A = \dfrac{3}{\sqrt{10}}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

30 According to a study done at a hospital, the average weight of a newborn baby is 3.39 kg, with a standard deviation of 0.55 kg. The weights of all the newborns in this hospital closely follow a normal distribution. Last year, 9256 babies were born at this hospital. Determine, to the *nearest integer*, approximately how many babies weighed more than 4 kg.

31 The table below shows the results of gender and music preference. Based on these data, determine if the events "the person is female" and "the person prefers classic rock" are independent of each other. Justify your answer.

	Rap	**Techno**	**Classic Rock**	**Classical**
Male	39	17	42	12
Female	17	37	36	15

32 Algebraically determine the solution set for the system of equations below.
$$y = 2x^2 - 7x + 4$$
$$y = 11 - 2x$$

33 When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.
a) Assuming an initial value of 11,000 bacteria, write a function, $p(t)$, that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope.
b) Using $p(t)$ from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

34 The function $v(x) = x(3-x)(x+4)$ models the volume, in cubic inches, of a rectangular solid for $0 \le x \le 3$. Graph $y = v(x)$ over the domain $0 \le x \le 3$.
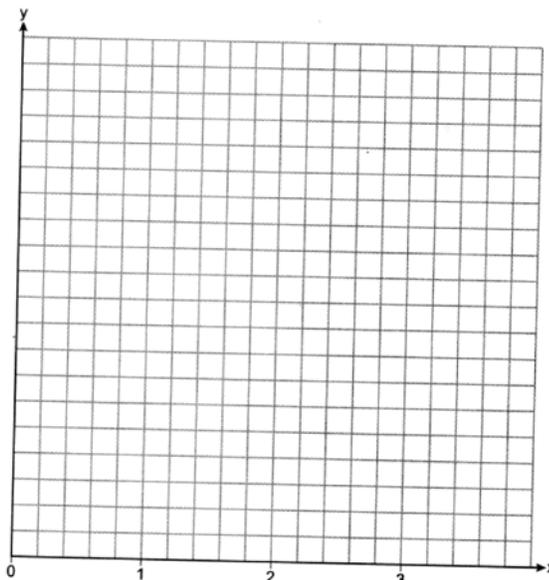
To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

35 Given $f(x) = 3x^3 - 4x^2 + 2x - 1$ and $g(x) = x - 4$, state the quotient and remainder of $\dfrac{f(x)}{g(x)}$, in the form $q(x) + \dfrac{r(x)}{g(x)}$. Is $x = 4$ a root of $f(x)$? Explain your answer.

36 State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.
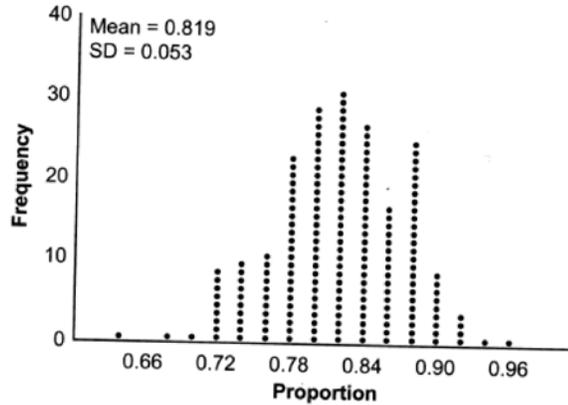
Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth*. The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

