## C - Expressions and Equations, Lesson 3, Solving Linear Equations (r. 2018)

## EXPRESSIONS AND EQUATIONS <br> Solving Linear Equations

Common Core Standard
A-REI.B. 3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Next Generation Standard
AI-A.REI. 3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
Note: Algebra I tasks do not involve solving compound inequalities.

## LEARNING OBJECTIVES

Students will be able to:

1) Solve one step and multiple step equations.
2) Explain each step involved in solving one step and multiple step equations.
3) Do a check to see if the solution is correct.

## Overview of Lesson

| Teacher Centered Introduction | Student Centered Activities |
| :--- | :--- |
| Overview of Lesson | guided practice 世Teacher: anticipates, monitors, selects, sequences, and <br> connects student work |
| - activate students' prior knowledge | - developing essential skills |
| - vocabulary | - Regents exam questions <br> - learning objective(s) <br> - big ideas: direct instruction <br> entry) |
| - modeling assessment assignment (exit slip, explain the math, or journal |  |

## VOCABULARY

balance
check
common sense
DIMS
four column strategy
four general rules
isolate
plug
proper notation
substitute

## BIG IDEAS <br> The Four General Rules (for solving equations and inequalities)



Isolate the Variable: The goal of solving any equation is to isolate the desired variable in either the left or right expression.
Keep It Balanced: During each step of the equation solving process, the left and right expressions must equal one another.
Reduce the Number of Terms: Any step that reduces the number of terms in an equation is usually a good step.
Use Proper Notation and Do a Check: You check your answers in algebra on two levels: first, you see if the answer actually makes sense, and then you plug your answer back into the problem to see if it works.

- Proper Notation involves making short notes that describe the action taken during each step of solving an equation. Academic language is sometimes required.
- Does It Make Sense (DIMS)

The first step in checking a solution is to use "common sense." For example, if your solution is $x=5$, and you are solving for a football player's weight in pounds, you have probably made a mistake because it does not make sense that a football player weighs only five pounds. On the other hand, if you are solving for the number of pennies in a nickel, it makes perfect sense.

- Plug (substitute) the answer back into the problem to see if it works. The second step in checking a solution is to substitute your solution into the original equation and solve the equation once again with your solution in it. If the left expression is equal to the right expression, the equation balances and your solution is correct.


## The Four Column Strategy

The four column strategy focuses on organizing and documenting each step in solving an equation or inequality. Emphasis is given to explaining each step and keeping the equal signs (or inequality signs) aligned in a vertical column. The vertical and horizontal lines are simply scaffolds that can be removed as students acquire understanding and skills in solving equations.
((keep the equation/inequality signs aligned vertically)

| Notes | Left Hand Expression | Sign | Right Hand Expression |
| :---: | :---: | :---: | :---: |
| Given | $2 x-6$ | $=$ | 2 |
| Add (6) | +6 |  | +6 |
|  | $2 x+0$ | $=$ | 8 |
| Divide (2) | $\frac{2 x}{2}$ | $=$ | $\frac{8}{2}$ |
| Answer | $x$ | $=$ | 4 |
| Check | $2(4)-6$ | $=$ | 2 |
|  | $8-6$ | $=$ | 2 |
|  | 2 | 2 |  |

## DEVELOPING ESSENTIAL SKILLS

Use the four general rules and the four column strategy to solve the following problems:
If $3(x-2)=2 x+6$, the value of $x$ is 12

| Notes | Left Hand Expression | Sign | Right Hand Expression |
| :---: | :---: | :---: | :---: |
| Given | $3(x-2)$ | $=$ | $2 x+6$ |
| Distributive <br> Property | $3 x-6$ | $=$ | $2 x+6$ |
| Subtract 2x | $-2 x$ |  | $=$ |
| Simplify | $x-6$ |  | $-2 x$ |
| Add 6 | +6 | $=$ | 6 |
| Solution | $x$ | $3(x-2)=2 x+6$ |  |
| Check | $3(12-2)=2(12)+6$ <br> $\quad 3(10)=24+6$ |  |  |
|  | $30=30$ |  |  |
|  |  |  |  |
|  |  |  |  |

What is the value of $x$ in the equation $\frac{3}{4} x+2=\frac{5}{4} x-6$ ? 16

| Notes | Left Hand Expression | Sign | Right Hand Expression |
| :---: | :---: | :---: | :---: |
| Given | $\frac{3}{4} x+2$ | $=$ | $\frac{5}{4} x-6$ |
|  | $3 x+8$ | $=$ | $5 x-24$ |
| Multiply by 4 | 8 | $=$ | $2 x-24$ |
| Subtract $3 x$ | 32 | $=$ | 2 x |
| Add 24 |  |  |  |


| Divide by 2 | $=$ | X |
| :---: | ---: | ---: |
| Check | $\frac{3}{4} x+2$ | $=\frac{54}{} x-6$ |
| $\frac{3}{4}(16)+2$ | $=\frac{5}{4}(16)-6$ |  |
|  | $\frac{48}{4}+2$ | $=\frac{80}{4}-6$ |
| $12+2$ | $=20-6$ |  |
| 14 | $=14$ |  |
|  |  |  |
|  |  |  |

