The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 17, 1968 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

Directions (1–18): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

1 The straight line whose equation is $5x + 2y - 3 = 0$ has a slope of

(1) $\frac{3}{2}$ (3) $\frac{5}{2}$

(2) $-\frac{3}{2}$ (4) $-\frac{5}{2}$

2 A boat can travel 9 miles per hour in still water. It travels at this speed upstream against a current of c miles per hour for h hours. The distance it travels is

(1) $9ch$ (3) $h(9 + c)$
(2) $9(c + h)$ (4) $h(9 - c)$

3 The sum of the first 6 terms of an arithmetic progression is 57. If the first term is 2, what is the sixth term of the progression?

(1) 15 (3) 19
(2) 17 (4) 21

4 The fraction $\frac{b + \frac{b}{a}}{a - \frac{1}{a}}$ is equivalent to

(1) b (3) $\frac{2ab}{a^2 - 1}$

(2) $\frac{b}{a - 1}$ (4) $\frac{a - 1}{b}$

5 In the formula $F = \frac{9}{5}C + 32$, at what temperature reading does $F = C$?

(1) $+8°$ (3) $+40°$
(2) $-8°$ (4) $-40°$

6 An arc of a circle is 24 inches long. If this arc subtends a central angle of 1.5 radians, then the length of the radius of this circle in inches is

(1) 36 (3) 3.6
(2) 16 (4) 1.6

7 If $\log_4 y = 3$, then the value of y is

(1) 12 (3) 3
(2) 64 (4) 4

8 Which is an identity?

(1) $\sin(A + B) = \sin A \sin B + \cos A \cos B$
(2) $\cos(A + B) = \sin A \cos B + \cos A \sin B$
(3) $\sin(A - B) = \sin A \cos B - \cos A \sin B$
(4) $\cos(A - B) = \cos A \cos B - \sin A \sin B$

9 If the graphs of the equations $x^2 + y^2 = 9$ and $y = 3$ are drawn on the same set of axes, what is the total number of points common to both graphs?

(1) 1 (3) 3
(2) 2 (4) 0

10 The discriminant of $3x^2 + \sqrt{2}x + 1 = 0$ is

(1) $\sqrt{14}$ (3) -10
(2) $i\sqrt{10}$ (4) 14

11 If one root of $x^2 + px + 8 = 0$ is -2, then the value of p is

(1) -6 (3) -4
(2) 6 (4) 4

12 The expression $\frac{\sin 2A}{2 \sin A}$ is equivalent to

(1) 1 (3) $\frac{\sin A}{2}$

(2) $\cos A$ (4) $\frac{1 - 2\cos^2 A}{2 \sin A}$

13 The reciprocal of $\left(\frac{1}{n} + 2\right)$ is

(1) $2n + 1$ (3) $\frac{n}{1 + 2n}$

(2) $n + 2$ (4) $\frac{2n + 1}{n}$

14 The period of the graph of the equation $y = 3 \cos 2x$ is

(1) π (3) 3

(2) 2π (4) 4π

15 The equation $2x = 5 + \sqrt{2x + 1}$ has

(1) both $\frac{3}{2}$ and 4 as roots

(2) $\frac{3}{2}$ as its only root

(3) neither $\frac{3}{2}$ nor 4 as a root

(4) 4 as its only root

16 The first two steps used in solving the equation $3x + (x + 4) = 4$ are

Step I $3x + (x + 4) = 4$

Step II $(3x + x) + 4 = 4$.

