The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 20, 1963 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express in degrees an angle of $\frac{\pi}{15}$ radians. 1.....

2 Find the numerical value of $\cos 120^\circ + \sin 270^\circ$. 2.....

3 Express $\tan^2 x$ in terms of $\cos^2 x$. 3.....

4 Given triangle ABC with $a = 2$, $B = 45^\circ$ and $A = 30^\circ$. Find the length of side b. 4.....

5 Find the numerical value of $\cos 32^\circ 14'$. 5.....

6 If $\log_5 x = 2$, what is the value of x? 6.....

7 If $\log 5b = 9.3451 - 10$, determine the value of $\log b$. 7.....

8 Find the positive value of $\tan (\arcsin \frac{2}{3})$. 8.....

9 In a circle whose radius is 3 inches, central angle AOB contains one-half radian. Find the number of inches in the length of minor arc AB. 9.....

10 As x increases between $\frac{\pi}{2}$ and π radians, which one of the following functions increases in value: $\sin x$, $\csc x$, $\cos x$? 10.....

11 Given $\log \tan A = 0.5130$, find to the *nearest minute* the smallest positive value of A. 11.....

12 Those angles in standard position which may be denoted by $-\frac{\pi}{4} + 2n\pi$, where n is an integer, form a set of coterminal angles. Write the number of the quadrant in which all angles of the set terminate. 12.....

13 In $\triangle ABC$, $a = 5$, $c = 6$ and $\cos B = \frac{1}{5}$. Find the value of b. 13.....

14 Express $\tan (x - 45°)$ in terms of $\tan x$. 14.....

15 Compute the area of triangle ABC in which $a = 3$, $b = 5$ and $c = 6$. 15.....

16 The bearing of point B from point A is 135 (S 45° E) and the bearing of point C from point A is 025 (N 25° E). Find the number of degrees in angle BAC. 16.....

Directions (17–27): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

17 For all values of x, $\sin 3x$ is equal to

(1) $\sin 2x + \sin x$
(2) $\sin (2x + x)$
(3) $3 \sin x$
(4) $\frac{1}{2} \sin 6x$

17.....

18 The expression $\sin^4 B - \cos^4 B$ is equivalent to

(1) $\sin^2 B - \cos^2 B$
(2) $1 - 2\cos^4 B$
(3) $\sin^2 B + \cos^2 B$
(4) $(\sin^2 B - \cos^2 B)^2$

18.....

19 A trigonometric function of θ which is identically equal to $\sin (270° + \theta)$ is

(1) $\sin \theta$
(2) $\cos \theta$
(3) $-\sin \theta$
(4) $-\cos \theta$

19.....

20 The expression $\sin 4x + \sin 2x$ is equivalent to

(1) $\sin 6x$
(2) $\sin 3x(\sin 2x + 1)$
(3) $2 \sin 3x \cos x$
(4) $2 \sin x \cos 3x$

20.....

21 Which pair of values of x is among the solutions of the equation $\tan^2 x = \frac{1}{3}$?

(1) 60° and −60°
(2) 30° and −30°
(3) 60° and 120°
(4) 30° and 240°

21.....

22 If $y = \frac{1}{2} \sin 3x$, the maximum value of y is

(1) 1 (2) $\frac{1}{2}$ (3) 3 (4) $\frac{3}{2}$

22.....

23 If $x \log a = y \log b$, then $\frac{x}{y}$ is equal to

(1) $\log \frac{a}{b}$
(2) $\frac{\log a}{\log b}$
(3) $\log \frac{b}{a}$
(4) $\frac{\log b}{\log a}$

23.....

324

24 In taking off, an airplane needs to clear a cliff 100 feet high and 900 feet from the point of takeoff. The constant angle of rising which makes the clearance most closely approximate 150 feet above the top of the cliff is

(1) arc tan $\frac{5}{18}$

(2) arc tan $\frac{1}{6}$

(3) arc tan $\frac{1}{9}$ + arc tan $\frac{1}{6}$

(4) arc tan $\frac{1}{9}$

24.....

25 The expression $\frac{\tan \theta + \sec \theta}{1 + \sin \theta}$ is equivalent to

(1) 1

(2) $\cos \theta$

(3) $\frac{1}{\cos \theta}$

(4) $\frac{1}{1 + \sin \theta}$

25.....

26 If $\cos \theta = \frac{5}{13}$ and $\tan \theta = -\frac{5}{12}$, then θ terminates in quadrant

(1) I (2) II (3) III (4) IV

26.....

27 In triangle ABC, $a = 5$, $b = 10$ and $B = 45°$. Then triangle ABC

(1) must be a right triangle
(2) must be an acute triangle
(3) must be an obtuse triangle
(4) may be either an acute or an obtuse triangle

27.....

Directions (28–30): For values of x for which the following statements are defined, indicate whether each of the statements is true for

(1) all real values of x,
(2) some but not all real values of x,
(3) no real values of x,

by writing on the line at the right the number 1, 2 or 3.

28 $2 \cos^2 x - \cos 2x = 1$ 28.....

29 $\cos x = \cos (-x)$ 29.....

30 $\sin \frac{1}{2} x < \sin x$ 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* On the same set of axes, *sketch* the graphs of $y = 3 \sin x$ and $y = \tan x$ as x varies from $-\pi$ to π radians. [Label each curve with its equation.] [4, 4]

b From the graphs made in answer to part *a*, determine the number of roots of $3 \sin x - \tan x = 0$ which lie between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ radians. [2]

32 Find all positive values of θ less than 360° which satisfy the equation $4 \cos 2\theta + 2 \sin \theta - 1 = 0$. [Express the values of θ to the *nearest degree*.] [10]

33 *a* Prove that the following equality is an identity: [5]

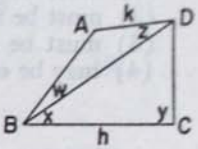
$$\cos (60° + A) + \cos (60° - A) = \cos A$$

b Starting with the formula $\tan \frac{x}{2} = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, where x is an acute angle, *derive* the formula $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$. [5]

34 Prove that the law of cosines is true for a triangle which has three acute angles. [10]

35 In the accompanying diagram, the letters w, x, y and z denote the measure of the indicated angles; h and k represent the lengths of line segments BC and AD, respectively. Show that

$$k = \frac{h \sin w \sin y}{\sin(x + y)\sin(w + z)}.$$ [10]



36 The three sides of a triangle are 35.0, 22.0 and 45.0. Find to the *nearest degree* the largest angle of the triangle. [10]

FOR TEACHERS ONLY

T

SCORING KEY

TRIGONOMETRY

Thursday, June 20, 1963 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 12

(2) $-\frac{3}{2}$

(3) $\frac{1-\cos^2 x}{\cos^2 x}$

(4) $2\sqrt{2}$

(5) 0.8459

(6) 25

(7) 8.6461—10

(8) $\frac{2\sqrt{5}}{5}$

(9) $\frac{3}{2}$

(10) csc x

(11) 72° 56′

(12) IV

(13) $\sqrt{51}$

(14) $\frac{\tan x - 1}{\tan x + 1}$

(15) $\sqrt{56}$ *or* $2\sqrt{14}$

(16) 110

(17) 2

(18) 1

(19) 4

(20) 3

(21) 2

(22) 2

(23) 4

(24) 1

(25) 3

(26) 4

(27) 3

(28) 1

(29) 1

(30) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) b 3 [2]

(32) 49°, 131°, 210°, 330° [10]

(36) 102 [10]