INTEGRATED ALGEBRA

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Wednesday, August 18, 2010—8:30 to 11:30 a.m., only

Student Name: Mr. Sibol
School Name: NS For Civil Rights

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.
Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

1. The school store did a study comparing the cost of a sweatshirt with the number of sweatshirts sold. The price was changed several times and the numbers of sweatshirts sold were recorded. The data are shown in the table below.

<table>
<thead>
<tr>
<th>Cost of Sweatshirt</th>
<th>$10</th>
<th>$25</th>
<th>$15</th>
<th>$20</th>
<th>$5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Sold</td>
<td>9</td>
<td>6</td>
<td>15</td>
<td>11</td>
<td>14</td>
</tr>
</tbody>
</table>

Which scatter plot represents the data?

(1) 

(2) 

(3) 

(4)

Use this space for computations.
2 What is the solution of $3(2m - 1) \leq 4m + 7$?

- (1) $m \leq 5$
- (2) $m \geq 5$
- (3) $m \leq 4$
- (4) $m \geq 4$

$6m - 3 \leq 4m + 7$

$2m \leq 10$

$m \leq 5$

3 Which set represents the intersection of sets $A$, $B$, and $C$ shown in the diagram below?

- (1) $\{3, 4, 5, 6, 7\}$
- (2) $\{2\}$
- (3) $\{2, 3, 4, 5, 6, 7\}$
- (4) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
4 The end of a dog’s leash is attached to the top of a 5-foot-tall fence post, as shown in the diagram below. The dog is 7 feet away from the base of the fence post.

\[ \sqrt{5^2 + 7^2} = \sqrt{74} \approx 8.6 \]

How long is the leash, to the nearest tenth of a foot?

(1) 4.9  (3) 9.0
(2) 8.6  (4) 12.0

(2) 8.6

5 What is the slope of the line passing through the points A and B, as shown on the graph below?

\[ \frac{5 - 6}{-3 - 3} = \frac{2}{-6} = -\frac{1}{3} \]

(1) -3  (3) 3
(2) -\frac{1}{3}  (4) \frac{1}{3}

(2) -\frac{1}{3}
6 The quotient of \((9.2 \times 10^6)\) and \((2.3 \times 10^2)\) expressed in scientific notation is

(1) \(4,000\)  
(2) \(40,000\)  
(3) \(4 \times 10^3\)  
(4) \(4 \times 10^4\)

\[
\frac{9.2 \times 10^6}{2.3 \times 10^2} = 4 \times 10^4
\]

7 In a recent town election, 1,860 people voted for either candidate A or candidate B for the position of supervisor. If candidate A received 55% of the votes, how many votes did candidate B receive?

(1) 186  
(2) 837  
(3) 1,023  
(4) 1,805

\[
45\% \times 1860 = 837
\]

8 Which expression is equivalent to \(121 - x^2\)?

(1) \((x - 11)(x - 11)\)  
(2) \((x + 11)(x - 11)\)  
(3) \((11 - x)(11 + x)\)  
(4) \((11 - x)(11 - x)\)

9 Given:

\[
U = \{1, 2, 3, 4, 5, 6, 7, 8\}
\]

\[
B = \{2, 3, 5, 6\}
\]

Set B is a subset of set U. What is the complement of set B?

(1) \{}  
(2) \(\{2, 3, 5, 6\}\)  
(3) \(\{1, 4, 7, 8\}\)  
(4) \(\{1, 2, 3, 4, 5, 6, 7, 8\}\)
10 Which graph can be used to find the solution of the following system of equations?

\[ \begin{align*}
  y &= x^2 + 2x + 3 \\
  2y - 2x &= 10 \\
  2y &= 2x + 10 \\
  y &= x + 5 
\end{align*} \]

Use this space for computations.

\[ x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1 \]
11 The width of a rectangle is 3 less than twice the length, $x$. If the area of the rectangle is 43 square feet, which equation can be used to find the length, in feet?

(1) $2x(x - 3) = 43$  
(2) $x(3 - 2x) = 43$  
(3) $2x + 2(2x - 3) = 43$  
(4) $x(2x - 3) = 43$

12 Which value of $x$ is the solution of $\frac{2x - 3}{x - 4} = \frac{2\pi}{3}$?

(1) $-\frac{1}{4}$  
(2) $\frac{1}{4}$  
(3) $-4$  
(4) 4

13 What is the perimeter of a regular pentagon with a side whose length is $x + 4$?

(1) $x^2 + 16$  
(2) $4x + 16$  
(3) $5x + 4$  
(4) $5x + 20$

14 Which equation represents a line parallel to the $y$-axis?

(1) $x = y$  
(2) $x = 4$  
(3) $y = 4$  
(4) $y = x + 4$
15 The diagram below shows the graph of $y = -x^2 - c$.

