ALGEBRA II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 25, 2025 — 9:15 a.m. to 12:15 p.m., only

Student Name ___Mr. S.bol___

School Name ___JMAP___

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 Which expression is equivalent to $2c\sqrt[3]{c}$?

(1) $2c^{\frac{4}{3}}$

(2) $2c^{\frac{3}{4}}$

(3) $(2c)^{\frac{4}{3}}$

(4) $(2c)^{\frac{3}{4}}$

$$2c^{\frac{3}{3}} \cdot c^{\frac{1}{3}}$$

$$2c^{\frac{4}{3}}$$

2 Which investigation technique is most often used to determine the cause and effect of a medication?

(1) observational study

(2) survey

(3) controlled experiment

(4) census

3 What is the solution to $5(2)^{19x} = 50$?

(1) $x = \dfrac{\log(50)}{19}$

(2) $x = \dfrac{\log_2(10)}{19}$

(3) $x = \dfrac{\log_2(45)}{19}$

(4) $x = \dfrac{5}{19}$

$$\log_2 2^{19x} = \log_2 10$$

$$\frac{19x}{19} = \frac{\log_2 10}{19}$$

4 The function $P(t) = 256{,}485(0.965)^t$ models the decreasing population of a city from 1999 to 2014, where t is the time in years since 1999. Which statement is *not* true?

(1) The function estimated the population was 256,485 in 1999.

(2) The decay rate was 0.35%.

(3) The decay factor is 0.965.

(4) The population declined over 15 years.

$$1 - .965 = .035 = 3.5\%$$

5 Four different surveys gathered data about the purchasing behaviors of pet owners. Pet owners from the same population were randomly selected. While collecting data, Chris surveyed 942 pet owners, John surveyed 410, Brooke surveyed 800, and Shane surveyed 100. Whose survey will likely have the *smallest* margin of error?

(1) Brooke   (3) John

(2) Chris   (4) Shane

6 Given i is the imaginary unit and $a = i^3$, $b = i^2$, and $c = i$, which expression is equivalent to $2ax^2 + 3bx - cx$?

(1) $-2ix^2 - 3x + ix$ (3) $-2ix^2 - 3x - ix$

(2) $-2ix^2 - 3ix$ (4) $-8ix^3 - 3x - ix$

$$2i^3x^2 + 3i^2x - ix$$
$$2(-i)x^2 + 3(-1)x - ix$$
$$-2ix^2 - 3x - ix$$

7 Which sequence has a common ratio of $\frac{1}{2}$?

(1) $-\frac{1}{4}a, -\frac{1}{8}a, -\frac{1}{16}a, -\frac{1}{32}a,...$ (3) $20a, \frac{39}{2}a, 19a, \frac{37}{2}a,...$

(2) $\frac{1}{32}a, \frac{1}{16}a, \frac{1}{8}a, \frac{1}{4}a,...$ (4) $22a, 22.5a, 23a, 23.5a,...$

$$\frac{-\frac{1}{8}\,\cancel{a}}{-\frac{1}{4}\,\cancel{a}} = \frac{-\frac{1}{16}\,\cancel{a}}{-\frac{1}{8}\,\cancel{a}} = \frac{1}{2}$$

8 The result of dividing $2x^3 + 6x^2 + 7x + 2$ by $x + 1$ is

(1) $2x^2 + 4x + 3 - \frac{1}{x+1}$ (3) $2x^2 + 8x - 15 + \frac{17}{x+1}$

(2) $2x^2 + 4x + 3 + \frac{5}{x+1}$ (4) $2x^2 + 8x + 15 - \frac{13}{x+1}$

$$-1 \,\big|\, 2 \quad 6 \quad 7 \quad 2$$
$$\qquad\quad -2 \;-4 \;-3$$
$$\overline{\quad 2 \quad 4 \quad 3 \;-1}$$

9 The probabilities that a randomly selected teenager uses social media websites F and I are shown below.

$$P(F) = 0.71$$
$$P(I) = 0.52$$
$$P(F \text{ or } I) = 0.77$$

Given this information, what is $P(F \text{ and } I)$, the probability that a randomly selected teenager uses both websites?

