ALGEBRA II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Tuesday, August 19, 2025 — 12:30 to 3:30 p.m., only

Student Name _____Mr. Sibol_____

School Name _____JMAP_____

> **The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

> **Notice ...**
>
> **A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 What is the seventh term of the sequence $-2, 6, -18, 54,...$?

(1) -1458　　　　　(3) 1458

(2) -4374　　　　　(4) 4374

$$a_7 = -2(-3)^{7-1} = -1458$$

2 Given $x \neq 0$, where $m(x) = 12x^{8a}$ and $p(x) = 3x^{2a}$, the expression $\frac{m(x)}{p(x)}$ is equivalent to

(1) $9x^{4a}$　　　　　(3) $4x^6$

(2) $4x^{6a}$　　　　　(4) $4x^4$

$$\frac{12x^{8a}}{3x^{2a}}$$

3 What is the inverse of $f(x) = 2x + 6$?

(1) $f^{-1}(x) = -2(x+3)$　　　　(3) $f^{-1}(x) = \frac{x}{2} - 3$

(2) $f^{-1}(x) = x - 3$　　　　(4) $f^{-1}(x) = \frac{x}{2} + 3$

$$x = 2y + 6$$
$$\frac{x-6}{2} = \frac{2y}{2}$$

4 The expression $\sqrt[3]{16x^6}$ is equivalent to

(1) $4x^3$ (3) $2x^2\sqrt[3]{2}$

(2) $4x^2$ (4) $2x^3\sqrt[3]{2}$

$$\sqrt[3]{8}\sqrt[3]{x^6}\sqrt[3]{2}$$

$$2x^2\sqrt[3]{2}$$

5 Mary would like to determine if there is an association between a student's height and their shoe size. She measures the height and shoe size of every 10ᵗʰ person entering her school. This is an example of

(1) a census   (3) a simulation

(2) an observational study (4) a controlled experiment

6 For all values for which the expressions are defined, which expression can *not* be rewritten as $(x - 6)(x + 2)$?

(1) $\dfrac{(x + 2)(x^2 - 2x - 24)}{(x + 4)}$ (3) $\dfrac{(x - 2)(x^2 - 4x - 12)}{(x - 6)}$

(2) $x(x + 2) - 6(x + 2)$ (4) $(x + 4)(x - 2) - 2(3x + 2)$

$$\frac{(x-2)(x-6)(x+2)}{x-6}$$

$$\frac{(x+2)(x-6)(x+4)}{x+4}$$

$$x^2 + 2x - 8 - 6x - 4$$
$$x^2 - 4x - 12$$
$$(x-6)(x+2)$$

7 The number of hours in the lifespan of a certain brand of light bulb is normally distributed with a mean of 2387 hours and a standard deviation of 238 hours. To the *nearest tenth of a percent*, what percent of light bulbs have a lifespan of greater than 2750 hours?

(1) 6.4%

(3) 43.6%

(2) 15.9%

(4) 93.6%

8 The solution set to the equation $\dfrac{2}{x^3} + \dfrac{1}{x} = \dfrac{6}{x^3}$ is

(1) $\{-2, 0, 2\}$

(3) $\{-2, 2\}$

(2) $\{2\}$

(4) $\{0, 2\}$

$$\frac{1}{x} = \frac{4}{x^3}$$

$$x^3 = 4x$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x(x+2)(x-2) = 0$$

$$0, -2, 2$$

9 What is the solution to $9(e^{x-2}) = 36$?

(1) $x = \dfrac{\ln(36)}{\ln(9e)} + 2$

(3) $x = \ln(4) + 2$

(2) $x = \ln(4) - 2$

(4) $x = \ln\left(\dfrac{4}{e}\right) + 2$

$$\ln e^{x-2} = \ln 4$$

$$(x-2)\ln e = \ln 4$$

$$x = \ln 4 + 2$$

10 Reynaldo got a score of 40 on his first test. If he gets a score of 100 on every additional test, which equation can be used to determine the number of additional tests, x, he would need to take in order to raise his test average to an 80?