37 A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer. Graph $A(x)$ and $B(x)$ in the interval $0 \le x \le 100$ on the set of axes below.
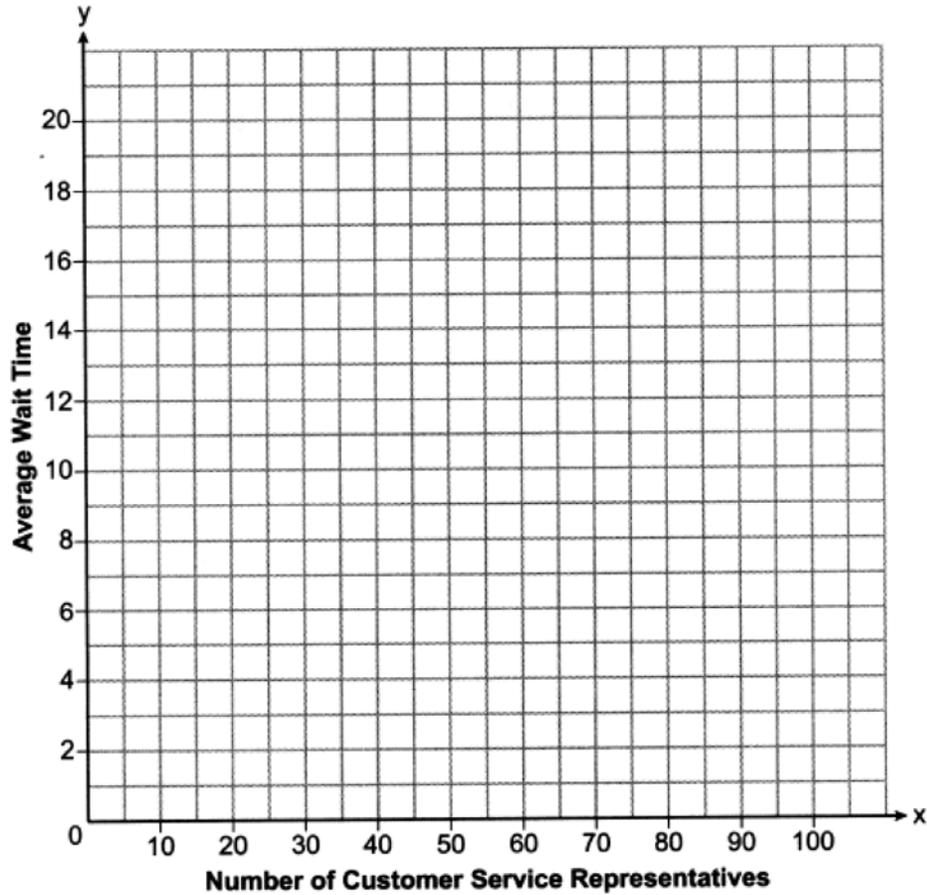
To the *nearest integer*, solve the equation $A(x) = B(x)$. Determine, to the *nearest minute*, $B(100) - A(100)$. Explain what this value represents in the given context.

0822aii
Answer Section

1 ANS: 3   PTS: 2   REF: 082201aii   NAT: S.IC.B.3
   TOP: Analysis of Data   KEY: type

2 ANS: 4

$$(x - 2i)(x - 2i) = x^2 - 4xi + 4i^2 = x^2 - 4xi - 4$$

   PTS: 2   REF: 082202aii   NAT: N.CN.A.2   TOP: Operations with Complex Numbers

3 ANS: 2   PTS: 2   REF: 082203aii   NAT: F.IF.C.7
   TOP: Graphing Trigonometric Functions   KEY: amplitude

4 ANS: 2   PTS: 2   REF: 082204aii   NAT: S.IC.B.3
   TOP: Analysis of Data   KEY: type

5 ANS: 4   PTS: 2   REF: 082205aii   NAT: F.TF.A.2
   TOP: Unit Circle

6 ANS: 2

$$2x^3 + x^2 - 18x - 9$$

$$x^2(2x + 1) - 9(2x + 1)$$

$$(x^2 - 9)(2x + 1)$$

$$(x + 3)(x - 3)(2x + 1)$$

   PTS: 2   REF: 082206aii   NAT: A.APR.B.2   TOP: Remainder Theorem

7 ANS: 4
   Translate the parent log function 2 to the right and reflect over the *x*-axis.

   PTS: 2   REF: 082207aii   NAT: F.IF.C.7   TOP: Graphing Logarithmic Functions

8 ANS: 1

   The product of the roots equals $(3 + i)(3 - i) = 9 - i^2 = 10 = \dfrac{c}{a}$. OR

$$(x - (3 + i))(x - (3 - i)) = 0$$

$$(x - 3 - i)(x - 3 + i) = 0$$

$$((x - 3) - i)((x - 3) + i) = 0$$

$$(x - 3)^2 - i^2 = 0$$

$$x^2 - 6x + 9 + 1 = 0$$

$$x^2 - 6x + 10 = 0$$

   PTS: 2   REF: 082208aii   NAT: A.REI.B.4   TOP: Complex Conjugate Root Theorem

9 ANS: 4

$$\frac{15000}{12000} = \frac{12000e^{.025t}}{12000}$$

$$1.25 = e^{.025t}$$

$$\ln 1.25 = \ln e^{.025t}$$
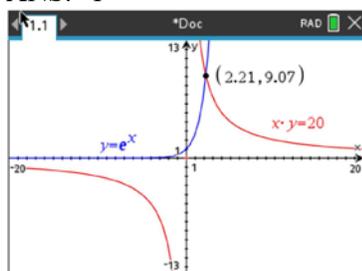
$$\ln 1.25 = .025t$$

$$\frac{\ln 1.25}{.025} = t$$

PTS: 2   REF: 082209aii   NAT: F.LE.A.4   TOP: Exponential Growth

10 ANS: 1

PTS: 2   REF: 082210aii   NAT: A.REI.D.11   TOP: Other Systems

11 ANS: 1

1) $A(20) > 0$; 2) $.5 \times .5 = .25$; 3) true; 4) $A(7) \approx 9.9$

PTS: 2   REF: 082211aii   NAT: F.LE.B.5   TOP: Modeling Exponential Functions

12 ANS: 3

The distance from the vertex to the focus, p, is 4. Since the focus is below the vertex, p is negative.

