The University of the State of New York

THE STATE EDUCATION DEPARTMENT

SPECIAL GEOMETRY (SMSG) EXAMINATION

Friday, June 18, 1976 — 1:15 to 4:15 p.m., only

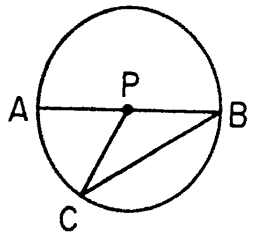
The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL YOU ARE TOLD TO DO SO

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

1 In the accompanying figure, P is the center of the circle, $\overline{AB}$ is a diameter of the circle, C is a point on the circle, and $m\overset{\frown}{AC} = 60$. Find $m\angle PCB$.

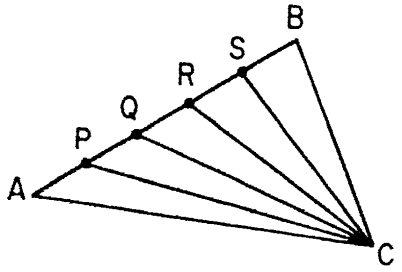


2 Write an equation of a line which contains the bisector of an angle formed by the x-axis and the y-axis of a two-dimensional coordinate system.

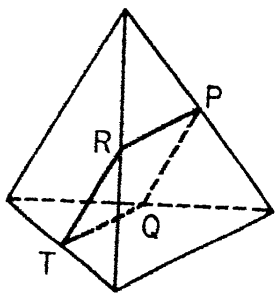
3 Find the surface area of a sphere with radius 7.

4 The altitude to the hypotenuse of right triangle ABC divides the hypotenuse into segments of lengths 1 and 4. Find the length of the *shorter* leg of right triangle ABC.

5 In the accompanying diagram, points P, Q, R, and S divide $\overline{AB}$ into five congruent segments. What is the ratio of the area of ΔAQC to the area of ΔBQC?



6 In the accompanying figure, all edges of the triangular pyramid are of length 8 and R, P, Q, and T are midpoints. Find the perimeter of quadrilateral $PQTR$.



7 Two similar triangles have areas of 40 and 90. If the perimeter of the smaller is 32, find the perimeter of the *larger*.

8 How many sides does a regular polygon have if the measure of an interior angle exceeds the measure of an exterior angle by 100?

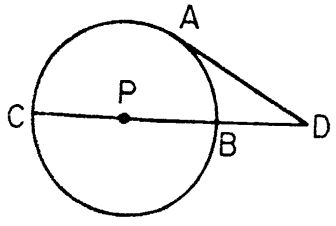
9 One endpoint of a segment has coordinates $(5,4)$ while the coordinates of the midpoint of the segment are $(-1,7)$. Find the coordinates of the other endpoint of the segment.

10 Find the distance between the points whose coordinates are $(2,-1,7)$ and $(5,0,4)$.

11 Plane E passes through the center of sphere S. The intersection of S and E is a circle with circumference 6π. Find the volume of the sphere.

12 A cone has a base with a radius of 6 inches and an altitude measuring 12 inches. What is the area of a cross section 2 inches from the vertex?

13 In the accompanying diagram, P is the center of the circle, $\overline{DA}$ is a tangent segment, $DB = BP = PC$, and $DA = \sqrt{27}$. Find the radius of the circle.



14 Find the area of a square inscribed in a circle whose diameter is 8.

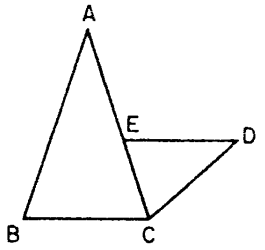
15 Circle C has a radius of 4. The center of circle C is at the origin. Write an equation of the line tangent to circle C at the point $(4,0)$.

16 In a circle of radius 10, find the length of an arc whose measure is 72.

17 For any real numbers a and b, $(a \neq b)$, if the coordinates of the point (a,b) are reversed, a new point is determined. What is the slope of the line joining these two points?

