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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, January 25, 1971 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

Directions (1–12): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

1 The algebraic expression $(2a + 1)^2 - 2(2a^2 - 1)$ is identical to
(1) $4a + 3$
(2) $2a + 3$
(3) 3
(4) -1

2 The complete set of values of x satisfying the inequality $3 < (x + 1) < 10$ consists of all x such that
(1) $x > 2$
(2) $x > 9$
(3) $x < 2$ or $x > 9$
(4) $2 < x < 9$

3 If $2 \cos B = \frac{a^2 + c^2 - b^2}{ac}$, where $a \neq 0$, $b \neq 0$, and $c \neq 0$, then b^2 equals
(1) $a + c - 2 \cos B$
(2) $2 \cos B - a - c$
(3) $a^2 + c^2 - 2ac \cos B$
(4) $2ac \cos B - a^2 - c^2$

4 If $n \neq 0$, then $n^{-\frac{1}{2}}$ is equal to
(1) $\sqrt{n}$
(2) 2
(3) $\frac{1}{n^{-2}}$
(4) $\frac{1}{\sqrt{n}}$

5 If $\cos x$ is twice the cosine of 30°, then x
(1) measures 30°
(2) measures 60°
(3) measures 90°
(4) does not exist

6 In triangle XYZ, if $y = 12$, $z = 18$, and angle $X = 30°$, the area is
(1) 27
(2) 54
(3) 108
(4) 216

7 For all values of A for which each expression is defined, which is *not* an example of an identity?
(1) $\sin A + \cos A = 1$
(2) $1 + \cot^2 A = \csc^2 A$
(3) $\cos A \cdot \tan A = \sin A$
(4) $\sec A \cdot \cos A = 1$

8 As x varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ radians, the value of $\cos x$
(1) increases and then decreases
(2) decreases and then increases
(3) decreases, only
(4) increases, only

9 In a circle whose radius is t, a central angle of x radians intercepts a minor arc whose length is y. Which is true?
(1) $x = ty$
(2) $y = xt$
(3) $t = xy$
(4) $x = \frac{t}{y}$

10 The value of $\sin \frac{\pi}{2} + \cos \pi$ is
(1) 1
(2) 2
(3) 0
(4) -1

11 A line $y = kx + 2$ is drawn perpendicular to the line $x + y = -7$. The value of k is
(1) 1
(2) -1
(3) $\frac{1}{2}$
(4) $-\frac{1}{2}$

12 Given that y varies inversely as x. The graph of this relation is
(1) a straight line
(2) an ellipse
(3) a hyperbola
(4) a parabola

13 When 8 is added to 4 times a certain number, the square root of the result is 6. Find the number.

14 For which real value(s) of x is $\frac{x^2 - 49}{2x^2 - 3x}$ undefined?

15 Write a quadratic equation in x whose roots are $-2 + \sqrt{5}$ and $-2 - \sqrt{5}$.

16 Express the product of $3i$ and $5 - 2i$ in the form $a + bi$.

17 Find the value of $\tan\left(\text{Arc sin } \frac{\sqrt{2}}{2}\right)$.

18 If $f(x) = 2x^2 - 5x + 3$, find the value of $f(\frac{1}{2})$.

19 In $\triangle ABC$, if $a = 12$, $\sin A = 0.4$, and $\sin B = 0.7$, find the value of b.

20 Factor: $2 \sin^2 \theta + 7 \sin \theta - 15$

21 If the graphs of $x^2 + y^2 = 16$ and $y = 3$ are plotted on the same axes, what is the total number of points which the two graphs have in common?

22 If 41 is divided by a number x, the quotient is 3 and the remainder is 2. Find the numerical value of x.

23 If $\cos x = \frac{1}{2}$ and x is a positive acute angle, find the value of $\cos^2 \frac{1}{2}x$.

24 Solve for x: $|2x - 3| = 11$

25 Express as a single fraction in *simplest form:*

$$\frac{1}{a^2 - 1} - \frac{1}{a - 1}$$

26 Solve the equation $3 \tan^2 x - 1 = 0$ for the smallest positive value of x.

27 Find to the *nearest* minute the positive acute angle A for which $\log \tan A = 10.5828 - 10$.

28 Solve for the real value of x: $\log_2 8 = x$

29 What are the x-intercepts of the graph of the parabola $y = x^2 - 5x + 6$?

30 Express $\cos(-170°)$ as a function of a positive acute angle.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find the values of $\sin \theta$ in the solution set of $\{\sin \theta \mid 8 \sin^2 \theta + 2 \sin \theta - 3 = 0\}$. [6]

b Find all the values of θ in the interval $0° < \theta < 360°$ which satisfy $8 \sin^2 \theta + 2 \sin \theta - 3 = 0$. [Express approximate values of θ to the *nearest degree*.] [4]