## REGENTS EXAM QUESTIONS (through June 2018)

## A.REI.B.3: Solving Linear Equations

55) Which value of $x$ satisfies the equation $\frac{7}{3}\left(x+\frac{9}{28}\right)=20$ ?
56) 8.25
57) 8.89
58) 19.25
59) 44.92
60) What is the value of $x$ in the equation $\frac{x-2}{3}+\frac{1}{6}=\frac{5}{6}$ ?
61) 4
62) 6
63) 8
64) 11
65) An equation is given below.

$$
4(x-7)=0.3(x+2)+2.11
$$

The solution to the equation is

1) 8.3
2) 8.7
3) 3
4) -3
5) Which value of $x$ satisfies the equation $\frac{5}{6}\left(\frac{3}{8}-x\right)=16$ ?
6) -19.575
7) -18.825
8) -16.3125
9) -15.6875
10) The value of $x$ which makes $\frac{2}{3}\left(\frac{1}{4} x-2\right)=\frac{1}{5}\left(\frac{4}{3} x-1\right)$ true is
11) -10
12) -2
13) $-9 . \overline{09}$
14) $-11 . \overline{3}$
15) Solve the equation below algebraically for the exact value of $x$.

$$
6-\frac{2}{3}(x+5)=4 x
$$

## SOLUTIONS

55) ANS: 1

Strategy: Use the four column method.

| Notes | Left Expression | Sign | Right Expression |
| :---: | :---: | :---: | :---: |
| Given | $\frac{7}{3}\left(x+\frac{9}{28}\right)$ | $=$ | 20 |
| Divide both <br> expressions by $\frac{7}{3}$ <br> (Division property of <br> equality) | $\frac{7}{3}\left(x+\frac{9}{28}\right)$ | $=$ | $\frac{70}{3}$ |
| Cancel and Simplify | $x+\frac{9}{28}$ | $=$ | $\frac{7}{3}$ |
| Subtract $\frac{9}{28}$ from <br> both expressions <br> (Subtraction property <br> of equality) | x | $=$ | $\frac{60}{7}-\frac{9}{28}$ |
| Simplify | x | $=$ | $\frac{231}{28}$ |
| Simplify | x | $=$ | 8.25 |


| Notes | Left Expression | Sign | Right Expression |
| :---: | :---: | :---: | :---: |
| Given | $\frac{7}{3}\left(x+\frac{9}{28}\right)$ | $=$ | 20 |
| Distributive Property | $\frac{7}{3} x+\frac{7}{3}\left(\frac{9}{28}\right)$ | $=$ | 20 |
| Cancellation | $\frac{7}{3} x+\frac{1}{3}\left(\frac{9}{4}\right)$ | $=$ | 20 |
| Simplification | $\frac{7}{3} x+\frac{3}{4}$ | $=$ | 20 |
| Subtract $\frac{3}{4}$ from <br> both expressions <br> (Subtraction Property <br> of Equality) | $\frac{7}{3} x$ | $=$ | $20-\frac{3}{4}$ |
| Simplification <br> Multiply both <br> expressions by 12 <br> (Multiplication <br> property of equality) | $\frac{12}{1}\left(\frac{7 x}{3}\right)$ | $=$ | $\frac{77}{4}$ |
| Cancel | $\frac{4}{1}\left(\frac{7 x}{1}\right)$ | $=$ | $\frac{12}{1}\left(\frac{77}{4}\right)$ |
| Siomplify | $\left.\frac{77}{1}\right)$ |  |  |
| Divide both <br> expressions by 28 <br> (Division property of <br> equality) | $\frac{28 x}{28}$ | $=$ | 231 |


| Simplify | $x$ | $=$ | 8.25 |
| :---: | :---: | :---: | :---: |

PTS: 2
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: fractional expressions
56) ANS: 1

Strategy: Use the four column method.

| Notes | Left Expression | Sign | Right Expression |
| :---: | :---: | :---: | :---: |
| Given: | $\frac{x-2}{3}$ | $=$ | $\frac{4}{6}$ |
| Multiply both <br> expressions by 6 <br> (Multiplication <br> property of equality) | $\frac{6}{1}\left(\frac{x-2}{3}\right)$ | $=$ | $\frac{6}{1}\left(\frac{4}{6}\right)$ |
| Cancel and Simplify | $\frac{2}{1}\left(\frac{x-2}{1}\right)$ | $=$ | $\frac{1}{1}\left(\frac{4}{1}\right)$ |
| Simplify | $2 x-4$ | $=$ | 4 |
| Add +4 to both <br> expressions <br> (Addition property of <br> equality) | $2 x$ | $=$ | 8 |
| Divide both <br> expressions by 2 <br> (Division property of <br> equality) | x | $=$ | 4 |