18 Find the area of a rhombus whose longer diagonal has length 24 and whose side has length 15.

19 In the accompanying figure, $AB = AC$ and $DE = DC$. If $\overline{DE} \parallel \overline{BC}$ and $m\angle D = 40$, find the measure of angle A.

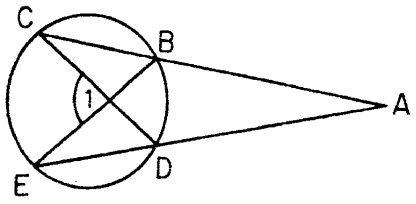


20 Find the side of an equilateral triangle whose altitude measures 12.

21 Chords $\overline{AC}$ and $\overline{QT}$ of a circle intersect at M. If MT is equal to the product of AM and MC, find QM.

22 Triangle ABC is an equilateral triangle with $\overline{BD}$ drawn so that D is between A and C. What is the *shortest* side of $\triangle ABD$?

23 In the accompanying diagram, $m\angle 1 = 80$ and $m\overset{\frown}{BD} = 60$. Find $m\angle A$.



Directions (24–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

24 The theorem "Every angle has exactly one bisector" implies
(1) existence, only
(2) uniqueness, only
(3) both existence and uniqueness
(4) neither existence nor uniqueness

25 The number of noncollinear points necessary to determine a plane is
(1) 5
(2) 2
(3) 3
(4) 4

26 The lengths of two adjacent sides of a parallelogram are represented by x and y and the angle included between these sides has a measure of 30. The area of the parallelogram may be represented by
(1) xy
(2) $2xy$
(3) $\frac{1}{2}xy$
(4) $\frac{\sqrt{3}}{2}xy$

27 A sphere of radius 4, centered at the point (0,0,0), intersects the y-axis at the point
(1) (0,4,0)
(2) (0,0,4)
(3) (4,0,0)
(4) (4,4,4)

28 If the center of the circumscribed circle of a triangle is in the exterior of two angles of the triangle, then the triangle
(1) may be a right triangle
(2) must be an obtuse triangle
(3) may be an acute triangle
(4) must be a scalene triangle

29 Which is *not* a convex set?
(1) a plane
(2) a half-plane
(3) an angle
(4) the interior of a triangle

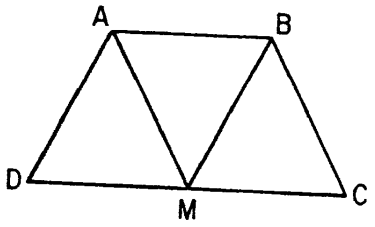
30 Soups *GEO* and *METRY* are each packed in right circular cylindrical cans. The *GEO* can has radius 4 and height 3 and the *METRY* can has radius 3 and height 4. If G represents the volume of one *GEO* can, and M the volume of one *METRY* can, which is true?
(1) $G = M$
(2) $G > M$
(3) $G < M$
(4) The relationship of G to M cannot be determined.

31 The union of the bisectors of the angles of a linear pair is
(1) a finite set
(2) a ray
(3) a right angle
(4) two intersecting lines

32 If the radius of a circle is increased by one foot, then the circumference of the circle is increased by

(1) 1 foot
(2) 2 feet
(3) π feet
(4) 2π feet

33 In the accompanying figure, $ABCD$ is a trapezoid and ΔADM, ΔMAB, and ΔBCM are equilateral triangles. If $AB = 2$, then what is the area of trapezoid $ABCD$?