(1) 0.06   (3) 0.46

(2) 0.19   (4) 0.96

$$.71 + .52 - .77 = .46$$

10 Consider $f(x) = (x - 2)^2(x + 3)$, and $g(x)$ as strictly defined in the table below.

x	g(x)	
−3	0	*x - intercept*
−2	1	
−1	−2	
0	−6	*y - intercept*
1	−1	
2	0	*X - intercept*

Which statement or statements must be true, based on the information given?

   I. Both $f(x)$ and $g(x)$ have the same x-intercepts.

   II. Both $f(x)$ and $g(x)$ have a y-intercept at $y = -6$.

(1) I, only   (3) I and II

(2) II, only   (4) neither I nor II

f(x) x-intercept

(x-2)²(x+3) = 0

x = 2, -3

f(x) y-intercept

(0-2)²(0+3)

4 · 3

12

11 Josie examines the graphs of $f(x) = 3^x - 8$ and $g(x) = \dfrac{1}{x^2 - 4}$. The number of solutions to $f(x) = g(x)$ is

(1) 1   (3) 3

(2) 2   (4) 0

12 Which binomial is a factor of $g^3 + 6g^2 + g - 14$?

(1) $g - 1$ (3) $g + 1$

(2) $g - 2$ (4) $g + 2$

$$-2 \,\big|\, \begin{array}{cccc} 1 & 6 & 1 & -14 \\ & -2 & -8 & 14 \\ \hline 1 & 4 & -7 & 0 \end{array}$$

13 Consider the recursively defined sequence below.

$$a_1 = 8$$
$$a_n = 2a_{n-1}$$

Which explicit formula represents the same sequence?

(1) $a_n = 2^n$ (3) $a_n = 2^{n+2}$

(2) $a_n = 2(4^n)$ (4) $a_n = 8^n$

14 What is the exact value of $\tan\left(-\dfrac{5\pi}{6}\right)$?

(1) $\dfrac{1}{\sqrt{3}}$ (3) $\sqrt{3}$

(2) $-\dfrac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

15 Given $m \neq 0$ and $\left(17^{\frac{1}{m}}\right)^n = 17^2$, what is n in terms of m?

(1) $2m$ (3) $\dfrac{m}{2}$

(2) $\dfrac{2}{m}$ (4) 2^m

$$\frac{n}{m} = 2$$
$$n = 2m$$

16 In order to qualify for a college tennis scholarship, Joe needs to win 90% of the matches he plays during his senior year of high school. If he has won 8 of the 10 matches that he has played, which equation can be used to determine how many more consecutive matches, x, Joe must win in order for his winning percentage to equal 90%?

(1) $\dfrac{8+x}{x} = 0.90$ (3) $\dfrac{8}{10} + x = 0.90$

(2) $\dfrac{8}{10+x} = 0.90$ (4) $\dfrac{8+x}{10+x} = 0.90$

17 Consider the system of equations below.

$$3x + 2y = 1$$
$$2y + z = 2 \longrightarrow y = \frac{2-z}{2}$$
$$2x - 2z = -6$$

What is the value of x? $x - z = -3 \longrightarrow z = x + 3$

(1) 1   (3) -4

(2) -1 (4) 4

$$3x + 2\left(\frac{2-z}{2}\right) = 1$$
$$3x + 2 - z = 1$$
$$3x - z = -1$$
$$3x - (x+3) = -1$$
$$2x = 2$$

18 The point $(2,-3)$ lies on the graph of the equation $y = f(x)$. Which point must lie on the graph of the equation $y = f(x - 4) + 1$?

(1) $(1,1)$ (3) $(3,-7)$

(2) $(-2,-2)$ (4) $(6,-2)$

$$(2+4, -3+1)$$

19 Which statement best describes the end behavior of the function $y = \log(x - 3)$?

(1) As $x \to -\infty, y \to -\infty$, and as $x \to \infty, y \to \infty$.

(2) As $x \to 3, y \to -\infty$, and as $x \to \infty, y \to \infty$.

(3) As $x \to -\infty, y \to 0$, and as $x \to \infty, y \to \infty$.

(4) As $x \to 3, y \to 0$, and as $x \to \infty, y \to \infty$.