(1) $\dfrac{40 + 100x}{x + 1} = 80$

(3) $\dfrac{40 + 100 + x}{x} = 80$

(2) $\dfrac{40 + 100x}{x} = 80$

(4) $\dfrac{40 + 100 + x}{x + 1} = 80$

11 Given $f(x) = \ln(x + 5)$, what is the *smallest* integer value of x for which $f(x)$ is defined?

(1) -5

(3) -1

(2) -4

(4) 0

12 Which expression is equivalent to $\dfrac{6x^3 + 7x^2 - 9x - 1}{2x - 1}$ when $x \neq \dfrac{1}{2}$?

(1) $3x^2 - 2x - 4$

(3) $3x^2 + 2x + 5 - \dfrac{6}{2x - 1}$

(2) $3x^2 + 5x - 7 - \dfrac{8}{2x - 1}$

(4) $3x^2 + 5x - 2 - \dfrac{3}{2x - 1}$

$$3x^2 + 5x - 2$$
$$2x-1 \overline{) 6x^3 + 7x^2 - 9x - 1}$$
$$\underline{6x^3 - 3x^2}$$
$$10x^2 - 9x$$
$$\underline{10x^2 - 5x}$$
$$-4x - 1$$
$$\underline{-4x + 2}$$
$$-3$$

13 A sketch for $p(x)$ is shown below, where $a > 0$ and $b > 0$.
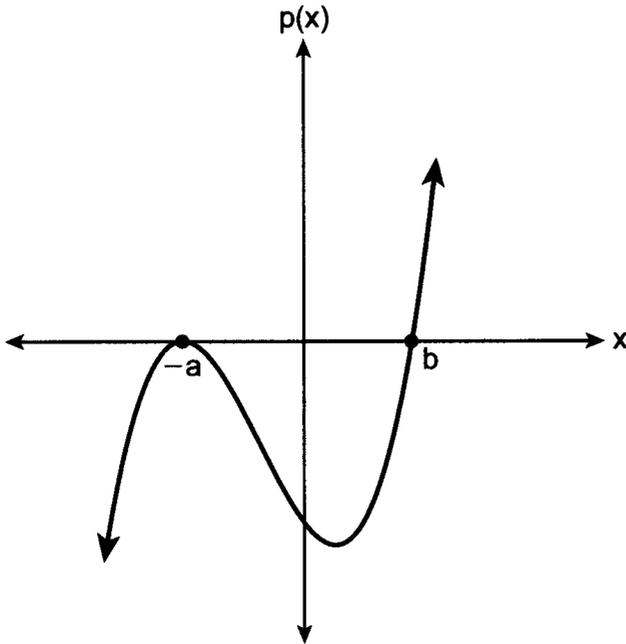
An equation for $p(x)$ could be

(1) $p(x) = (x + a)(x - b)$ (3) $p(x) = (x - a)(x + b)$

(2) $p(x) = (x + a)^2(x - b)$ (4) $p(x) = (x - a)^2(x + b)$

14 If $f(x) = \frac{1}{2}x^3 + 3x^2 - 4x$ and $g(x) = 5\log_3(x + 10)$, then which value, rounded to the *nearest tenth*, is *not* a solution to $f(x) = g(x)$?

(1) -6.9 (3) 2.2
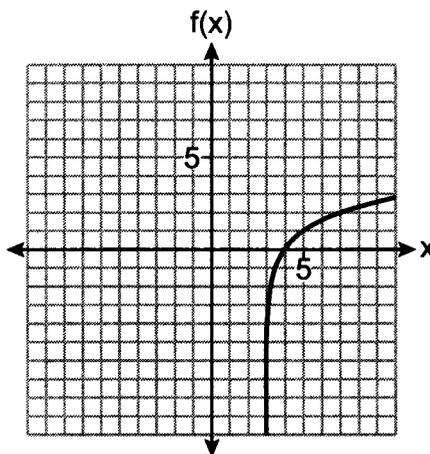
(2) -1.4 (4) 9.8

15 The graph of $f(x)$ is shown below.
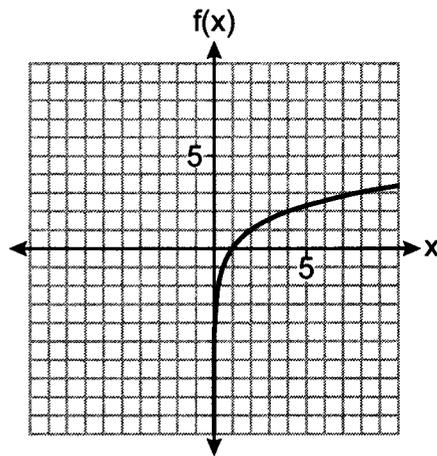
Which graph represents $f(x + 3)$?

(1)

(3)