(1) $3 + \sqrt{3}$
(2) 10
(3) $3\sqrt{3}$
(4) $3\sqrt{2}$

34 If $x^2 + y^2 = 25$ is an equation of a circle, then an equation of a concentric circle is

(1) $(x - 3)^2 + (y - 1)^2 = 10$
(2) $(x - 3)^2 + (y - 1)^2 = 25$
(3) $(x - 3)^2 + (y - 1)^2 = 25$
(4) $x^2 + y^2 = 10$

35 If an isosceles triangle has perimeter 7 and sides whose lengths are integers, then the length of the base

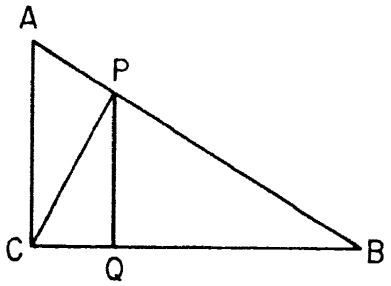
(1) must be 1
(2) must be 3
(3) could be 1 or 3
(4) could be 5

Answers to the following questions are to be written on paper provided by the school.

Part II

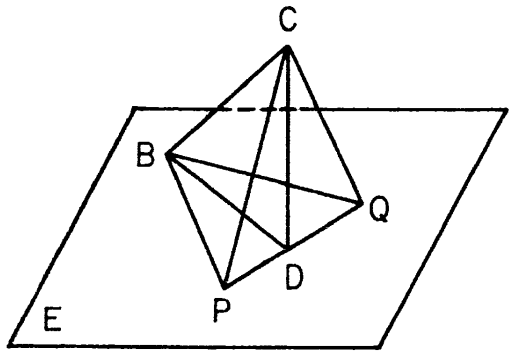
Answer four questions from this part. Show all work unless otherwise directed.

36 *Given:* $\overline{AC} \perp \overline{CB}$
$\overline{PQ} \perp \overline{CB}$
$\overline{CP} \perp \overline{AB}$



Prove. *a* $\angle QPC \cong \angle PCA$ [5]
b $PC^2 = AC \cdot PQ$ [5]

37 *Given:* B, P, D, and Q are points in plane E, plane CBD is the perpendicular bisecting plane of $\overline{PQ}$, $\overline{CD} \perp \overline{BD}$



Prove: *a* $\overline{CD} \perp E$ [5]
b $\angle BCP \cong \angle BCQ$ [5]

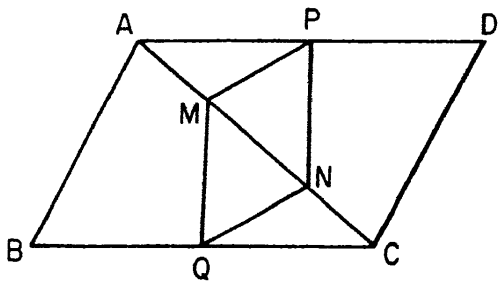
35 *a* Graph the isosceles triangle whose coordinates are $A(0,1)$, $B(10,3)$, and $C(4,7)$. [1]

b Find the slope of base $\overline{AB}$. [2]

c Write an equation of the line containing the altitude of ΔABC to base $\overline{AB}$. [4]

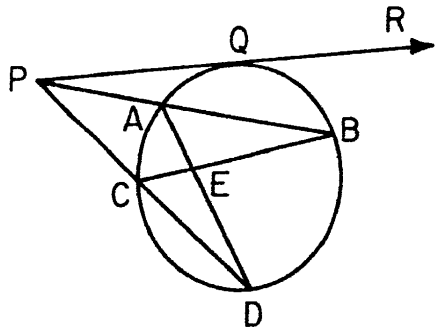
d Find the length of the median to $\overline{AC}$. [3]

39 *Given:* Parallelogram $ABCD$, P is the midpoint of $\overline{AD}$, Q is the midpoint of $\overline{BC}$, $\overline{AM} \cong \overline{CN}$



Prove: $PMQN$ is a parallelogram [10]

