The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

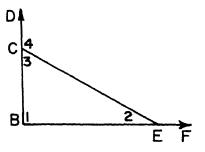Thursday, January 22, 1981 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in radical form.

1 In the accompanying diagram, $\overrightarrow{BCD}$, $\overrightarrow{BEF}$, $m\angle 1 = 90$, and $m\angle 2 = 30$. Find the number of degrees in the measure of angle 4.

2 Determine the value of $b * (a * c)$ within the following system.

*	a	b	c	d
a	c	a	b	d
b	a	b	c	d
c	b	c	a	d
d	d	d	d	b

3 The ratio of the measures of the angles of a triangle are 2:3:7. What is the measure in degrees of the largest angle?

4 The slope of line $\overleftrightarrow{AB}$ is $\frac{3}{5}$ and the slope of line $\overleftrightarrow{CD}$ is $\frac{9}{k}$. If $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$, what is the value of k?
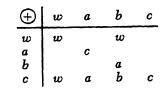
5 Using the accompanying table, find the inverse element of 2.

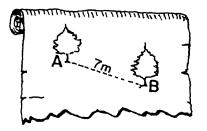| #	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

6 Write another statement that is logically equivalent to $\sim(p \wedge \sim q)$.

7 In triangle PQR, the measure of angle P is 59° and an exterior angle at Q measures 117°. What is the *shortest* side of the triangle?

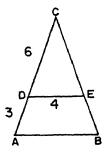8 If the following table defines a commutative operation, find the value of $b \oplus c$.

$\oplus$	w	a	b	c
w	w		w	
a		c		
b			a	
c	w	a	b	c

9 The accompanying diagram shows a pirate's map. A treasure is buried at a distance of 3 meters from the foot of tree A and 5 meters from the foot of tree B. If $AB = 7$ meters, what is the total number of possible places in which the treasure might be buried?

10 In the accompanying figure of triangle ABC, $\overline{DE} \parallel \overline{AB}$, $DE = 4$, $CD = 6$, and $DA = 3$. Find AB.

11 The length of a side of an equilateral triangle is 10. What is the length, in radical form, of an altitude of the triangle?

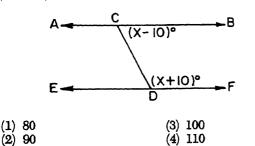12 Lines ℓ and m are perpendicular. The slope of line ℓ is $\frac{1}{3}$. What is the slope of line m?

13 Find, in radical form, the length of the line segment that joins the points $A(4,7)$ and $B(7,9)$.

14 If one of the roots of the equation $x^2 - x + q = 0$ is 3, what is the value of q?

15 Find the number of combinations of 40 things taken 38 at a time.

16 How many different four-digit permutations are there from the numeral 1981?

17 A pair of dice is rolled once. What is the probability that both dice will *not* show the same number?

Directions (18–34): For each question chosen, write in the space provided on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

18 In the accompanying diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{EF}$, and the measure in degrees of the interior angles on the same side of transversal $\overline{CD}$ are $(x + 10)$ and $(x - 10)$. What is the value of x?

(1) 80   (3) 100
(2) 90   (4) 110

19 Line segment $\overline{AB}$ has midpoint M. If the coordinates of A are $(-3,2)$ and the coordinates of M are $(-1,5)$, what are the coordinates of B?
(1) $(1,10)$ (3) $(0,7)$
(2) $(1,8)$ (4) $(-5,8)$

20 In a right triangle, the altitude to the hypotenuse has length 6. If the altitude divides the hypotenuse into segments with lengths 4 and x, the value of x is
(1) 6   (3) 9
(2) 8   (4) 10

21 If the lengths of two sides of a triangle are 5 and 7, the length of the third side may *not* be
(1) 12   (3) 3
(2) 7   (4) 5

22 An equation of the line which passes through the point $(0,2)$ and which has a slope of 4 is
(1) $x = 2y + 4$ (3) $x = 4y + 2$
(2) $y = 2x + 4$ (4) $y = 4x + 2$

23 Which statement is true?
(1) $\vee_x \ x^2 + 2x = 0$ (3) $\vee_x \ x + 1 = x$
(2) $\exists_x \ x^2 + 2x = 0$ (4) $\exists_x \ x + 2 \neq 2 + x$