(2)

(4)
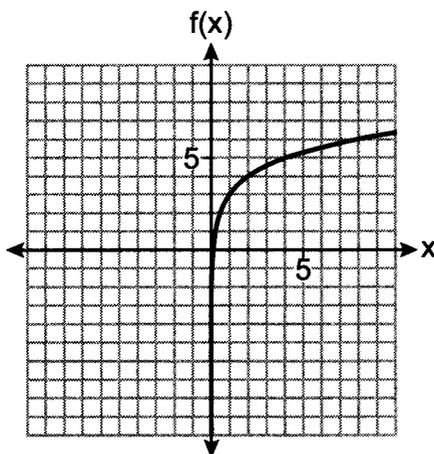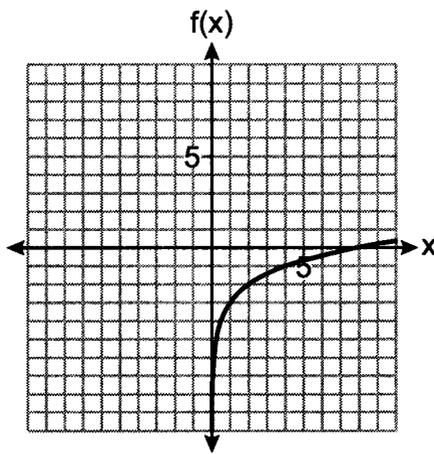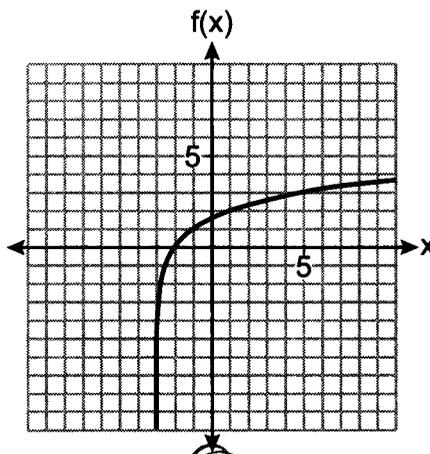
16 What is one solution to the system of equations shown below?

$$x^2 + y^2 = 20$$
$$y = x - 6$$

(1) $x = 2$ (3) $y = -4$

(2) $(4, -2)$ (4) $(4, 2)$

$$X^2 + (x-6)^2 = 20$$
$$X^2 + X^2 - 12X + 36 = 20$$
$$2X^2 - 12X + 16 = 0$$
$$X^2 - 6X + 8 = 0$$
$$(X - 4)(X - 2) = 0$$
$$4, 2$$

$$y = 4 - 6 = -2$$

17 At a high school, 10th-grade students were recently asked if they walk to school and if they eat breakfast. The survey results are summarized in the table below.

	Walks to School	**Doesn't Walk to School**
Eats Breakfast	7	53
Doesn't Eat Breakfast	10	30

What is the probability that a randomly selected 10th-grade student from the school walks to school or eats breakfast?

(1) 0.07   (3) 0.77

(2) 0.70   (4) 0.84

$$\frac{17 + 60 - 7}{100} = \frac{70}{100}$$

18 A vehicle's depreciation rate is 9.2% per year. If a vehicle costs $34,950, then which recursive formula models the value of the vehicle n years after it was purchased?

(1) $a_n = 34{,}950(1.092)^n$

(2) $a_n = 34{,}950(0.921)^n$

(3) $a_0 = 34{,}950$

$a_n = 1.092a_{n-1}$

(4) $a_0 = 34{,}950$

$a_n = 0.908a_{n-1}$

19 When factored completely, $(3x - 1)^2 - 5(3x - 1) + 6$ is equivalent to

(1) $(3x - 3)(3x - 4)$

(2) $3x(3x - 7)$

(3) $3(x - 1)(3x - 4)$

(4) $(3x + 1)(3x - 2)$

$U = 3x - 1$

$U^2 - 5U + 6$

$(U - 3)(U - 2)$

$(3x - 1 - 3)(3x - 1 - 2)$

$(3x - 4)(3x - 3)$

$3(3x - 4)(x - 1)$

20 Given $E(t) = 26(2)^{\frac{t}{20}}$ represents the mass, in grams, of a substance after t minutes in a laboratory, which statement or statements must be true?