32 *a* Starting with a formula for $\cos 2x$, *derive* the formula for $\cos \frac{A}{2}$ in terms of $\cos A$. [Assume A is an angle in the first quadrant.] [5]

b For all values of x for which the expression is defined, show that the following is an identity: [5]

$$\frac{\cos 2x + 1}{\sec^2 x - \tan^2 x} = 2 \cos^2 x$$

33 Given the function $\{(x,y) \mid y = x^2 - 6\}$ whose domain is $\{x \mid -4 \leq x \leq 4\}$.

a Draw the graph of this function. [6]

b What is the range? [2]

c From the graph, estimate to the *nearest tenth* the solution set of $x^2 - 6 = 0$. [2]

34 Write an equation or a system of equations which can be used to solve *each* of the following problems. In each case state what the variable or variables represent. [*Solution of the equations is not required.*]

a A man in a motorboat finds it takes him two hours longer to travel 48 miles upstream than it takes him to return. If the rate of the boat in still water is 10 miles per hour, find the rate of the stream. [5]

b The sum of the digits of a two digit number is 7. If the number is divided by the units digit, the result is 26. Find the number. [5]

35 Using logarithms, compute K to the *nearest integer*: [10]

$$K = \frac{27.6\ (1.63)^2}{\sqrt[3]{0.0636}}$$

36 Answer *either a or b* but *not* both:

a The diagonals of a parallelogram are respectively 7.0 inches and 10.0 inches long, and they intersect at an angle of 65°. Find the length of one of the longer sides of the parallelogram to the *nearest tenth* of an inch. [5,5]

OR

b Two forces of 410 pounds and 670 pounds, respectively, act upon a body at an acute angle with each other. The angle between the resultant force and the 410-pound force is 37° 20′. Find to the *nearest ten minutes* the angle formed by the 410-pound and 670-pound forces. [4,6]

*37 Three numbers whose common difference is 5 are in an arithmetic progression. If the first number is left unchanged, and 1 is subtracted from the second, and 2 is added to the third, the resulting three numbers are in a geometric progression. What are the three original numbers? [*Only an algebraic solution will be accepted.*] [5,5]

* This question is based upon an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

Monday, January 25, 1971 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 1–12, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 1

(2) 4

(3) 3

(4) 4

(5) 4

(6) 2

(7) 1

(8) 1

(9) 2

(10) 3

(11) 1

(12) 3

(13) 7

(14) 0 and $\frac{3}{2}$

(15) $x^2 + 4x - 1 = 0$

(16) $6 + 15i$

(17) 1

(18) 1

(19) 21

(20) $(2 \sin \theta - 3)(\sin \theta + 5)$

(21) 2

(22) 13

(23) $\frac{3}{4}$

(24) 7 and —4

(25) $\frac{-a}{a^2 - 1}$ *or* $\frac{-a}{(a + 1)(a - 1)}$

(26) 30

(27) 75° 21′

(28) 3

(29) 2 and 3 *or* (2,0) and (3,0)

(30) —cos 10° *or* —sin 80°

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* $\sin\theta = \frac{1}{2}$, $\sin\theta = -\frac{3}{4}$ [6]

b 30°, 150°, 229°, 311° [4]

(33) *b* $\{y \mid -6 \leq y \leq 10\}$ [2]

c 2.3, 2.4, *or* 2.5
−2.3, −2.4, *or* −2.5 [2]

(34) *a* Let x = rate of the stream in miles per hour

$$\frac{48}{10 - x} = \frac{48}{10 + x} + 2 \quad [5]$$

b Let t = tens digit
Let u = units digit

$$t + u = 7$$
$$\frac{10t + u}{u} = 26 \quad [5]$$

(35) 184 [10]

(36) *a* Analysis [5]
7.2 [5]

b Analysis [4]
59° 10′ [6]

*(37) Analysis [5]
4, 9, 14 [5]

DO YOU KNOW...

. . . that most questions used on Regents examinations have been tried out in advance in representative classrooms throughout the State?

Each year more than 40,000 pupils in about 300 schools "pretest" questions intended for use in future Regents examinations. When committees of classroom teachers meet to assemble Regents examinations, the information obtained from this pretesting is to aid them in determining which questions are appropriate, which questions need revision, and which questions should be eliminated.