24 The roots of $3x^2 + 9x - 2 = 0$ are
(1) $\dfrac{-9 \pm \sqrt{105}}{6}$ (3) $\dfrac{-9 \pm \sqrt{57}}{6}$

(2) $\dfrac{9 \pm \sqrt{105}}{6}$ (4) $\dfrac{9 \pm \sqrt{57}}{6}$

25 Which is an equation of a parabola whose axis of symmetry is $x = -2$?
(1) $y = x^2 - 2$ (3) $y = 2x^2 - 8x + 5$
(2) $y = x^2 + 4x + 1$ (4) $y = -x^2 + 4x + 3$

26 Which statement is true of the slope of the straight line that passes through the points $(5,2)$ and $(-1,2)$?
(1) It has no slope.
(2) It has a slope of zero.
(3) It has a slope of 3.
(4) It has a slope of $\frac{1}{3}$.

27 The set $\{1,3,5,7,...\}$ is closed under which operation?
(1) addition   (3) multiplication
(2) subtraction   (4) division

28 Which is *always* false?
(1) $p \wedge \sim p$ (3) $\sim(p \vee q)$
(2) $p \vee \sim p$ (4) $\sim(p \rightarrow q)$

29 Given the true statements: "If a boy plays high school football, he must be passing in three subjects," and "Bob is not passing three subjects." It follows that
(1) Bob plays on the football team
(2) Bob does not play on the football team
(3) few boys try out for the team
(4) no conclusion can be reached

30 Which must have the same truth value as $p \rightarrow q$?
(1) $q \rightarrow p$
(3) $\sim p \rightarrow q$
(2) $\sim p \rightarrow \sim q$
(4) $\sim q \rightarrow \sim p$

31 Which point lies on the circle $x^2 + y^2 = 49$?
(1) (5,24)
(3) (−7,0)
(2) (−4,3)
(4) (0,0)

32 Two squares have areas of 9 square centimeters and 16 square centimeters, respectively. What is the ratio of the perimeter of the smaller square to the perimeter of the larger square?
(1) 81:256
(3) 3:4
(2) 9:16
(4) $\sqrt{3}$:2

33 A parallelogram *must* be a rectangle if the diagonals
(1) are congruent
(3) bisect the angles
(2) are perpendicular
(4) bisect each other

34 If the midpoints of the sides of a quadrilateral are joined consecutively, the resulting figure will *always* be a
(1) rhombus
(3) rectangle
(2) square
(4) parallelogram

Directions (35): Leave all construction lines on the answer sheet.

35 *On the answer sheet*, construct the perpendicular bisector of side $\overline{AB}$ of triangle ABC.

Answers to the following questions are to be written on paper provided by the school.
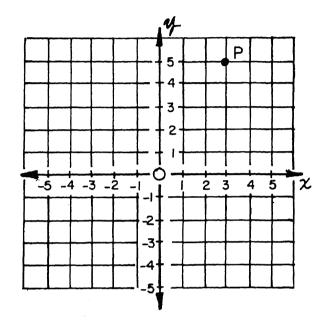
Part II

Answer three questions from this part. Show all work unless otherwise directed.

36 The vertices of triangle ABC are $A(1,1)$, $B(10,4)$, and $C(7,7)$.

    a Find the slope of $\overleftrightarrow{AB}$.   [2]

    b If $D(7,k)$ is a point on $\overline{AB}$, find k.   [3]

    c Write an equation for $\overleftrightarrow{AC}$.   [2]

    d If E is a point on $\overline{AC}$ such that $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$, find the coordinates of E.   [3]

37 There are 7 marbles in an urn: 4 white, 2 red, and 1 blue. The marbles are alike in all ways except color. A 3-marble selection is randomly made.

    a How many such 3-marble selections are possible?   [2]

    b How many of these selections contain marbles which are all the same color?   [3]

    c What is the probability that if a 3-marble selection is made it will contain all marbles of the same color?   [2]

    d What is the probability that the 3-marble selection will contain one marble of each color?   [3]