$$y = -\frac{1}{4(4)}(x-2)^2 + 1$$

PTS: 2   REF: 082212aii   NAT: G.GPE.A.2   TOP: Graphing Quadratic Functions

13 ANS: 1

$$\left(a\sqrt[3]{2b^2}\right)\left(\sqrt[3]{4a^2b}\right) = a\sqrt[3]{8a^2b^3} = 2ab\sqrt[3]{a^2}$$

PTS: 2   REF: 082213aii   NAT: N.RN.A.2   TOP: Operations with Radicals

KEY: with variables, index > 2

14 ANS: 3   PTS: 2   REF: 082214aii   NAT: F.IF.C.7

TOP: Graphing Exponential Functions

15 ANS: 2

$$\frac{x^2+3x}{x^2+5x+6} = \frac{x(x+3)}{(x+2)(x+3)}$$

PTS: 2   REF: 082215aii   NAT: A.APR.D.6   TOP: Rational Expressions

KEY: factoring

16 ANS: 4
1) is a correct formula, but not recursive

PTS: 2   REF: 082216aii   NAT: F.LE.A.2   TOP: Sequences
KEY: recursive

17 ANS: 2

$$\begin{array}{r} x^2 + 2x + 4 \\ x - 2 \overline{\smash{\big)}\, x^3 - 0x^2 + 0x - 2} \\ \underline{x^3 - 2x^2} \\ 2x^2 + 0x \\ \underline{2x^2 - 4x} \\ 4x - 2 \\ \underline{4x - 8} \\ 6 \end{array}$$

PTS: 2   REF: 082217aii   NAT: A.APR.D.6   TOP: Rational Expressions
KEY: division

18 ANS: 3

$$\frac{4}{k^2 - 8k + 12} = \frac{k(k-6) + (k-2)}{k^2 - 8k + 12}$$ $k = 6$ is extraneous

$$4 = k^2 - 6k + k - 2$$

$$0 = k^2 - 5k - 6$$

$$0 = (k - 6)(k + 1)$$

$$k = 6, -1$$

PTS: 2   REF: 082218aii   NAT: A.REI.A.2   TOP: Solving Rationals

19 ANS: 1

2) $(x^4 - x^2 y^2 + y^4) \neq (x^2 - y^2)(x^2 - y^2)$; 3) $x^6 + y^6 \neq (x^3 + y^3)^2$; 4) $\dfrac{x^6 + y^6}{x^2 + y^2} \neq x^6 + y^6 - (x^2 + y^2)$

PTS: 2   REF: 082219aii   NAT: A.APR.C.4   TOP: Polynomial Identities

20 ANS: 4   PTS: 2   REF: 082220aii   NAT: F.IF.B.4
TOP: Graphing Trigonometric Functions

21 ANS: 1   PTS: 2   REF: 082221aii   NAT: F.BF.B.6
TOP: Sigma Notation   KEY: represent

22 ANS: 2   PTS: 2   REF: 082222aii   NAT: A.CED.A.1
TOP: Modeling Rationals

23 ANS: 3

$$x = -\frac{2y}{5} + 4 \quad y = -\frac{5}{2}(6) + 10 = -5$$

$$5x = -2y + 20$$

$$2y = -5x + 20$$

$$y = -\frac{5}{2}x + 10$$

PTS: 2   REF: 082223aii   NAT: F.BF.B.4   TOP: Inverse of Functions
KEY: linear

24 ANS: 1

$$0.5^{\frac{1}{0.0803}} \approx 0.000178$$

PTS: 2   REF: 082224aii   NAT: A.SSE.B.3   TOP: Modeling Exponential Functions

25 ANS:

$$\frac{60 - 20}{4 - 2} = \frac{40}{2} = 20$$

PTS: 2   REF: 082225aii   NAT: F.IF.B.6   TOP: Rate of Change

26 ANS:

$$x^3 - 2x^2 - 9x + 18 = x^2(x - 2) - 9(x - 2) = \left(x^2 - 9\right)(x - 2) = (x + 3)(x - 3)(x - 2)$$

PTS: 2   REF: 082226aii   NAT: A.SSE.A.2   TOP: Factoring Polynomials
KEY: factoring by grouping