20 The black bear population for a certain area of the Adirondacks can be modeled by the equation $B = 5835.943(1.026)^t$, where t is measured in years since 2010. Kieran would like to rewrite this model in terms of a 5-year growth rate. Kieran's model is best represented by

(1) $B = 5835.943(1.005147)^{\frac{t}{5}}$

(2) $B = 5835.943(1.005147)^{5t}$

(3) $B = 5835.943(1.136938)^{\frac{t}{5}}$

(4) $B = 5835.943(1.136938)^{5t}$

$$\left((1.026)^5\right)^{\frac{t}{5}} \approx 1.136938^{\frac{t}{5}}$$

21 Which expression or expressions are equal to 0 for all real numbers?

I. $(x^2 + y^2)^2 + (x^2 + y^2)^2 - 2(x^2 + y^2)^2$ $\rightarrow$ $(x^2+y^2)^2(1+1-2)$

II. $(x^2 + y^2)^2 - (x^2 - y^2)^2$ $= 4x^2y^2$

III. $(x^2 + y^2)^2 - (x^2 - y^2)^2 - (2xy)^2$

(1) I, only

(2) III, only

(3) I and II, only

(4) I and III, only

$x^4 + 2x^2y^2 + x^4 - (x^4 - 2x^2y^2 + y^4) - 4x^2y^2$

$4x^2y^2 - 4x^2y^2$

22 The equation $\dfrac{1}{x} - \dfrac{1}{5} = \dfrac{x}{5}$ has

(1) rational solutions

(2) irrational solutions

(3) imaginary solutions

(4) no solutions

$$\frac{1}{x} = \frac{x+1}{5}$$

$$x^2 + x = 5$$

$$x^2 + x - 5 = 0$$

$$b^2 - 4ac$$

$$1^2 - 4(1)(-5)$$

$$1 + 20$$

$$21 \rightarrow \text{not a perfect square}$$

23 For $x \neq \pm 4y$, the expression $\dfrac{x^2 + 3xy - 28y^2}{16y^2 - x^2}$ is equivalent to

(1) $-1 - \dfrac{7}{4}y$

(2) $\dfrac{x - 7y}{4y - x}$

(3) $\dfrac{x + 7y}{x + 4y}$

(4) $\dfrac{-x - 7y}{x + 4y}$

Handwritten work:
$$\dfrac{-(x + 7y)(x - 4y)}{(4y + x)(4y - x)}$$

24 Which equation represents a parabola with a focus of $(-2,1)$ and directrix of $y = 5$?

(1) $(x + 2)^2 = -8(y - 3)$

(2) $(x + 2)^2 = 5(y - 1)$

(3) $(x + 2)^2 = -8(y - 1)$

(4) $(x + 2)^2 = 8(y - 3)$

Handwritten work:
$$P = \dfrac{5-1}{2} = 2$$

$$\text{vertex is } (-2, 3)$$

The directrix is above the focus, so the parabola opens downward

$$y = -\dfrac{1}{4(2)}(x+2)^2 + 3$$

$$x - 3 = -\dfrac{1}{8}(x+2)^2$$

$$-8(y-3) = (x+2)^2$$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Data collected showing the depth of the water in a bay during a 24-hour period are shown in the graph below.
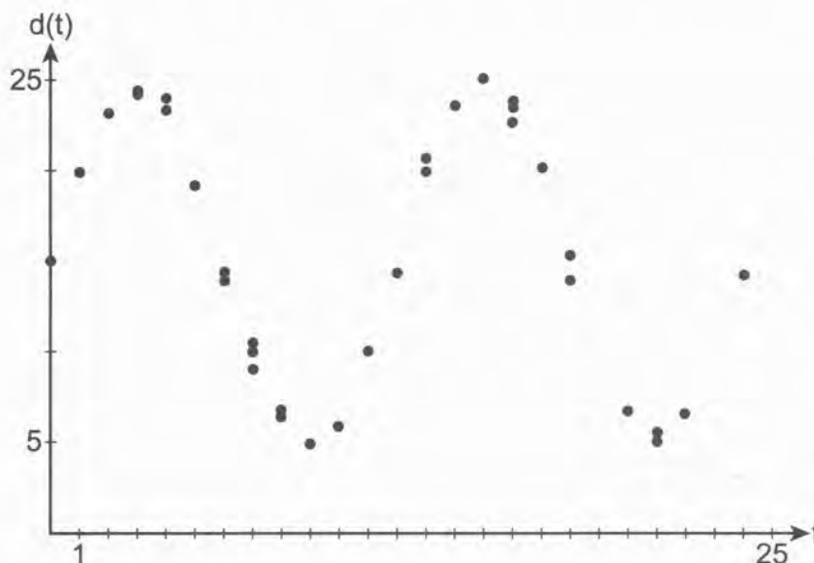
The depth of the water can be modeled with a trigonometric function of the form $d(t) = A \sin\left(\frac{\pi}{6}t\right) + C$. Estimate the value of A, to the *nearest integer*. Justify your answer.