Which diagram shows the graph of $y = x^2 - c$?
16 Which point lies on the line whose equation is \(2x - 3y = 9\)?

(1) \((-1, -3)\)\hfill (3) \((0, 3)\)
(2) \((-1, 3)\)\hfill (4) \((0, -3)\)

\[2(1) - 3(-3) = 9\]
\[0 + 9 = 9\]

17 Which phrase best describes the relationship between the number of miles driven and the amount of gasoline used?

(1) causal, but not correlated
(2) correlated, but not causal
(3) both correlated and causal
(4) neither correlated nor causal

18 The height, \(y\), of a ball tossed into the air can be represented by the equation \(y = -x^2 + 10x + 3\), where \(x\) is the elapsed time. What is the equation of the axis of symmetry of this parabola?

(1) \(y = 5\)
(2) \(y = -5\)
(3) \(x = 5\)
(4) \(x = -5\)

\[x = \frac{-b}{2a} = \frac{-10}{2(-1)} = 5\]

19 In the diagram below, \(MATH\) is a rectangle, \(GB = 4.6\), \(MH = 6\), and \(HT = 15\).

What is the area of polygon \(MBATH\)?

(1) 34.5
(2) 55.5
(3) 90.0
(4) 124.5

The shaded area = \(\text{Whole} - \text{Unshaded} = \text{Rectangle} - \text{Triangle} = \text{LW} - \frac{1}{2} \times \text{base} \times \text{height} = 15 \times 6 - \frac{1}{2} \times 15.4.6 = 90 - 34.5 = 55.5\)
20 This year, John played in 10 baseball games. In these games he had hit the ball 2, 3, 0, 1, 3, 2, 4, 0, 2, and 3 times. In the first 10 games he plays next year, John wants to increase his average (mean) hits per game by 0.5. What is the total number of hits John needs over the first 10 games next year to achieve his goal?

\[
\frac{2 + 3 + 0 + 1 + 3 + 2 + 4 + 0 + 2 + 3}{10} = \frac{20}{10} = 2.0
\]

\[
\frac{x}{10} = 2.5
\]

(1) 5  (3) 20
(2) 2  (4) 25

21 What is the value of the y-coordinate of the solution to the system of equations \(2x + y = 8\) and \(x - 3y = -3\)?

\[
\begin{align*}
2x + y &= 8 \\
x - 3y &= -3
\end{align*}
\]

\[
\begin{align*}
2x + y &= 8 \\
2x - 6y &= -6
\end{align*}
\]

\[
2x - 6y = -6
\]

\[
y = 14
\]

\[
x = 2
\]

(1) -2  (3) 3
(2) 2  (4) -3

22 Which set-builder notation describes \{-3, -2, -1, 0, 1, 2\}? 

(1) \[x \mid -3 \leq x < 2, \text{ where } x \text{ is an integer}\]
(2) \[x \mid -3 < x \leq 2, \text{ where } x \text{ is an integer}\]
(3) \[x \mid -3 < x < 2, \text{ where } x \text{ is an integer}\]
(4) \[x \mid -3 \leq x \leq 2, \text{ where } x \text{ is an integer}\]
23 Corinne calculated the area of a paper plate to be 50.27 square inches. If the actual area of the plate is 55.42 square inches, what is the relative error in calculating the area, to the nearest thousandth?

\[
\frac{55.42 - 50.27}{55.42} \approx 0.093
\]

(1) 0.092  (2) 0.093  (3) 0.102  (4) 0.103

24 The probability that it will snow on Sunday is \( \frac{3}{5} \). The probability that it will snow on both Sunday and Monday is \( \frac{3}{10} \). What is the probability that it will snow on Monday, if it snowed on Sunday?

\[
P(M) = \frac{\frac{3}{10}}{\frac{3}{5}} = \frac{3}{2}
\]

(1) \( \frac{9}{50} \)  (2) 2  (3) \( \frac{1}{2} \)  (4) \( \frac{9}{10} \)
Which graph represents an exponential equation?

(1) Linear

(2) Quadratic

(3) Absolute Value

(4) Exponential
26 Right triangle $ABC$ has legs of 8 and 15 and a hypotenuse of 17, as shown in the diagram below.