PTS: 2
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: fractional expressions
57) ANS: 1

$$
\begin{aligned}
4(x-7) & =0.3(x+2)+2.11 \\
4 x-28 & =.3 x+2.71 \\
4 x-.3 x & =2.71+28 \\
3.7 x & =30.71 \\
x & =8.3
\end{aligned}
$$

PTS: 2
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: decimals
58) ANS: 2

$$
\begin{aligned}
& \frac{5}{6}\left(\frac{3}{8}-x\right)=16 \\
& 5\left(\frac{3}{8}-x\right)=96 \\
& \frac{3}{8}-x=\frac{96}{5} \\
& -x=\frac{96}{5}-\frac{3}{8} \\
& -x=18.825 \\
& x=-18.825
\end{aligned}
$$

PTS: 2
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: fractional expressions
59) ANS: 4

Solve for x :

$$
\frac{2}{3}\left(\frac{1}{4} x-2\right)=\frac{1}{5}\left(\frac{4}{3} x-1\right)
$$

Multiply by 3 to clear the first fraction.

$$
\begin{aligned}
\left(\frac{3}{1}\right) \frac{2}{3}\left(\frac{1}{4} x-2\right) & =\left(\frac{3}{1}\right) \frac{1}{5}\left(\frac{4}{3} x-1\right) \\
2\left(\frac{1}{4} x-2\right) & =\frac{3}{5}\left(\frac{4}{3} x-1\right)
\end{aligned}
$$

Multiply by 5 to clear the remaining fraction.

$$
\begin{gathered}
\text { (5) } 2\left(\frac{1}{4} x-2\right)=\left(\frac{5}{1}\right) \frac{3}{5}\left(\frac{4}{3} x-1\right) \\
10\left(\frac{1}{4} x-2\right)=3\left(\frac{4}{3} x-1\right)
\end{gathered}
$$

Use distributive property to clear parentheses.

$$
\frac{10}{4} x-20=4 x-3
$$

Multiply by 4 to clear fraction.

$$
\begin{gathered}
\text { (4) } \frac{10}{4} x-(4) 20=(4) 4 x-(4) 3 \\
10 x-80=16 x-12
\end{gathered}
$$

Transpose and solve for x .

$$
\begin{aligned}
& -6 x=68 \\
& \frac{-6 x}{-6}=\frac{68}{-6} \\
& x=-11 . \overline{33}
\end{aligned}
$$

PTS: 2
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: fractional expressions
60) ANS:

Answer: $\frac{4}{7}$
Strategy: Solve algebraically (without a calculator).

| Notes | Left Expression | Sign | Right Expression |
| :---: | :---: | :---: | :---: |
| Given | $6-\frac{2}{3}(x+5)$ | $=$ | $4 x$ |
| Multiply by 3 | $18-2(x+5)$ | $=$ | $12 x$ |
| Distributive Property | $18-2 x-10$ | $=$ | $12 x$ |
| Add 2x | $18-10$ | $=$ | $14 x$ |
| Simplify | 8 | $=$ | $14 x$ |
| Divide by 14 | $\frac{8}{14}$ | $=$ | x |
| Simplify | $\frac{4}{7}$ | $=$ | $x$ |

Check.is Optional

| Notes | Left <br> Expression | Sign | Right Expression |
| :---: | :---: | :---: | :---: |
| Given | $6-\frac{2}{3}(x+5)$ | $=$ | 4 x |
| Evaluate for $x=\frac{4}{7}$ | $6-\frac{2}{3}\left(\frac{4}{7}+5\right)$ | $=$ | $4\left(\frac{4}{7}\right)$ |
| Get a Common Denominator <br> Inside Parentheses | $6-\frac{2}{3}\left(\frac{4}{7}+\frac{35}{7}\right)$ | $=$ | $4\left(\frac{4}{7}\right)$ |
| Do Addition Inside <br> Parentheses | $6-\frac{2}{3}\left(\frac{39}{7}\right)$ | $=$ | $4\left(\frac{4}{7}\right)$ |
| Remove Parentheses Using <br> Multiplication of Fractions | $6-\frac{78}{21}$ | $=$ | $\frac{16}{7}$ |
| Get a Common Denominator | $\frac{126}{21}-\frac{78}{21}$ | $=$ | $\frac{48}{21}$ |
| Simplify | $\frac{48}{21}$ | $=$ | $\frac{48}{21}$ |

PTS: 3
NAT: A.REI.B. 3 TOP: Solving Linear Equations
KEY: fractional expressions

