The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

## THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

# COURSE III

**Thursday,** January 27, 2000 — 9:15 a.m. to 12:15 p.m., only

**Notice . . .**

Scientific calculators must be available to all students taking this examination.

**The formulas which you may need to answer some questions in this examination are found on page 2. The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

# Formulas

## Pythagorean and Quotient Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$\cot^2 A + 1 = \csc^2 A$$

$$\tan A = \frac{\sin A}{\cos A}$$
$$\cot A = \frac{\cos A}{\sin A}$$

## Functions of the Sum of Two Angles

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$
$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

## Functions of the Difference of Two Angles

$$\sin (A - B) = \sin A \cos B - \cos A \sin B$$
$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## Functions of the Double Angle

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 2 \cos^2 A - 1$$
$$\cos 2A = 1 - 2 \sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

## Functions of the Half Angle

$$\sin \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \tfrac{1}{2}A = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \tfrac{1}{2}A = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

## Area of Triangle

$$K = \tfrac{1}{2}ab \sin C$$

## Standard Deviation

$$\text{S.D.} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

## Part I

**Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.** [60]

1 Express $\frac{7\pi}{6}$ radians in degrees.

2 Express the sum of $3 + \sqrt{-49}$ and $2 + \sqrt{-121}$ in simplest $a + bi$ form.

3 Solve for the positive value of $x$: $|2x - 3| = 11$

4 Solve for $x$: $3^{2x+1} = 27^x$

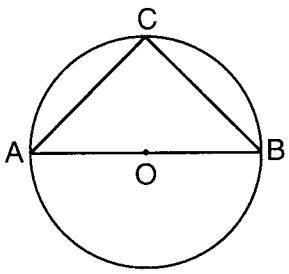
5 Which negative real number is *not* in the domain of $\frac{3}{x^2-4}$?

6 Evaluate: $\sum_{n=1}^{3} n^2$

7 If $x + 2$ is a factor of $x^2 + bx + 10$, what is the value of $b$?

8 In $\triangle ABC$, $a = 16$, $c = 14$, and $m\angle B = 30$. What is the area of $\triangle ABC$?

9 In the accompanying diagram, isosceles triangle $ABC$ is inscribed in circle $O$ with diameter $\overline{AOB}$. Find $m\angle CAB$.




10 In $\triangle ABC$, $a = 2$, $\sin A = \frac{2}{3}$, and $\sin B = \frac{5}{6}$. Find the length of side $b$.

11 If $f(x) = 2\cos^2 x + \sin x - 1$, find the value of $f\left(\frac{\pi}{2}\right)$.

12 Express $\frac{\frac{x}{3} - 1}{\frac{x^2}{3} - 3}$ in simplest form.

13 Find the value of $5x^0 + x^{-\frac{1}{2}} - x^{\frac{1}{2}}$ when $x = 16$.

14 Express $\frac{\cos 2A + \sin^2 A}{\cos A}$ as a single trigonometric function for all values of $A$ for which the fraction is defined.

15 In circle $O$, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$, $AE = 3$ inches, $BE = 8$ inches, and $CE$ is 2 inches longer than $DE$. What is the length of $\overline{DE}$, expressed in inches?

16 What is the probability of getting *exactly* two heads in three tosses of a fair coin?

*Directions* (17–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

17 The numerical value of $\sin \frac{3\pi}{2} + \cos \frac{\pi}{4}$ is

(1) $1 + \frac{\sqrt{2}}{2}$ (3) $-1 + \frac{\sqrt{2}}{2}$

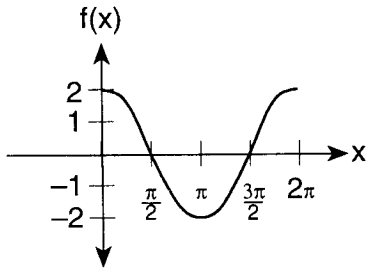
(2) $\frac{\sqrt{2}}{2}$ (4) $-1$

18 If $\tan A = \frac{2}{3}$ and $\tan B = \frac{1}{2}$, what is the value of $\tan (A + B)$?

(1) $\frac{1}{8}$ (3) $\frac{1}{4}$

(2) $\frac{7}{8}$ (4) $\frac{7}{4}$

19 Which trigonometric function is shown in the graph below?




(1) $f(x) = 2 \sin x$
(2) $f(x) = 2 \cos x$
(3) $f(x) = \cos 2x$
(4) $f(x) = \sin 2x$

20 What is the solution set of the equation $\sqrt{x^2-3x+3} = 1$?

(1) {1}
(2) {2}
(3) {1,2}
(4) { }

21 If $\sin \theta = -\frac{3}{5}$ and $\cos \theta > 0$, what is the value of $\tan \theta$?

(1) $\frac{3}{4}$
(2) $-\frac{3}{4}$
(3) $\frac{4}{3}$
(4) $-\frac{4}{3}$

22 The roots of the equation $x^2 - 7x + 15 = 0$ are

(1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real, irrational, and unequal