$$\frac{25 - 5}{2} = 10$$

26 Algebraically determine the solution to the equation below.

$$\sqrt{x - 2} + x = 4$$

$$\sqrt{x-2} = 4-x$$

$$x-2 = 16-8x+x^2$$

$$0 = x^2-9x+18$$

$$0 = (x-6)(x-3)$$

$$x = \cancel{6}, 3$$

27 Factor the expression completely.

$$(x - 1)^2 + 5(x - 1) - 6$$

$$U = x-1$$

$$u^2+5u-6$$

$$(u+6)(u-1)$$

$$(x-1+6)(x-1-1)$$

$$(x+5)(x-2)$$

**28** The results of a survey of the students at the local high school regarding the topic "What I Do to Relax" are displayed in the table below.

	Read	Listen to Music	Exercise	Total
Female	87	94	21	202
Male	68	110	18	

If a student from this survey is selected at random, determine the exact probability that the person claims to relax by listening to music given that the person is female.

$$\frac{94}{202}$$

29 The graph of $y = f(x)$ is shown below. The cubic function has a leading coefficient of 1.
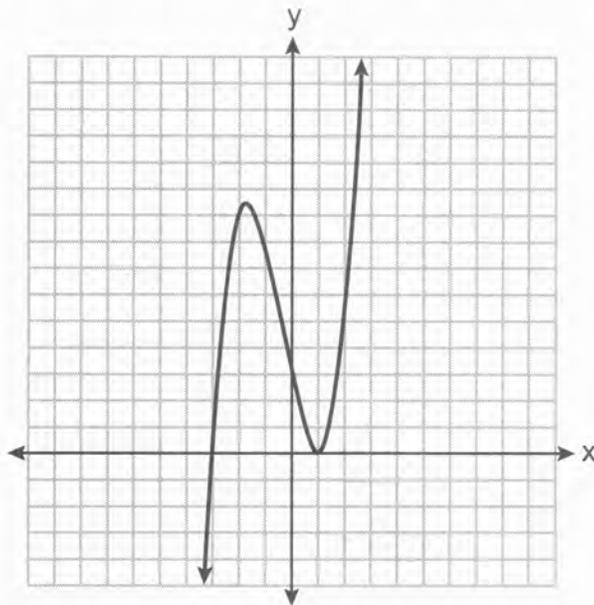
Write an equation for $f(x)$.

$$F(x) = (x + 3)(x - 1)^2$$

30 Given $f(x) = \frac{2}{3}x + 6$, write the equation of $f^{-1}(x)$.

$$x = \frac{2}{3}x + 6$$
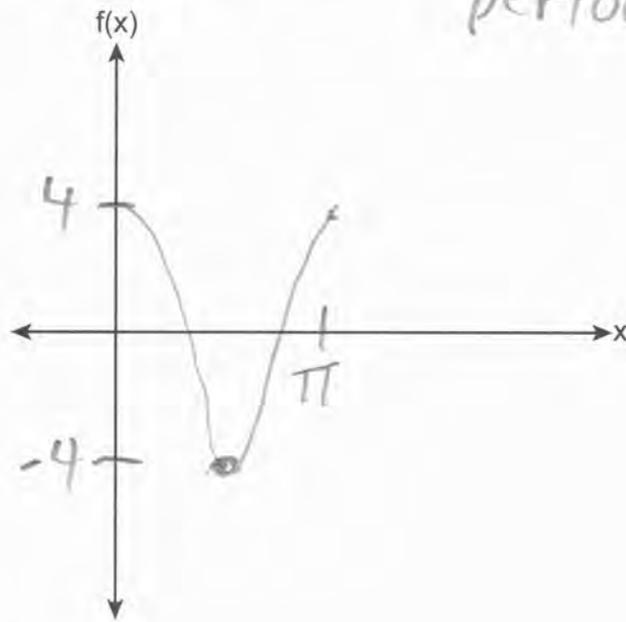
$$x = \frac{2}{3}y + 6$$