![Diagram of right triangle ABC with legs 8 and 15 and hypotenuse 17]

The value of the tangent of $\angle B$ is

1. $0.4706$  
2. $0.5333$  
3. $0.8824$  
4. $1.8750$

27 What is $\frac{2 + x}{5x} - \frac{x - 2}{5x}$ expressed in simplest form?

1. $0$  
2. $\frac{2}{5}$  
3. $\frac{4}{5x}$  
4. $\frac{2x + 4}{5x}$
28 How many different four-letter arrangements are possible with the letters G, A, R, D, E, N if each letter may be used only once?

(1) 15
(2) 24
(3) 360
(4) 720

29 What is an equation of the line that passes through the points (1,3) and (8,5)?

(1) \( y + 1 = \frac{2}{7}(x + 3) \)
(2) \( y - 5 = \frac{2}{7}(x - 8) \)
(3) \( y - 1 = \frac{2}{7}(x + 3) \)
(4) \( y + 5 = \frac{2}{7}(x - 8) \)

30 An example of an algebraic expression is

(1) \( x + 2 \)
(2) \( y = x + 2 \)
(3) \( y < x + 2 \)
(4) \( y = x^2 + 2x \)
Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 Express in simplest form: \( \frac{45a^4b^3 - 90a^3b}{15a^2b} \)

\[
\frac{45a^4b^3}{15a^2b} - \frac{90a^3b}{15a^2b} = \frac{3a^2b^2}{b} - 6a
\]
32 Joseph typed a 1,200-word essay in 25 minutes. At this rate, determine how many words he can type in 45 minutes.

\[
\frac{1200}{25} \cdot \frac{x}{45} \Rightarrow 25x = 54000 \\
x = 2160
\]

33 Express \(-3\sqrt{48}\) in simplest radical form.

\[ -3 \sqrt{16 \cdot 3} = -12\sqrt{3} \]
Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]

34 The number of songs fifteen students have on their MP3 players is:
120, 124, 132, 145, 200, 255, 260, 292, 308, 314, 342, 407, 421, 435, 452

State the values of the minimum, 1st quartile, median, 3rd quartile, and maximum.

120 145 252 407 452

Using these values, construct a box-and-whisker plot using an appropriate scale on the line below.
Find the volume, in cubic centimeters, and the surface area, in square centimeters, of the rectangular prism shown below.

\[ \text{Vol} = lwh \]
\[ = 10 \times 2 \times 4 \]
\[ = 80 \]

\[ \text{SA} = 2lh + 2hw + 2lw \]
\[ = 2 \times 10 \times 2 + 2 \times 4 \times 2 + 2 \times 10 \times 4 \]
\[ = 40 + 16 + 80 \]
\[ = 136 \]
36 Find the roots of the equation $x^2 = 30 - 13x$ algebraically.

\[
\begin{align*}
  x^2 + 13x - 30 &= 0 \\
  (x + 15)(x - 2) &= 0 \\
  -15 & \quad 2
\end{align*}
\]
37. On the set of axes below, solve the following system of inequalities graphically.

\[
\begin{align*}
y &< 2x + 1 \\
y &\geq -\frac{1}{3}x + 4
\end{align*}
\]

State the coordinates of a point in the solution set.

The coordinates of a point in the solution set are \((6, 4)\).
Each of the hats shown below has colored marbles placed inside. Hat A contains five green marbles and four red marbles. Hat B contains six blue marbles and five red marbles. Hat C contains five green marbles and five blue marbles.

If a student were to randomly pick one marble from each of these three hats, determine from which hat the student would most likely pick a green marble. Justify your answer.

\[ P(G) = \frac{5}{9} \quad \frac{0}{10} \quad \frac{5}{10} \]

Hat A

Determine the fewest number of marbles, if any, and the color of these marbles that could be added to each hat so that the probability of picking a green marble will be one-half in each of the three hats.

Hat A - 1 not green \[ \frac{5}{10} = \frac{1}{2} \]

Hat B - 11 green \[ \frac{11}{22} = \frac{1}{2} \]

Hat C - none \[ \frac{5}{10} = \frac{1}{2} \]
A hot-air balloon is tied to the ground with two taut (straight) ropes, as shown in the diagram below. One rope is directly under the balloon and makes a right angle with the ground. The other rope forms an angle of 50° with the ground.

Determine the height, to the nearest foot, of the balloon directly above the ground.

\[
\sin 50^\circ = \frac{x}{110}
\]

\[x \approx 84\]

Determine the distance, to the nearest foot, on the ground between the two ropes.

\[
\cos 50^\circ = \frac{y}{110}
\]

\[y \approx 71\]