I. The initial mass of the substance is 26 grams.

II. The mass of the substance doubles every 20 days. minutes

III. The mass of the substance after 3 hours is approximately 29 grams.

$26 \cdot 2^{\frac{180}{20}} = 13{,}312$

(1) I, only

(2) III, only

(3) I and II, only

(4) I and III, only

21 For $x > 0$, which expression is equivalent to $\sqrt[3]{9x^2} \cdot \sqrt{9x}$?

(1) $9^5 x^{\frac{7}{2}}$

(2) $9^6 x^3$

(3) $9^{\frac{1}{6}} x^{\frac{1}{3}}$

(4) $9^{\frac{5}{6}} x^{\frac{7}{6}}$

$9^{\frac{1}{3}} \cdot x^{\frac{2}{3}} \cdot 9^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

$9^{\frac{5}{6}} x^{\frac{7}{6}}$

22 The number of people who have read an article grows exponentially throughout the day and can be modeled by the function $N(t) = 2(1.0098)^t$, where t represents the number of minutes since the article has been posted.

Which equation best represents the number of people who have read the article in terms of the growth rate per second?

(1) $N(t) = 2(1.000163)^{\frac{t}{60}}$

(2) $N(t) = 2(1.000163)^{60t}$

(3) $N(t) = 2(1.79524)^{\frac{t}{60}}$

(4) $N(t) = 2(1.79524)^{60t}$

$\left(1.0098^{\frac{1}{60}}\right)^{60t}$

$P = \frac{3 - -5}{2} = 4$

23 Which equation represents a parabola with focus $(2,-5)$ and directrix $y = 3$?

(1) $(x - 2)^2 = -16(y + 1)$
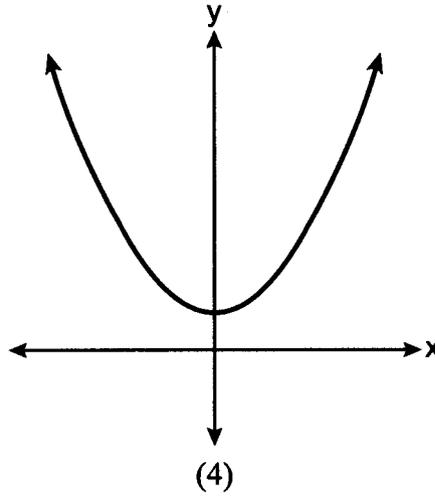
(2) $(x - 2)^2 = -16(y - 1)$
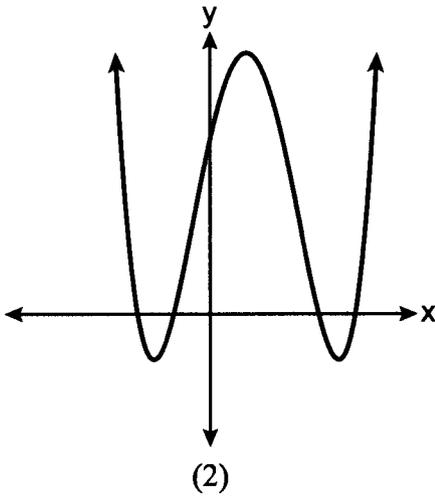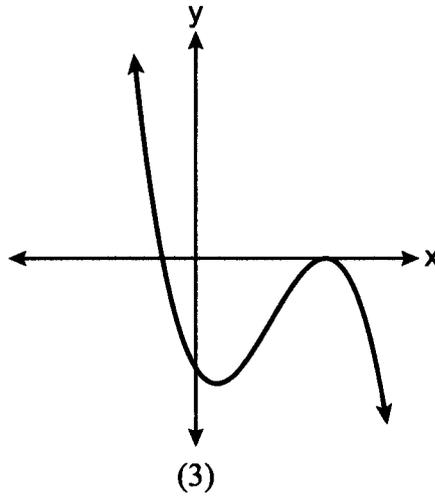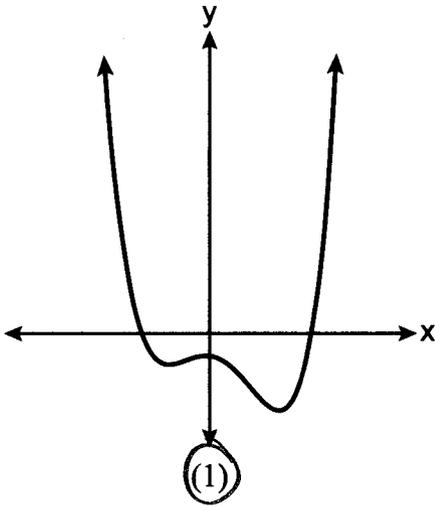
(3) $(x + 2)^2 = -16(y - 1)$