23 How many distinct triangles can be formed if $a = 20$, $b = 30$, and $m\angle A = 30$?

(1) 1
(2) 2
(3) 3
(4) 0

24 Which expression is *not* an isometry?

(1) $r_{y=x}$
(2) $T_{-2,4}$
(3) $D_{-2}$
(4) $R_{0,90°}$

25 What is one solution of the equation $(\sin x + \cos x)^2 = 2$?

(1) $\frac{\pi}{4}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{2}$
(4) 0

26 Which equation does *not* represent a function?

(1) $y = 2x$
(2) $y = x^2 + 10$
(3) $y = \frac{10}{x}$
(4) $x^2 + y^2 = 9$

27 In which quadrant does the sum of $2 + 3i$ and $3 - 5i$ lie?

(1) I
(2) II
(3) III
(4) IV

28 If the coordinates of point $P$ are (2,–3), then $(R_{90} \circ R_{180})(P)$ is

(1) (–2,3)
(2) (–2,–3)
(3) (3,–2)
(4) (–3,–2)

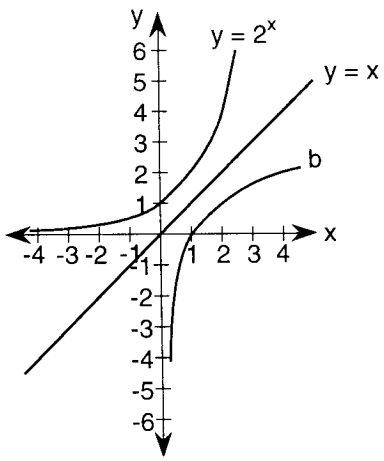
29 What is the value of $\sin\left(\text{Arc cos } \frac{1}{x}\right)$?

(1) $\frac{\sqrt{1-x^2}}{x}$
(2) $\frac{\sqrt{1+x^2}}{x}$
(3) $\frac{\sqrt{x^2-1}}{x}$
(4) $\frac{x}{\sqrt{x^2+1}}$

30 In a normal distribution, what is the greatest percent of the data that falls within 2 standard deviations of the mean?

(1) 95
(2) 81.5
(3) 68
(4) 34

31 In the diagram below, figure $b$ is the reflection of $y = 2^x$ in the line $y = x$.




Which is an expression for the equation of figure $b$?

(1) $y = (-2)^x$
(2) $y = 2^{-x}$
(3) $y = \log_2 x$
(4) $y = \log_x 2$

32 What is the solution set of the inequality $x^2 + 3x - 10 > 8$?

(1) $\{x|-6 < x < 3\}$ (3) $\{x|-3 < x < 6\}$

(2) $\{x|x < -6 \text{ or } x > 3\}$ (4) $\{x|x < -3 \text{ or } x > 6\}$

33 What is the third term in the expansion of $(x + 2y)^5$?

(1) $10x^3y^2$ (3) $80x^2y^3$

(2) $40x^3y^2$ (4) $20x^2y^3$

34 If $f(x) = x^2$, what is the value of $f(i^3)$?

(1) 1 (3) $i$

(2) $-1$ (4) $-i$

35 If $x$ varies inversely as $y$ and $x = 12$ when $y = 3$, what is the value of $x$ when $y = 9$?

(1) 36 (3) $\frac{1}{4}$

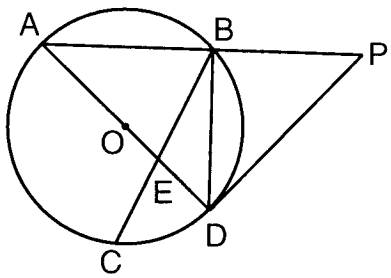
(2) $\frac{1}{3}$ (4) 4

**Answers to the following questions are to be written on paper provided by the school.**

**Part II**

**Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.** [40]

36 In the accompanying diagram of circle $O$, $\overline{AOED}$ is a diameter, $\overline{PD}$ is a tangent, $\overline{PBA}$ is a secant, chords $\overline{BD}$ and $\overline{BEC}$ are drawn, $m\angle DAB = 43$, and $m\angle DEC = 72$.




Find:

*a* $m\angle BDP$ [2]

*b* $m\overset{\frown}{AB}$ [2]

*c* $m\overset{\frown}{AC}$ [2]

*d* $m\angle P$ [2]

*e* $m\angle CBD$ [2]

37 *a* In the month of February at a ski resort, the probability of snow on any day is $\frac{3}{4}$.

(1) What is the probability that snow will fall on every day of a 5-day trip to that resort in February? [2]

(2) What is the probability that snow will fall on *at least* 3 days of that 5-day trip in February? [4]

*b* Using the scores in the table below, find the standard deviation to the *nearest tenth*. [4]

| Scores | Frequency |
|---|---|
| 60 | 2 |
| 65 | 6 |
| 70 | 4 |
| 75 | 8 |
| 80 | 5 |

GO RIGHT ON TO THE NEXT PAGE.