38 Solve for x and y algebraically and check.
$$x^2 + y^2 = 25$$
$$x - y = 1 \qquad [8,2]$$

39 *a* Draw the graph of the equation $y = 2x^2 + 4x - 3$, using all integral values of x from $x = -4$ to $x = 2$.   [6]

    b On the same set of axes used in part *a*, draw the graph of the equation $y = 3$.   [2]

    c Write the coordinates of the points of intersection of the graphs in parts *a* and *b*.   [1,1]

40 Given: (M,X), where $M = \{0,1,2,3,4\}$ and X is multiplication mod 5 (clock 5).

    a Construct a multiplication table for M as defined.   [4]

    b Use the table constructed in part *a* to answer the following:

      (1) What is the identity element?   [2]

      (2) What is the inverse element of 3?   [2]

      (3) Find all values of y when $y^2 = 1$.   [1,1]

41 In the accompanying graph, P is a point whose coordinates are $(3,5)$.

    a Describe fully the locus of points at a distance d units from P.   [3]

    b Describe fully the locus of points at a distance of one unit from the y-axis.   [4]

    c How many points satisfy the conditions in parts *a* and *b* simultaneously for the following values of d?

      (1) $d = 2$ [1]

      (2) $d = 4$ [1]
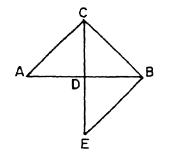
      (3) $d = 5$ [1]

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Answers to the following questions are to be written on paper provided by the school.

Part III

Answer one question from this part. Show all work unless otherwise directed.

42 Given: $\overline{AC} \cong \overline{CB}$, $\overline{CDE} \perp \overline{ADB}$, $\overline{EB} \parallel \overline{AC}$.

Prove:

a $\angle ACD \cong \angle BCD$ [4]

b $\overline{CD} \cong \overline{DE}$ [6]

43 The points $A(-1,4)$, $B(3,-2)$, $C(0,-4)$, and $D(-4,2)$ form a quadrilateral. Prove $ABCD$ is a rectangle. [10]

44 Given the following sentences:

If Brian is a junior, then he will be taking several Regents exams this week.

If he takes several Regents exams this week, he will not be on vacation.

Either he is a junior or a senior.

He is on vacation.

Let J represent: "Brian is a junior."

Let R represent: "He will be taking several Regents exams this week."

Let T represent: "He is on vacation."

Let S represent: "He is a senior."

a Using $J,R,T,S,$ and proper connectives, express each sentence in symbolic form. [4]

b Using laws of inference, show that Brian is a senior. [6]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE II

Thursday, January 22, 1981 — 1:15 to 4:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 18–34, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 120	(11) $5\sqrt{3}$ *or* $\sqrt{75}$	(21) 1	(31) 3
(2) b	(12) $-\frac{3}{2}$	(22) 4	(32) 3
(3) 105	(13) $\sqrt{13}$	(23) 2	(33) 1
(4) 15	(14) -6	(24) 1	(34) 4
(5) 2	(15) 780	(25) 2	(35) construction
(6) $\sim p \vee q$ *or* $p \to q$ *or* $\sim q \to \sim p$	(16) 12	(26) 2	
(7) $\overline{PQ}$ *or* PQ *or* r	(17) $\frac{5}{6}$	(27) 3	
(8) b	(18) 2	(28) 1	
(9) 2	(19) 2	(29) 2	
(10) 6	(20) 3	(30) 4	

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(36) *a* $\frac{1}{3}$ [2]

   b 3   [3]

   c $y = x$ [2]

   d (5,5)   [3]

(37) *a* 35   [2]

   b 4   [3]

   c $\frac{4}{35}$ [2]

   d $\frac{8}{35}$ [3]

(38) $x = -3,\ y = -4$
   $x = 4,\ y = 3$ [8]

(39) *c* (1,3), (−3,3)   [1,1]

(40) *a*

x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

[4]

   b (1) 1   [2]
      (2) 2   [2]
      (3) 1,4   [1,1]

(41) *a* $(x - 3)^2 + (y - 5)^2 = d^2$

         or   [3]

   A circle with center at
   (3,5) having a radius
   of length *d*

   b $x = 1,\ x = -1$
        or   [4]

   Two lines parallel to the
   y-axis and one unit from it

   c (1) 1   [1]
      (2) 3   [1]
      (3) 4   [1]

Part III

(44) *a* $J \rightarrow R$
      $R \rightarrow \sim T$ [4]
      $J \vee S$
      T