(4) $(x - 2)^2 = 16(y + 1)$

vertex $(2, -1)$

$y = \frac{-1}{4(4)}(x - 2)^2 - 1$

$y + 1 = \frac{-1}{16}(x - 2)^2$

$-16(y + 1) = (x - 2)^2$

24 Which graph shows a fourth-degree polynomial function with exactly two imaginary roots?

(1)

(2)

(3)

(4)

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Seniors at a high school were surveyed to see if they preferred a hoodie or a jacket for Spirit Day and if they wanted a design on the back or the front. The survey results are summarized in the table below.
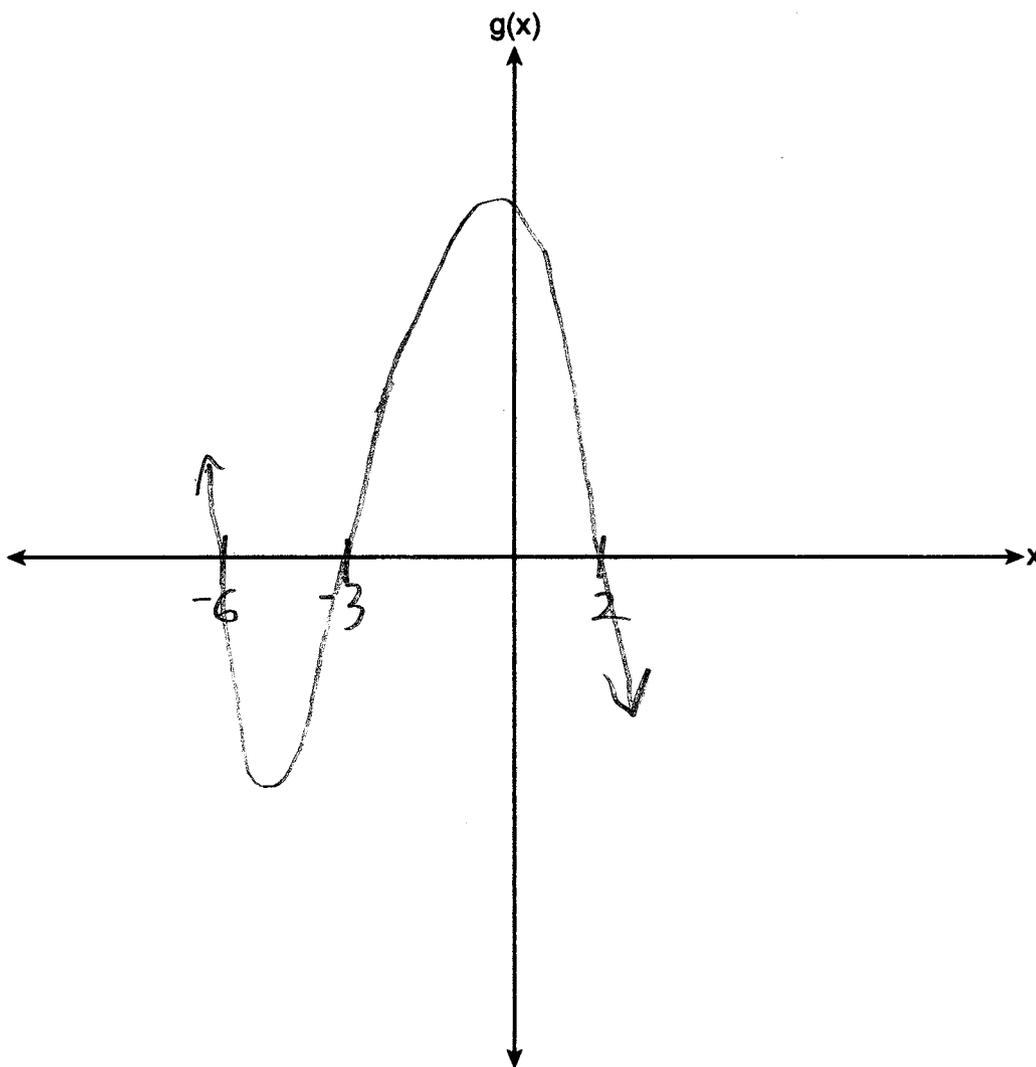
	Hoodie	Jacket
Back	45	15
Front	27	13