$$3x = 2y + 18$$

$$3x - 18 = 2y$$

$$\frac{3}{2}x - 9 = y$$

$$\frac{3}{2}x - 9 = f^{-1}(x)$$

31 On the coordinate plane below, sketch *at least one cycle* of the function $f(x) = 4\cos(2x)$. Label the axes with an appropriate scale.

period $= \dfrac{2\pi}{2} = \pi$

32 In a recent online contest with a large number of randomly selected human players, the computer player won 67% of the time. The game-design company claims that the computer player can beat human players 70% of the time. The company runs a simulation of a large number of games, with the same number of human players, assuming that the computer wins 70% of the time. The simulation is approximately normal with a mean of 0.705 and a standard deviation of 0.045.

Does the contest result provide evidence to contradict the designer's claim? Use the simulation results to justify your answer.

$$.705 \mp 2(.045)$$
$$.615 - .795$$

No, because .67 falls within the confidence interval

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve algebraically for x: $\dfrac{2}{x} = \dfrac{2x+3}{x-4}$. Express your answers in simplest $a + bi$ form.

$$2x^2 + 3x = 2x - 8$$

$$2x^2 + x + 8 = 0$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(8)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{63}}{4}$$

$$= \frac{-1 \pm 3i\sqrt{7}}{4}$$

$$= -\frac{1}{4} \pm \frac{3i\sqrt{7}}{4}$$

34 A highly selective college reports that the mean score earned by accepted students on the Mathematics Level 2 subject test is 750 with a standard deviation of 20 and that the scores are approximately normally distributed.

Given this information, determine the interval representing the middle 95% of student scores.

$$750 \mp 2(20)$$

$$710 - 790$$

To the *nearest whole percent*, determine the percentage of accepted students who scored a 760 or less.

$$50\% + 19.1\% \approx 69\%$$

35 For $c(x) = 3x^2 - 4x + 7$ and $d(x) = x - 2$, determine $c(x) \cdot d(x) - [d(x)]^3$ as a polynomial in standard form.

$$(3x^2 - 4x + 7)(x - 2) - (x - 2)^3$$

$$3x^3 - 6x^2 - 4x^2 + 8x + 7x - 14 - (x^3 - 6x^2 + 12x - 8)$$

$$3x^3 - 10x^2 + 15x - 14 - x^3 + 6x^2 - 12x + 8$$

$$2x^3 - 4x^2 + 3x - 6$$

36 Christopher works for a defense contractor and earned $85,000 his first year. For each additional year he will receive a 2.5% raise.

Write a geometric series formula, C_n, for Christopher's total earnings over n years.

$$C_n = \frac{85000 - 85000(1.025)^n}{1 - 1.025}$$

Use this formula to find Christopher's total earnings, to the *nearest hundred dollars*, over his first 10 years of employment.