27 ANS:

$$\sqrt{4x + 1} = 11 - x \qquad \text{20 is extraneous.}$$

$$4x + 1 = 121 - 22x + x^2$$

$$0 = x^2 - 26x + 120$$

$$0 = (x - 6)(x - 20)$$

$$x = 6, 20$$

PTS: 2   REF: 082227aii   NAT: A.REI.A.2   TOP: Solving Radicals
KEY: extraneous solutions

28 ANS:

$$\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{10}{8}}} \right)^{-4} = y^n \quad n = -\dfrac{7}{2}$$

$$\left(y^{\frac{7}{8}} \right)^{-4} = y^n$$

$$y^{-\frac{7}{2}} = y^n$$

PTS: 2   REF: 082228aii   NAT: A.APR.D.6   TOP: Expressions with Negative Exponents
KEY: variables

29 ANS:

$$\cot A = \dfrac{\cos A}{\sin A}$$

$$-3 = \dfrac{\dfrac{3}{\sqrt{10}}}{\sin A}$$

$$\sin A = \dfrac{3}{-3\sqrt{10}} = -\dfrac{1}{\sqrt{10}}$$

PTS: 2   REF: 082229aii   NAT: F.TF.C.8   TOP: Determining Trigonometric Functions

30 ANS:
$0.133696 \times 9256 \approx 1237$

PTS: 2   REF: 082230aii   NAT: S.ID.A.4   TOP: Normal Distributions
KEY: predict

31 ANS:
No, because $P(F \,/\, CR) \neq P(F)$

$$\dfrac{36}{42 + 36} \neq \dfrac{17 + 37 + 36 + 15}{39 + 17 + 42 + 12 + 17 + 37 + 36 + 15}$$

$$\dfrac{36}{78} \neq \dfrac{105}{215}$$

$$\dfrac{6}{13} \neq \dfrac{21}{43}$$

PTS: 2   REF: 082231aii   NAT: S.CP.A.4   TOP: Conditional Probability

32 ANS:

$$2x^2 - 7x + 4 = 11 - 2x \qquad y = 11 - 2\left(\frac{7}{2}\right) = 4 \qquad \left\{\left(\frac{7}{2}, 4\right), (-1, 13)\right\}$$

$$2x^2 - 5x - 7 = 0$$

$$(2x - 7)(x + 1) = 0 \qquad y = 11 - 2(-1) = 13$$

$$x = \frac{7}{2}, -1$$

PTS: 2   REF: 082232aii   NAT: A.REI.C.7   TOP: Quadratic-Linear Systems

33 ANS:

a) $p(t) = 11000(2)^{\frac{t}{20}}$;   b) $\dfrac{1000000}{11000} = \dfrac{11000(2)^{\frac{t}{20}}}{11000}$

$$\log\frac{1000}{11} = \log 2^{\frac{t}{20}}$$

$$\log\frac{1000}{11} = \frac{t \cdot \log 2}{20}$$

$$\frac{20\log\dfrac{1000}{11}}{\log 2} = t$$

$$t \approx 130.13$$

PTS: 4   REF: 082233aii   NAT: F.LE.A.4   TOP: Exponential Growth

34 ANS:

 12.6

PTS: 4   REF: 082234aii   NAT: F.IF.C.7   TOP: Graphing Polynomial Functions

35 ANS:

$$x - 4 \overline{\smash{\big)}\ 3x^3 - 4x^2 + 2x - 1} \quad \begin{array}{c} 3x^2 + 8x + 34 \end{array}$$

$$\begin{array}{r} 3x^3 - 12x^2 \\ \hline 8x^2 + 2x \\ 8x^2 - 32x \\ \hline 34x - 1 \\ 34x - 136 \\ \hline 135 \end{array}$$

$3x^2 + 8x + 34 + \dfrac{135}{x - 4}$ $x = 4$ is not a root of $f(x)$ because $\dfrac{f(x)}{g(x)}$ has a remainder.
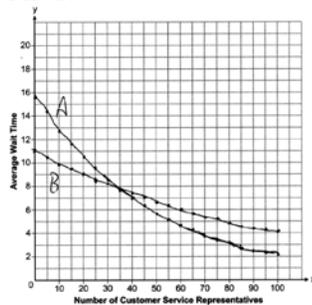
PTS: 4   REF: 082235aii   NAT: A.APR.D.6   TOP: Rational Expressions
KEY: division

36 ANS:
$.819 \pm 2 \cdot .053 = .713 - .925$. Since .70 does not fall within the 95% interval.

PTS: 4   REF: 082236aii   NAT: S.IC.A.2   TOP: Analysis of Data

37 ANS:

35; $B(100) - A(100) \approx 2$, which represents the difference of the average wait time when there are 100 CSRs between the plans.

PTS: 6   REF: 082237aii   NAT: A.REI.D.11   TOP: Other Systems