60

Determine the exact probability that a randomly selected senior from the survey preferred a hoodie, given that the senior wanted a design on the back.

$$\frac{45}{60}$$

26 Sketch $g(x) = -x^3 - 7x^2 + 36$ on the axes below, including appropriate end behavior and zeros.

27 Express $8xi^{10} - 4yi^{19} + 2yi^3 - 6xi$ in simplest form, where i is the imaginary unit.

$$8x(-1) - 4y(-i) + 2y(-i) - 6xi$$

$$-8x + 4yi - 2yi - 6xi$$

$$-8x + 2yi - 6xi$$

28 The job satisfaction rating at a company is approximately normally distributed with a mean of 12. About 95% of the scores are between 8 and 16. What is the standard deviation of this distribution? Justify your answer.

$$\frac{16-8}{4} = 2$$

29 An angle, θ, is drawn in standard position and terminates in Quadrant III. Given $\cos \theta = -\dfrac{\sqrt{10}}{10}$, determine the value of $\tan \theta$.

$$\sin^2 \theta + \left(-\frac{\sqrt{10}}{10}\right)^2 = 1$$

$$\sin^2 \theta + \frac{10}{100} = \frac{100}{100}$$

$$\sin^2 \theta = \frac{90}{100}$$

$$\sin \theta = \frac{-3}{\sqrt{10}} \frac{\sqrt{10}}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-3\sqrt{10}/10}{-\sqrt{10}/10} = 3$$

30 Solve algebraically for all values of x.

$$\sqrt{x+5} - x = 3$$

$$\sqrt{x+5} = x+3$$

$$x+5 = x^2 + 6x + 9$$

$$0 = x^2 + 5x + 4$$

$$0 = (x+4)(x+1)$$

$$-\cancel{4}, -1$$

31 Use the geometric series formula to determine the total 30-year earnings for an employee whose first-year salary is $42,000 and earns an annual raise of 3%, rounded to the *nearest thousand dollars*.

$$S_{30} = \frac{42000 - 42000(1.03)^{30}}{1 - 1.03}$$

$$\approx 1,998,000$$

32 Algebraically determine the solution(s) to the equation $2x^2 = 2x - 1$, in simplest $a + bi$ form.

$$2x^2 - 2x + 1 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{2 \pm \sqrt{4-8}}{4}$$

$$x = \frac{2 \pm 2i}{4}$$

$$x = \frac{1}{2} \pm \frac{i}{2}$$

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 The gross domestic product (GDP) per capita measures worldwide economic output per person. The GDP per capita, y, in dollars, x years after 1990 is listed in the table below.

x	y
1	9680
6	10,201
18	13,713
25	15,552
29	16,976

(a) Based on these data, write an exponential regression equation to model the GDP per capita, in dollars, x years after 1990. Round all coefficients to the *nearest hundredth*.

$$y = 9290.57(1.02)^x$$

(b) Use the rounded equation from part *a* to algebraically determine, to the *nearest tenth of a year*, the number of years after 1990 when GDP per capita was $15,000.

$$15000 = 9290.57(1.02)^x$$

$$\frac{15000}{9290.57} = 1.02^x$$

$$24.2 \approx x$$

34 Consider the function $f(x)$ below. Is $(x + 3)$ a factor of $f(x)$? Justify your answer.

$$f(x) = x^3 + 3x^2 - 2x - 6$$

$$x^2(x+3) - 2(x+3)$$

$$(x^2 - 2)(x+3)$$

yes

Determine all zeros of $f(x)$.

$$(x^2 - 2)(x+3) = 0$$

$$\pm\sqrt{2}, \, -3$$

35 Solve the system algebraically:

$$2a + b - c = -4$$
$$4a + b + c = 3$$
$$-2a - 3b + 2c = 11$$

$$4a + 2b - 2c = -8$$
$$4a + b + c = 3$$
$$-4a - 6b + 4c = 22$$

$$-4b + 2c = 14 \qquad -5b + 5c = 25$$
$$-2b + c = 7 \qquad -6 + c = 5$$
$$c = 2b + 7 \qquad c = b + 5$$

$$2b + 7 = b + 5$$
$$b = -2$$

$$c = -2 + 5 = 3$$

$$2a - 2 - 3 = -4$$
$$2a - 5 = -4$$
$$2a = 1$$
$$a = \frac{1}{2}$$

36 Given: $f(x) = 5x^2 + 3x - 12$ and $g(x) = 2x - 1$.

Express $4g(x) - [f(x + 1)]$ as a polynomial in standard form.