38 In parallelogram $ABCD$, $AD = 8$, $AB = 12$, and diagonal $BD = 15$.

*a* Find $m\angle BAD$ to the *nearest degree*. [6]

*b* Using the answer obtained in part *a*, find the area of parallelogram $ABCD$ to the *nearest tenth*. [4]

39 *a* Find the period of the graph of $y = 3 \sin 2x$. [2]

*b* On graph paper, sketch the graph of $y = 3 \sin 2x$ for one period. [4]

*c* On the same set of axes, sketch the image of the graph drawn in part *b* after it is reflected in the $x$-axis. Label the graph *c*. [2]

*d* Write an equation for the graph sketched in part *c*. [2]

40 Find, to the *nearest degree*, all positive values of $\theta$ less than 360° that satisfy the equation $2 \tan^2 \theta - 2 \tan \theta = 3$. [10]

41 *a* Given: $\log 2 = x$
$\log 3 = y$

Express in terms of $x$ and $y$:

(1) $\log \frac{2}{3}$ [2]

(2) $\log 12$ [3]

*b* Using logarithms, find $w$ to the *nearest hundredth*.

$$5^{2w} + 9 = 40 \quad [5]$$

42 *a* For all values of $\theta$ for which the expressions are defined, prove the following is an identity:

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1 \quad [5]$$

*b* For all values of $x$ for which the expression is defined, solve for $x$:

$$\frac{3}{x+3} + \frac{2}{x-4} = \frac{4}{3} \quad [5]$$

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# SEQUENTIAL MATH – COURSE III

**Thursday,** January 27, 2000 — 9:15 a.m. to 12:15 p.m., only

| | |
|---|---|
| **Part I Score** | ..... |
| **Part II Score** | ..... |
| **Total Score** | ..... |
| **Rater's Initials:** | ..... |

## ANSWER SHEET

Pupil ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School .....

**Your answers to Part I should be recorded on this answer sheet.**

**Part I**

**Answer 30 questions from this part.**

| | | | |
|---|---|---|---|
| 1 ..... | 11 ..... | 21 ..... | 31 ..... |
| 2 ..... | 12 ..... | 22 ..... | 32 ..... |
| 3 ..... | 13 ..... | 23 ..... | 33 ..... |
| 4 ..... | 14 ..... | 24 ..... | 34 ..... |
| 5 ..... | 15 ..... | 25 ..... | 35 ..... |
| 6 ..... | 16 ..... | 26 ..... | |
| 7 ..... | 17 ..... | 27 ..... | |
| 8 ..... | 18 ..... | 28 ..... | |
| 9 ..... | 19 ..... | 29 ..... | |
| 10 ..... | 20 ..... | 30 ..... | |

**Your answers for Part II should be placed on paper provided by the school.**

**The declaration below should be signed when you have completed the examination.**

**I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.**

_____

**Signature**

Tear Here

Tear Here

Tear Here

# FOR TEACHERS ONLY

**The University of the State of New York**

**REGENTS HIGH SCHOOL EXAMINATION**

## THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

# COURSE III

**Thursday,** January 27, 2000 — 9:15 a.m. to 12:15 p.m., only

## SCORING KEY

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the student's work by making insertions or changes of any kind. Use checkmarks to indicate student errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

**Part I**

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 17–35, allow credit if the student has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 210

(2) $5 + 18i$

(3) 7

(4) 1

(5) –2

(6) 14

(7) 7

(8) 56

(9) 45

(10) 2.5

(11) 0

(12) $\frac{1}{x+3}$

(13) $1\frac{1}{4}$

(14) $\cos A$

(15) 4

(16) $\frac{3}{8}$

(17) 3

(18) 4

(19) 2

(20) 3

(21) 2

(22) 1

(23) 2

(24) 3

(25) 1

(26) 4

(27) 4

(28) 4

(29) 3

(30) 1

(31) 3

(32) 2

(33) 2

(34) 2

(35) 4

## Part II

Please refer to the Department's publication *Guide for Rating Regents Examinations in Mathematics*, 1996 Edition. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(36) *a* 43 [2]
*b* 94 [2]
*c* 130 [2]
*d* 47 [2]
*e* 25 [2]

(37) *a* (1) $\frac{243}{1024}$ [2]
(2) $\frac{918}{1024}$ [4]
*b* 6.3 [4]

(38) *a* 95 [6]
*b* 95.6 [4]

(39) *a* $\pi$ [2]
*d* $y = -3 \sin 2x$ [2]

(40) 61, 141, 241, 321 [10]

(41) *a* (1) $x - y$ [2]
(2) $2x + y$ [3]
*b* 1.07 [5]

(42) *b* $-\frac{5}{4}$, 6 [5]