The principle which justifies deriving Step II from Step I is the

(1) associative law (3) addition axiom

(2) distributive law (4) commutative law

17 In $\triangle ABC$, $a = 1$, $b = 1$, and $C = 120°$. The value of c is

(1) 1 (3) $\sqrt{3}$

(2) $\sqrt{2}$ (4) 0

18 Using the data $a = 7$, $b = 10$, and $A = 40°$, $\triangle ABC$

(1) must be a right triangle

(2) must be an acute triangle

(3) must be an obtuse triangle

(4) may be either an acute or an obtuse triangle

19 Determine the value of cos 28° 8′.

20 Express the sum of $\sqrt{-36}$ and $2\sqrt{-4}$ in terms of i.

21 In acute triangle ABC, $a = 6$, $b = 4$, and $\sin B = \frac{1}{3}$. Find the number of degrees in angle A.

22 Express in degrees the value of x between 180° and 270° which satisfies the equation $\tan x = \cot x$.

23 If t varies directly as $\sqrt{x}$ and if $t = 2$ when $x = 25$, what is the value of t when $x = 100$?

24 Find the value of $4x^0 + x^{\frac{2}{3}}$ if $x = 8$.

25 Express cos (−340°) as a function of a positive acute angle.

26 Solve for $\sin x$:

$$\sin x - 2 \cos y = 0.2$$
$$2 \sin x + \cos y = 0.3$$

27 Express $3 \sin^2 A - \sin A - 2$ as a product of factors.

28 If $\cos \theta = \frac{7}{25}$ and θ is in the 4th quadrant, what is the numerical value of $\sin \theta$?

29 Find the positive value of $\tan \left(\text{arc } \sin \frac{\sqrt{2}}{2}\right)$.

30 Write the third term of a geometric progression whose first term is a and whose common ratio is $-\frac{1}{3}$.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for all positive values of θ less than 360° (which are in the interval $0° < \theta < 360°$): [10]

$$5 \cos 2\theta + 7 \sin \theta + 1 = 0$$

32 Solve the following system of equations and check your solutions in both equations: [8,2]

$$x^2 + 5y^2 = 14$$
$$x - y = 2$$

33 *a* On the same set of axes, sketch the graphs of $y = \tan x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [4,4]

b Determine the number of values of x between 0 and 2π radians which satisfy the equation $\tan x = \cos 2x$. [2]

34 Using logarithms, find to the *nearest hundredth* the value of $\sqrt[3]{\dfrac{92.3 \cos 47° 10'}{1810}}$. [10]

35 Write an equation or a system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a The tens digit of a certain two-digit number is 3 more than the units digit. The sum of the two-digit number and the number obtained when these digits are reversed is 99. Find the original number. [5]

b There are 24 boys and 12 girls at a party. How many boys should leave so that the number of boys remaining will be 60% of the total number of people still at the party? [5]

36 Answer *either a* or *b*, but *not* both:

a Two boys from a surveying class establish a base line AB on a level field. The boy at point A is 50 feet from the boy at point B. Each one sights a stake at point C. The boy at A measures $\angle CAB$ to be 78° 40′. The boy at B measures $\angle CBA$ to be 92° 50′. Find to the *nearest foot* the distance from A to C. [4,6]

OR

b At 2 p.m. on a certain day, a ship sailed from a port on a course 71° 20′ (N 71° 20′ E) at a speed of 15 miles per hour. At 3 p.m., a second ship left the same port on a course 131° 10′ (S 48° 50′ E) at a speed of 20 miles per hour. Find, to the *nearest mile*, the distance between the ships at 5 p.m. [5,5]

37 Answer *both a* and *b*:

a Given acute triangle ABC. Show that

$$\cot B = \frac{c - b \cos A}{b \sin A}.$$

[Hint: Draw altitude from C.] [6]

b Prove the identity: [4]

$$\frac{\sin \theta \cot \theta + \cos^2 \theta}{1 + \cos \theta} = \cos \theta$$

FOR TEACHERS ONLY

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SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 1–18, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 4

(2) 4

(3) 2

(4) 2

(5) 4

(6) 2

(7) 2

(8) 3

(9) 1

(10) 3

(11) 2

(12) 2

(13) 3

(14) 1

(15) 4

(16) 1

(17) 3

(18) 4

(19) 0.8819

(20) $10i$

(21) 30

(22) 225

(23) 4

(24) 8

(25) cos 20° *or* sin 70°

(26) 0.16

(27) $(3 \sin A + 2)(\sin A - 1)$

(28) $-\frac{24}{25}$

(29) 1

(30) $\frac{a}{9}$

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 210°, 330° [10]

(32) $x = 3, y = 1$
$x = \frac{1}{3} \quad y = -\frac{5}{3}$ [8]
Check [2]

(33) *b* 2 [2]

(34) 0.33 [10]

(35) *a* u = units digit, t = tens digit
$t = u + 3$
$10t + u + 10u + t = 99$ [5]

b x = number of boys leaving
$24 - x = .60(36 - x)$ [5]

(36) *a* Analysis [4]
338 [6]

b Analysis [5]
43 [5]