$$4(2x-1) - (5(x+1)^2 + 3(x+1) - 12)$$

$$8x - 4 - (5(x^2 + 2x + 1) + 3x + 3 - 12)$$

$$8x - 4 - (5x^2 + 10x + 5 + 3x - 9)$$

$$8x - 4 - (5x^2 + 13x - 4)$$

$$-5x^2 - 5x$$

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The height, in feet, of the tides along the coastlines can be measured with water levels oscillating between low tide and high tide. The graph below shows the height of the tides, $y = B(t)$, in feet, in Daytona Beach, t hours after midnight on a day in July.

Height in Feet vs **Hours after Midnight**
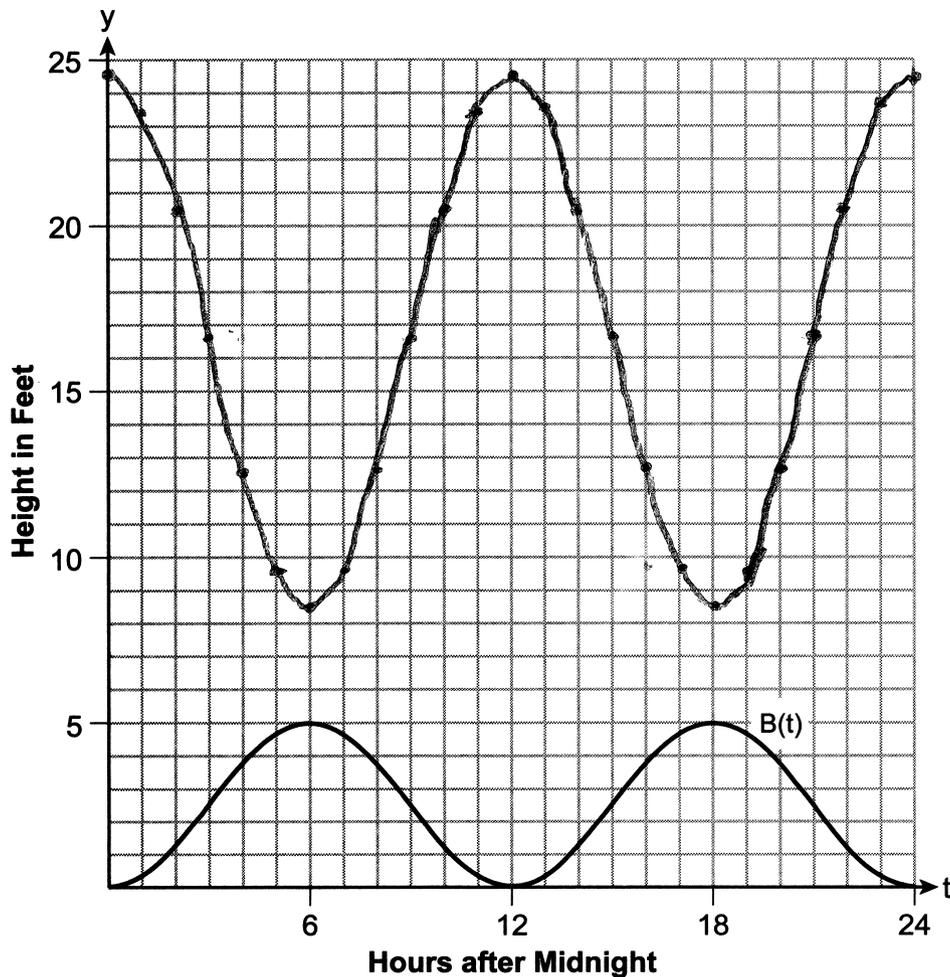
State the period of $B(t)$, in hours.

12

Write an equation for $B(t)$ in the form $B(t) = a\cos(bt) + c$.

$$B(t) = -2.5 \cos \frac{\pi}{6}t + 2.5$$

In Derby, Australia, the height of the tide, in feet, can be modeled by the function $D(t) = 8\cos\left(\dfrac{\pi}{6}t\right) + 16.5$. On the grid provided on the previous page, graph $y = D(t)$ on the domain $0 \le t \le 24$.

State the height, in feet, of the low tide in Derby.

8.5