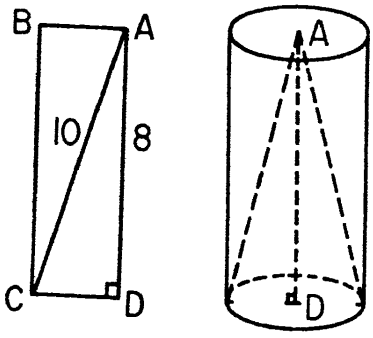
40 In the accompanying plane figure, $\overrightarrow{PR}$ is tangent to the circle at Q. $\overrightarrow{PAB}$ and $\overrightarrow{PCD}$ are secants; chords $\overline{AD}$ and $\overline{BC}$ intersect at E; $m\overset{\frown}{AQ} = x + 15$; $m\overset{\frown}{QB} = 3x + 3$; $m\overset{\frown}{BD} = 4x + 8$; $m\overset{\frown}{DC} = 3x + 19$; $m\overset{\frown}{CA} = 2x - 10$.



a Write an equation to solve for x, and find x. [4]
b Find $m\angle BPD$ [2]
c Find $m\angle BED$ [2]
d Find $m\angle ADC$ [2]

☞ GO RIGHT ON TO THE NEXT PAGE.

41 In the accompanying figure, $ABCD$ is a rectangle with $AC = 10$ and $AD = 8$. The solid shown is generated by rotating the rectangle about $\overline{AD}$ as an axis.



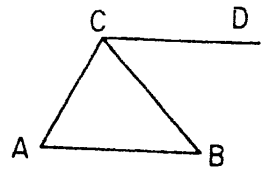
a What is the lateral surface area of the cylinder generated? [*Answer may be left in terms of* π.] [4]

b What is the volume of the cone generated? [*Answer may be left in terms of* π.] [2]

c Find the volume of that portion of the cylinder which lies outside the cone. [*Answer may be left in terms of* π.] [4]

42 Choose *two* questions from *a*, *b*, and *c*.

a *Given:* Scalene $\triangle ABC$
$\overline{CD} \perp \overline{AB}$



Prove: $\overrightarrow{CB}$ does not bisect $\angle ACD$ [5]

b Eight tropical fish and gravel were put into a rectangular fishtank 20 inches by 15 inches, raising the level of the water by one-tenth of one inch. What was the total volume of the fish and the gravel? [5]

c *Given:* $\triangle USA$; point A is between S and N; point U is between A and F; Y is any point on $\overline{AU}$

Prove: $\angle FUS > \angle SNY$ [5]

Part I Score: .....
Rater's Initials:
.....

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ANSWER SHEET

Pupil ..... Teacher .....

School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions in this part.

1	10	19 .....
2	11	20 .....
3	12	21 .....
4	13	22 .....
5	14	23 .....
6	15	24 .....
7	16	25 .....
8	17	26 .....
9	18	27 .....

Questions 28 through 35 should be answered on the back of this page.

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28 .....

29 .....

30 .....

31 .....

32 .....

33 .....

34 .....

35 .....

Your answers for Part II should be placed on paper provided by the school.

FOR TEACHERS ONLY

SCORING KEY

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Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.] For questions 24–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 30

(2) $x = y$ *or* $x = -y$

(3) 196π

(4) $\sqrt{5}$

(5) $\frac{2}{3}$

(6) 16

(7) 48

(8) 9

(9) $(-7, 10)$

(10) $\sqrt{19}$

(11) 36π

(12) π

(13) 3

(14) 32

(15) $x = 4$

(16) 4π

(17) -1

(18) 216

(19) 40

(20) $8\sqrt{3}$

(21) 1

(22) $\overline{AD}$

(23) 20

(24) 3

(25) 3

(26) 3

(27) 1

(28) 2

(29) 3

(30) 2

(31) 3

(32) 4

(33) 3

(34) 4

(35) 3

[OVER]

Part II

38 *b* $\frac{1}{5}$ [2]

c $y - 7 = -5(x - 4)$ [4]

d $\sqrt{65}$ [3]

40 *a* $13x + 35 = 360$
$x = 25$ [4]

b 34 [2]

c 74 [2]

d 20 [2]

41 *a* 96π [4]

b 96π [2]

c 192π [4]

42 *b* 30 cu. in. [5]