$$\frac{85000 - 85000(1.025)^{10}}{1 - 1.025} \approx 952,300$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Cesium-137 decay can be modeled with the formula $A(t) = A_0 e^{kt}$, where $A(t)$ represents the mass remaining in grams after t years and A_0 represents the initial mass. A sample of 500 grams of cesium-137 takes approximately 60.34 years to decay to 125 grams. Use this sample with the given formula to determine the constant k, to the *nearest thousandth*.
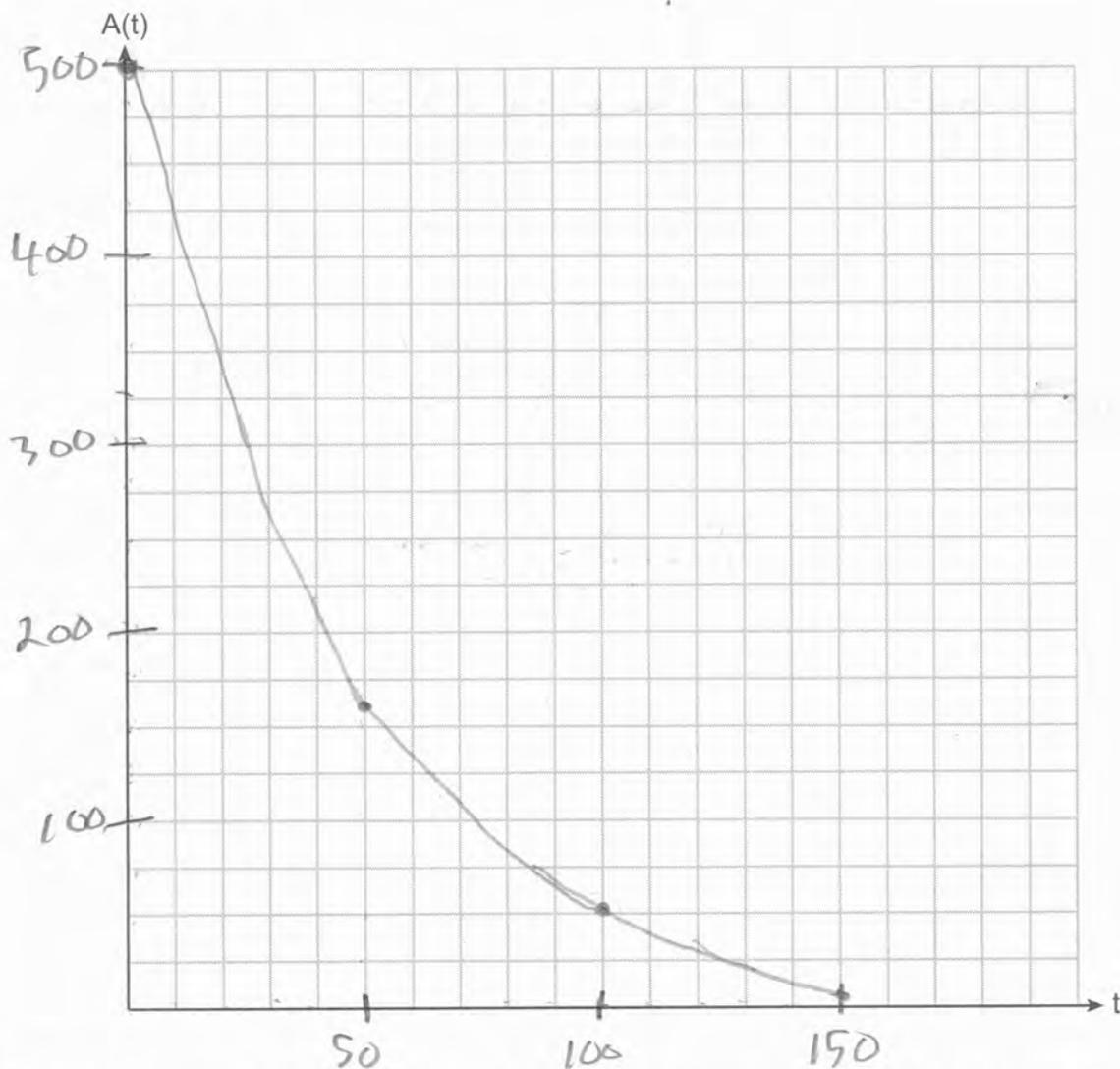
$$125 = 500 e^{k(60.34)}$$

$$k \approx -.023$$

Use this value for k to write a function, $A(t)$, that will find the mass of the 500-gram sample remaining after any amount of time, t, in years.

$$A(t) = 500 e^{-.023t}$$

Graph $A(t)$ on the graph below from $t = 0$ to $t = 150$ years.

Use $A(t)$ to calculate the average rate of change in grams per year, from $t = 0$ to $t = 60$ years, to the *nearest tenth*.

$$\frac{A(60) - A(0)}{60 - 0} \approx -6.2$$

Explain what this value means in the given context.

The mass of the element decreases about 6.2g per year.